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Adjustment distance in TM mode electromagnetic induction

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SUMMARY

The concept of adjustment distance—the range over which induced currents in 2- or 3-D structures re-adjust themselves to their 'normal' 1-D configurations outside the anomalous domain—first introduced into the subject of magnetotellurics a decade ago by Ranganayaki & Madden, is re-examined in the light of an analytical solution obtained previously by the authors using a generalized thin sheet model of the TM mode 'coast effect'. Ranganayaki & Madden defined the adjustment distance to be $r := (\tau\lambda)^{1/2}$ where τ and λ are respectively the integrated conductivity and integrated resistivity of the two layers in the generalized thin sheet. While on physical grounds this parameter is a reasonable estimate for many practical applications, its independence of the frequency of the inducing field is less than satisfactory. The parameter d given by the analytic solution is frequency dependent, but reduces to r for long periods. A more detailed comparison of the two parameters shows that r is always an upper bound of d under conditions likely to occur in nature and may even represent a significant overestimate of adjustment distance when the conductance of the surface layer is high, such as for an ocean. An alternative parameter for adjustment distance developed by Fainberg & Singer is found to be in good agreement with d, however, and for the model investigated varies with period in a similar manner over its range of validity. For more moderate values of surface conductance such as those found over continental regions, Ranganayaki & Madden's parameter serves as an accurate measure of adjustment distance over all periods of interest.

Key words: adjustment distance, coast effect, magnetotellurics.

1 INTRODUCTION

Following the pioneering work of Price (1949), many authors have used a mathematical thin sheet, defined to be the limit of a layer of (finite) conductance τ as its thickness $\epsilon \to 0$, as an idealization of the surface layer of the Earth when solving problems in geo-electromagnetic induction (see Ashour 1973, for a review). The effect of the layer is then reduced to a modification of the boundary condition applicable at the surface of the Earth. Across the thin sheet the horizontal electric field E is continuous but, because of the presence of a surface current flowing in the sheet, the horizontal magnetic field B is discontinuous by an amount $\mu\tau E$ where μ is the permeability.

In a very important paper published a decade ago, Ranganayaki & Madden (1980) (hereafter referred to as R–M) introduced into the analysis of geomagnetic problems the new concept of a *generalized* thin sheet, in which Price's conductive sheet was underlain by a resistive layer also of infinitesimal thickness. The resistive layer acts as a powerful inhibitor of the vertical flow of currents into the conductive part of the sheet and, by virtue of the charge distributions set up, causes the horizontal electric field to be discontinuous across the resistive sheet while B remains essentially continuous. Thus with one generalized boundary condition it becomes possible to take into account surficial conductive anomalies (e.g. oceans) and sub-surface resistive features (e.g. the oceanic crust).

In the TE mode with 2-D models, the resistive sheet has no effect because the electric field and the associated induced currents are everywhere horizontal. In the TM mode, however, it exerts a considerable influence if the conductive sheet is anomalous by impeding the vertical current flow into and out of the conductive sheet and thereby increasing the 'adjustment distance' r over which the induced current system readjusts to its normal 1-D pattern. R–M emphasized the importance of taking this distance into account when making interpretations of MT data and derived the approximate expression for r given by $r = (\tau\lambda)^{1/2}$ where τ is the conductance of the thin sheet and λ is its integrated resistivity. The same parameter had arisen quite naturally in an earlier discussion of the distortion of electromagnetic fields by surface anomalies (Berdichevsky & Dmitriev 1976), but the concepts of 'adjustment distance' and 'generalized thin sheet' were first introduced by R–M. Given the values of $\tau = 10^4$ S and $\lambda = 10^8$ $\Omega\,m^2$ considered by R–M to be representative of an ocean and its underlying crust the adjustment distance is 1000 km, and even if a lower figure of 10^6 $\Omega\,m^2$ is taken for the oceanic crust (Drury 1981) it is still 100 km. However, more recent data obtained by Cox *et al.* (1986) confirm the high value of integrated resistivity λ quoted by R–M. For land-based measurements the story is not quite as bad because the surface layer has a lower conductance, but this is partly counterbalanced by the increased integrated resitivity of the continental crust. R–M pointed out the depressing implications of their findings to magnetotelluric studies—the doubt they cast on the validity of attempting 1-D interpretations of MT data even from sites far removed from a coastline, and the possibility that numerical grids for forward model calculations would have to be designed to cover larger areas of the region under investigation than previously thought. Although the above remarks have been made with the 2-D coast effect in mind, they apply equally well to other 2- or 3-D conductive anomalies with an underlying resistive crust such as sedimentary basins. A review of problems associated with adjustment distance has been given by Jones (1983).

A detracting feature of R–M's parameter r is its lack of any frequency dependence; intuitively it seems unlikely that anomalous telluric fields of high frequency should require the same adjustment distance as those with very long periods. Fainberg & Singer (1987), referred to as F–S in the ensuing discussion, investigated the problem of adjustment distance from a somewhat different point of view by representing a conductive anomaly in a surface sheet of conductance τ on top of a layered half-space whose top resistive layer has integrated resistivity λ, by a grounded electric dipole; this is because the components of the Green's tensor which forms the kernel of the integral equation satisfied by the tangential electric field in an inhomogeneous surface sheet (Vasseur & Weidelt 1977) are, in fact, given by the electric field components of such a dipole. Using asymptotic approximations of the field expressions valid for ranges greater than the penetration depth in the underlying medium, they found that a reliable estimate of impedance was given by the usual (1-D) Tikhonov–Cagniard formula at distances greater than $s := |\lambda/(Z + 1/\tau)|^{1/2}$ where Z is the impedance of the layered structure underneath the surface layer. Thus their analysis showed that adjustment distance is indeed dependent on frequency but reduces to the R–M value r when $|Z| \ll 1/\tau$. More recently Menvielle & Tarits (1988) have also considered the role of anomalous fields in 3-D models but their expression for the adjustment distance also suffered from the fact that it was independent of frequency.

In this paper we hope to shed further light on the frequency dependence of adjustment distance by returning to an analytical solution of the TM mode coast effect obtained a few years ago by Dawson, Weaver & Raval (1982), hereafter referred to as D–W–R. An alternative expression for adjustment distance will be extracted from this solution and the conditions under which it differs from r will be examined. Our analysis only applies to a specific 2-D model, of course, but it is a relevant one which involves the effect of two large surface anomalies on 1-D interpretations of magnetotelluric data. It also has the advantage of being an analytic solution in closed form from which a precise algebraic expression for the horizontal decay of the anomalous electric field can be obtained.

Menvielle (1987) has included a nice summary of these various approaches to the calculation of adjustment distance as part of a more comprehensive review of the effect of conductivity heterogeneities on induced fields. He concluded that r would be a good approximation to the F–S parameter s for frequencies below 10^{-2} Hz in most geophysical situations. No detailed calculations were presented, however, and no estimate of the D–W–R adjustment distance for comparison with r and s was attempted.

2 SOLUTION OF THE COAST EFFECT MODEL

The mathematical model of the coast effect investigated by D–W–R is shown in Fig. 1. A (uniform) medium of conductivity σ and (vacuum) permeability μ_0 occupies the half-space $z > 0$ in a right-handed rectangular coordinate system (x, y, z). The surface $z = 0$ contains a generalized thin sheet divided along the line $y = 0$ into two halves of (uniform) conductances τ_1 (land) in $y < 0$ and τ_2 (ocean) in $y > 0$. Its integrated resistivity λ is taken to be the same in both halves, i.e. the underlying resistive crust is assumed to be uniform, a condition which was also present in the analysis of F–S. The regional or 'normal' magnetic field, which comprises both the source field and the field of the induced currents at $y = \pm\infty$, is assumed to be uniform in the non-conducting atmosphere above the Earth, directed parallel to the x-axis in space and oscillating with angular frequency ω in time. It can, therefore, be written as $B_0 e^{i\omega t}\hat{\mathbf{x}}$ where $\hat{\mathbf{x}}$ is a unit vector in the x-direction. Our notation differs from that used by D–W–R; in particular it should be noted that λ denoted the *conductance* of the sheet normalized by σ in the paper by D–W–R rather than the integrated resistivity as here, and that the roles of the subscripts 1 and 2 are reversed.

Since there is no dependence on the variable x among the model parameters, the field components are functions of y and z only and Maxwell's equations with displacement

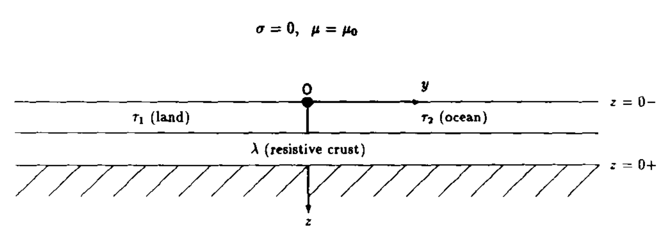

Figure 1. The mathematical model.

currents neglected decouple into the three scalar equations

$$\partial X/\partial y = -\mu_0\sigma W, \qquad \partial X/\partial z = \mu_0\sigma V,$$
$$\partial V/\partial z - \partial W/\partial y = i\omega X. \tag{1}$$

Here we have written the respective magnetic and electric fields in their component forms $\mathbf{B} = [X(y, z), 0, 0]e^{i\omega t}$ and $\mathbf{E} = [0, V(y, z), W(y, z)]e^{i\omega t}$. That the problem is transverse magnetic (TM) with the magnetic field everywhere horizontal and in the x-direction is determined by the prescribed form of the normal field and the 2-D nature of the model. From (1) we obtain immediately

$$\frac{\partial^2 X}{\partial y^2} + \frac{\partial^2 X}{\partial z^2} = i\alpha^2 X \tag{2}$$

as the differential equation governing the behaviour of the field inside the Earth. Here we have defined

$$\alpha := \sqrt{\omega\mu_0\sigma} \equiv \sqrt{2}/\delta \tag{3}$$

with δ denoting the skin depth in the underlying half-space of conductivity σ. Above the surface of the Earth where $\sigma = 0$, it follows from the first pair of equations in (1) and the nature of the field at infinity that

$$X = B_0. \tag{4}$$

Across the conductive part of the generalized thin sheet V is continuous but there is a discontinuity in X proportional to the density of the surface current in the sheet; across the resitive part V is discontinuous while X remains the same. As shown by D–W–R this leads to the generalized boundary condition

$$X(y, 0+) - (\tau/\sigma)X'(y, 0+) - \lambda r\, \partial^2 X(y, 0+)/\partial y^2 = B_0 \tag{5}$$

on the underside of the generalized thin sheet. In the derivation of (5) D–W–R assumed that both τ and λ were uniform. In the model this condition is satisfied except at the origin where τ has a jump discontinuity. Thus a singularity in the field at the origin can be expected.

As $y \to \pm\infty$, X must approach the 1-D solutions

$$X_1(z) := X(-\infty, z) = \frac{B_0 \exp(-z\alpha\sqrt{i})}{1 + \tau_1\alpha\sqrt{i}/\sigma}, \tag{6}$$

$$X_2(z) := X(+\infty, z) = \frac{B_0 \exp(-z\alpha\sqrt{i})}{1 + \tau_2\alpha\sqrt{i}/\sigma}, \tag{7}$$

which satisfy (2) and (5) when all y dependence is removed. The problem is therefore better formulated in terms of an anomalous field

$$X(y, z) - X_2(z) =: f(y, z) = f_+(y, z) + f_-(y, z) \tag{8}$$

where $f_+(y, z) = 0$ for $y < 0$ and $f_-(y, z) = 0$ for $y > 0$. It is also convenient to introduce the normalized parameters

$$\tau' := \tau/\sigma, \qquad \lambda' := \lambda\sigma. \tag{9}$$

Then f continues to satisfy (2), but is subject to the boundary conditions

$$f_+(\infty, z) = 0, \qquad f_-(-\infty, z) = X_1(z) - X_2(z), \qquad f(y, \infty) = 0 \tag{10}$$

and according to (5)

$$(1 - i\alpha^2\tau_1'\lambda')f_-(y, 0+) - \tau_1'f_-'(y, 0+) + \tau_1'\lambda'f_-''(y, 0+) = (\tau_1 - \tau_2)\alpha\sqrt{i}X_2(0) \tag{11}$$

for $y < 0$, and

$$(1 - i\alpha^2\tau_2'\lambda')f_+(y, 0+) - \tau_2'f_+'(y, 0+) + \tau_2'\lambda'f_+''(y, 0+) = 0 \tag{12}$$

for $y > 0$. Moreover, the first of conditions (10) can be stated more precisely by the assertion that $f_+(y, z) = O(e^{-cy})$ as $y \to \infty$ (where $c > 0$ is some real constant) in accordance with the usual diffusive decay of an anomalous field.

The boundary value problem in f with mixed boundary conditions on the surface $z = 0$ was solved by D–W–R using the Wiener–Hopf technique. The procedure followed was to take the Fourier transform of $f(y, z)$ with respect to y to obtain a function $F(\eta, z)$ which is analytic in the strip $-c < \mathscr{I}m\, \eta < 0$ of the complex η-plane and whose solution in $z > 0$ is

$$F(\eta, z) = F(\eta, 0) \exp(-z\sqrt{\eta^2 + i\alpha^2}). \tag{13}$$

The functions F_- and F_+ whose sum is F are analytic in the respective regions $\mathscr{I}m\, \eta < 0$ and $\mathscr{I}m\, \eta > -c$ and satisfy the Fourier transforms of boundary conditions (11) and (12). It is not necessary to concern ourselves further with the intricacies of the calculation here as full details have been published by D–W–R. We need only to quote the form of their final solution for the horizontal electric on the surface $z = 0-$.

First we define for $j = 1, 2$ the parameters

$$\chi_j = \frac{1}{2\lambda'} - \left(\frac{1}{4\lambda'^2} - \frac{1}{\tau_j'\lambda'} + i\alpha^2\right)^{1/2}, \qquad \nu_j = \left(\frac{\chi_j}{\lambda'} - \frac{1}{\tau_j'\lambda'}\right)^{1/2} \tag{14}$$

noting that

$$\chi_j = \pm\sqrt{\nu_j^2 + i\alpha^2}. \tag{15}$$

If $\mathscr{R}e\, \chi_j < 0$ the minus sign must be chosen in (15) and it then turns out that the function to be factorized in the Wiener–Hopf technique has a pole in the lower half-plane. If this pole lies nearer to the real axis than the branch point at $\eta = -\alpha\sqrt{i}$ it limits the domain of analyticity of F and thereby determines the value of c. By residue theory the pole also introduces an extra term when the final solution given by an infinite integral along a line parallel to the real axis is transformed by Cauchy's theorem into an integral along the branch cut from infinity to the branch point. Otherwise, if $\mathscr{R}e\, \chi_j > 0$, no such pole exists, the domain of analyticity is limited by the branch point so that $c = -\alpha/\sqrt{2}$, and the final solution contains no residue term. The sign of $\mathscr{R}e\, \chi_j$ is therefore important and is determined by the values of the parameters τ_j', λ' and α.

For the time being we shall assume that the residue term is indeed present deferring our investigation of the conditions under which the condition $\mathscr{R}e\, \chi_j < 0$ holds until the next section. The solution for the *anomalous* horizontal electric field $V_\alpha(y) := V(y, 0-) - V(\pm\infty, 0-)$ on the Earth's surface on either side of the origin is then given by [equation

(4.21) in D–W–R]

$$V_a(y) = \frac{(\tau_2'/\tau_1' - 1)B_0}{1 + \tau_2'\alpha\sqrt{i}} \sqrt{\frac{i\omega}{\mu_0\sigma}} \left[\frac{\lambda'\chi_1 \exp[-h(-\nu_1) + iy\nu_1]}{(1 - \tau_1'\chi_1)(1 - 2\lambda'\chi_1)} + \frac{1}{\pi}\int_0^\infty G_1(u) \exp(y\sqrt{u^2 + i\alpha^2})\, du \right] \quad (16)$$

for $y < 0$ and

$$V_a(y) = \frac{(1 - \tau_1'/\tau_2')B_0}{1 + \tau_1'\alpha\sqrt{i}} \sqrt{\frac{i\omega}{\mu_0\sigma}} \left[\frac{\lambda'\chi_2 \exp[h(-\nu_2) - iy\nu_2]}{(1 - \tau_2'\chi_2)(1 - 2\lambda'\chi_2)} + \frac{1}{\pi}\int_0^\infty G_2(u) \exp(-y\sqrt{u^2 + i\alpha^2})\, du \right] \quad (17)$$

for $y > 0$. Note that these solutions are also valid on an ocean floor because the electric field is continuous across the upper conductive part of the generalized thin sheet.

The function h appearing in the first term is obtained from the Wiener–Hopf factorization; G_1 and G_2 also involve h but the precise form of these functions is not required here. Indeed, it has been found by numerical evaluation that, for the range of parameter values likely to occur under natural conditions, the integrals are completely dominated by the algebraic terms on the right-hand sides of (16) and (17) as we shall see in Section 3. Thus, remarkably, the anomalous electric fields can be expressed very accurately by relatively simple algebraic expressions when the contributions from the residues of the poles are present.

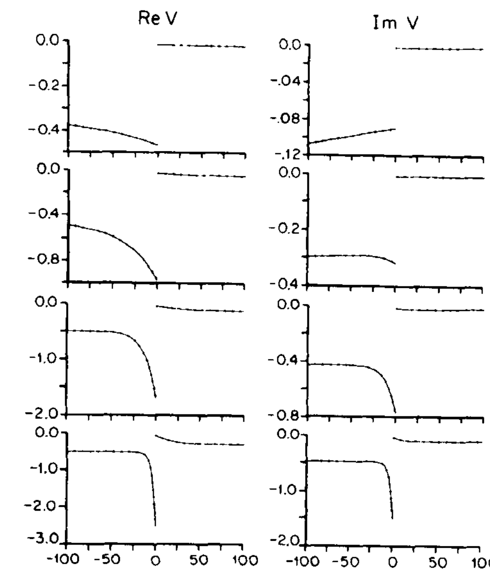

Figure 2. Variation of the real and imaginary parts of the total electric field $V(y, 0-)$ along the surface of a coast effect model in which $\tau_1 = 400$ S, $\tau_2 = 1.6 \times 10^4$ S and $\sigma = 0.1$ S. Solid lines represent the exact solutions; the dots indicate values calculated using only the algebraic residue terms. The four sets of graphs from the top downwards are for $\lambda = 10^9\ \Omega\,\mathrm{m}^2$ and periods $T = 10$, 10^2, 10^3 and 10^4 s respectively. The electric fields V are dimensionless being expressed in units of $\omega\delta B_0$ and the y values along the abscissa are given in units of skin depth δ.

3 ADJUSTMENT DISTANCE

In this section it is convenient to drop the subscripts j since our discussion is quite general and there is no reason to distinguish between the regions $y < 0$ and $y > 0$. We investigate first the conditions under which $\mathcal{Re}\,\chi < 0$. Introducing the dimensionless conductance and integrated resistivity, $\bar{\tau} = \tau'/\delta$ and $\bar{\lambda} = \lambda'/\delta$ respectively, where δ is the skin depth defined in (3), and putting $\beta = \tau'/2\lambda' \equiv \bar{\tau}/2\bar{\lambda}$, we have

$$\tau'\chi = \beta - \sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2} \quad (18)$$

whence the condition $\mathcal{Re}\,\chi < 0$ becomes

$$\mathcal{Re}\sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2} > \beta. \quad (19)$$

Now since

$$\mathcal{Re}\sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2} = [\tfrac{1}{2}\beta^2 - \beta + \tfrac{1}{2}\sqrt{\beta^2(\beta-2)^2 + 4\bar{\tau}^4}]^{1/2}$$

and both β and $\mathcal{Re}\sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2}$ are positive, it follows that (19) is satisfied if

$$\sqrt{\beta^2(\beta-2)^2 + 4\bar{\tau}^4} > \beta(\beta + 2)$$

which is satisfied in turn if $\bar{\tau}^4 > 2\beta^3$. Expressed in terms of the original model parameters this condition becomes $\tau\lambda^3\sigma^2\alpha^4 > 1$, or

$$\tau\lambda^3\sigma^4/T^2 > 10^{14}/64\pi^4 \approx 1.60 \times 10^{10} \quad (20)$$

where $T = 2\pi/\omega$ is the period of the magnetic variations. While the right-hand side of (20) may seem very large, for the values of τ and λ for oceanic regions quoted in Section 1 and with a conductivity of $\sigma = 0.1\ \mathrm{S\,m^{-1}}$ assigned to the deep crust and upper mantle, the condition becomes $T < 7.91 \times 10^3\ \mathrm{s} \approx 2.19$ hr which covers the normal range of periods used in MT studies. Over continental regions a value of $\tau = 400$ S is more typical for surface rocks (Schmucker 1970) but the integrated resistivity of the underlying resistive crust is then more likely to be about $10^9\ \Omega\,\mathrm{m}^2$ which was the figure chosen by R–M. Even if we reduce the conductivity of the deep crust to $\sigma = 0.01/\mathrm{S\,m^{-1}}$ the condition continues to hold for periods less than about 139 hr! In fact it appears that (20) will practically always be satisfied under naturally prevailing conditions so that the algebraic terms in (16) and (17) do indeed occur in the solution for the anomalous field.

Given their presence it remains to be decided how dominant the algebraic terms are in comparison with the infinite integrals which form the remaining part of the solution. In Figs 2 and 3 we have plotted variations of the *total* electric field V along the surface [given by the solutions (16) and (17) with the 1-D solutions at infinity added on] for the parameter values $\tau_1 = 400$ S, $\tau_2 = 1.6 \times 10^4$ S and $\sigma = 0.1$ S representing the coast effect, values of λ ranging from 10^6 to $10^9\ \Omega\,\mathrm{m}^2$, and various periods T. The superimposed dotted curves were calculated by ignoring the contributions from the integrals in solutions (16) and (17) and using only the algebraic terms representing the residues. The extraordinary accuracy of the dotted curves for most of the parameters considered is clearly visible from the graphs. Only when λ takes its lowest value of $10^6\ \Omega\,\mathrm{m}^2$ (Fig. 3) and the period is long (10^4 s) is there clearly a discrepancy although some very slight deviation from the solid curve for $\mathcal{Im}\,V$ can also be discerned when $T = 10^3$ s (in fact the deviation for this period is much more apparent in other components of the field on the underside $z = 0+$ of the

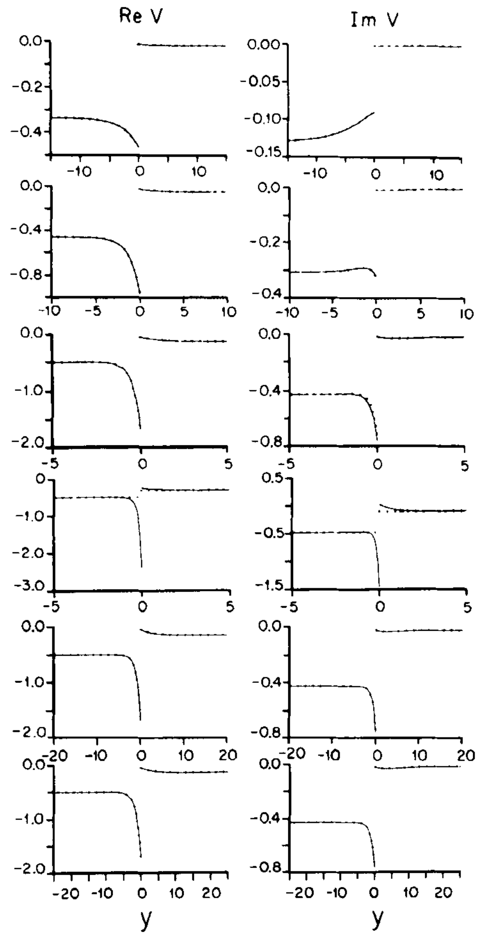

Figure 3. The first four sets of graphs from the top downwards are the same as Fig. 2 except that $\lambda = 10^6\ \Omega\ \mathrm{m}^2$; the final pair is for $\lambda = 5 \times 10^6\ \Omega\ \mathrm{m}^2$ and $T = 10^3$ and 10^4 s respectively.

generalized thin sheet which are not shown here). As soon as λ is raised to 5×10^4, however, there is virtually complete agreement again even at the longest period. We may safely conclude that the algebraic terms do indeed give accurate values of $V(y, -0)$ for most realistic models of the coast effect. Caution need only be exercised when the integrated resistivity is very low and the period of the field is of the order of hours rather than minutes.

A suitable measure of adjustment distance is the horizontal range above the 1-D structures at the sides of the model over which the anomalous field is attenuated by a factor $1/e$. This definition is the horizontal analogue of skin depth which is usually used as a measure of the depth of penetration in the vertical direction. Disregarding the integral terms in (16) and (17) we see that

$$V_a(y)/V_a(0\pm) = \exp(-i\,|y|\,\nu) \tag{21}$$

with the appropriate value $V_a(0\pm)$ chosen at the origin for $y < 0$ or $y > 0$. The adjustment distance d is therefore given by

$$d = -(\mathcal{I}m\ \nu)^{-1}. \tag{22}$$

We have confined our attention to the anomalous electric field because the magnetic field is constant on the surface of the Earth. If MT measurements are made on an ocean floor, however, the measured magnetic field corresponds to the field on the underside of the generalized thin sheet (recall that the magnetic field is constant across the resistive part of the sheet) which also has an anomalous part. The solution for the anomalous magnetic components $X_a(y) := X(y, 0+) - X(\pm\infty, 0+)$ were found by D–W–R to possess exactly the same dependence on y as the anomalous electric fields (in fact the latter were obtained from the former by differentiation) so that the adjustment distance for magnetic field variations on the sea-floor will continue to be given by d.

It follows at once from (18) that $\mathcal{I}m\ \chi < 0$ and since the present discussion is based on the premise that $\mathcal{R}e\ \chi < 0$ also, we deduce from (14) that

$$\nu = -i\left(\frac{1}{\tau'\lambda'} - \frac{\chi}{\lambda'}\right)^{1/2}$$

whence

$$\mathcal{I}m\ \nu = -(\tau'\lambda')^{-1/2}\ \mathcal{R}e\sqrt{1 - \tau'\chi}.$$

Replacing the normalized integrated conductivity and resistivity by their actual values and substituting in (22) we obtain

$$d = \frac{\sqrt{\tau\lambda}}{\mathcal{R}e\sqrt{1 - \tau\chi/\sigma}} \tag{23}$$

as our frequency dependent expression for the adjustment distance. In terms of r and the dimensionless parameters β and $\bar{\tau}$ introduced above in equation (18), we can rewrite (23) as

$$r/d = \mathcal{R}e\,(1 - \beta + \sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2})^{1/2}. \tag{24}$$

Note that if

$$|\beta - \sqrt{\beta^2 - 2\beta + 2i\bar{\tau}^2}| \ll 1 \tag{25}$$

then $r/d \approx 1$ and our expression for d reduces to the adjustment distance r of R–M. Unfortunately it does not appear possible to express this condition in simpler terms, but one special case is worth noting. Suppose $\beta < 1$ and $\sqrt{2\bar{\tau}\bar{\lambda}} \ll 1$; then since $\bar{\lambda} = \bar{\tau}/2\beta$ we may neglect second order terms in $\bar{\tau}/\sqrt{\beta}$ and (24) becomes

$$r/d \approx \mathcal{R}e\,(1 - \beta + i\sqrt{2\beta - \beta^2})^{1/2} = \sqrt{1 - \beta/2}. \tag{26}$$

If β itself is small then $r/d \approx 1 - \beta/4$ and d will be very close to r.

Little additional insight is gained by taking the real part in (24) and writing out the full algebraic expression for r/d. It is necessary to resort to numerical calculations to extract further information.

4 NUMERICAL RESULTS

Illustrative calculations have been made for values of the various parameters that represent extreme limits of conditions likely to be found in nature. A true picture probably falls somewhere in the middle of the cases to be discussed here. For the conductance of an ocean we have assumed the conductivity of seawater to be 4 S m^{-1} and the average ocean depth to be 4 km giving $\tau_2 = 1.6 \times 10^4$ S. The conductance of a surface layer of rocks on the landward side of a coastal boundary is taken to be $\tau_1 = 400$ S as in Section

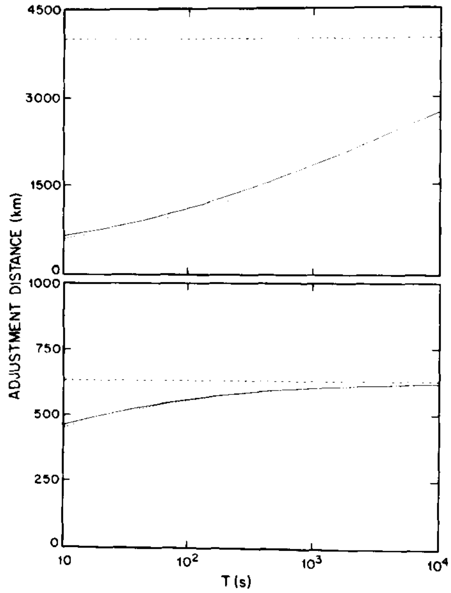

Figure 4. Variation of the adjustment distance d in km (solid line) with period T in s for a generalized thin sheet of integrated resistivity $10^9\,\Omega\,m^2$, and conductances 1.6×10^4 S (upper diagram) and 400 S (lower diagram). Graphs depicting the corresponding variations of the parameter s due to F–S (dotted line) and the horizontal (broken) lines giving the values r of the adjustment distance of R–M are also shown.

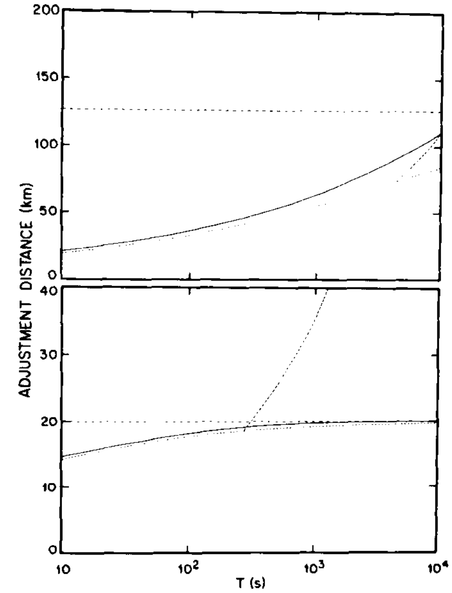

Figure 5. The same as Fig. 4 but for a thin sheet of integrated resistivity $10^6\,\Omega\,m^2$. The values of s as given by the plotted values of $|\lambda/(Z+1/\tau)|^{1/2}$ become inaccurate at periods greater than 4250 s in the upper diagram ($\tau=1.6\times10^4$ S) and 280 s in the lower diagram ($\tau=400$ S). The dashed curves diverging from the main graphs at these points indicate the variation of penetration depth $1/\alpha$, the range beyond which the asymptotic expansions used in the F–S theory are valid.

3. Two different values of the integrated resistivity of the crustal layer will be used, $\lambda=10^9\,\Omega\,m^2$ and $\lambda=10^6\,\Omega\,m^2$, representing the estimated upper and lower bounds of the range of values expected to occur in nature. As an average value of the conductivity of the deep structure we shall take $\sigma=0.1\,S\,m^{-1}$.

Figure 4 shows the variation with period of the adjustment distance d defined by (23) for a model in which $\lambda=10^9\,\Omega\,m^2$. Curves for both $y<0$ (conductance τ_1) and for $y>0$ (conductance τ_2) have been plotted, along with the corresponding curves for the adjustment distance s due to F–S and horizontal straight lines representing the adjustment distance r of R–M. The frequency dependence of the adjustment distance is immediately apparent from the curves and, as expected, d diminishes with decreasing period. Caution must be exercised, however, when interpreting the results for short periods because the thin sheet model may cease to be a valid representation of the real Earth. For example, the ocean depth of 4 km must always be small compared with the skin depth in seawater which effectively imposes a lower bound of about 1600 s on the range of admissible periods when considering a deep ocean. The agreement, both in magnitude and trend, between the frequency-dependent parameters s and d is remarkable considering the completely different ways in which they were derived. The theory of F–S is more general, of course, having been developed for a layered substructure; the fact that it agrees so well with a result obtained from an exact analytic solution for a particular problem is convincing evidence of its general validity.

Figure 5 depicts the same variations when the integrated resistivity takes the smaller value $\lambda=10^6\,\Omega\,m^2$. Here the adjustment distances are much reduced, of course, because vertical currents flow more freely with a less resistive layer and the induced current system can more readily adjust to its 'normal', purely horizontal flow away from the coastline. Another difference is that the F–S formula breaks down at longer periods (above 4250 s when $\tau=1.6\times10^4$ S and above 280 s when $\tau=400$ S) because at these periods the penetration depth $1/\alpha$ in the underlying half-space becomes greater than s, and the asymptotic expansions on which the theory of F–S is based are then only valid at ranges beyond $1/\alpha$ which increases with $\sqrt{T}$, as indicated by the steeply diverging dashed curves in Fig. 5.

The most important conclusion to be drawn from Figs 4 and 5, however, is that for all period ranges of interest d is smaller, and over an ocean (or on the sea-floor) generally much smaller, than the adjustment distance of R–M. For example, if we take 1 hr as a typical period and choose the lower value of λ for the oceanic crust (Fig. 5), the adjustment distance on the sea-floor is predicted to be only about 87 km rather than the 126 km given by r. This is reassuring for it means that MT measurements with ocean bottom instruments may not be as contaminated with 'coast effect' noise as previously predicted. At the same period on the landward side of the coast the value of d coincides almost exactly with r, and only at shorter periods is d visibly less than r in both Figs 4 and 5. In fact, for the parameters we have chosen over land, $\sqrt{2\bar{\tau}\bar{\lambda}}$ becomes $2\pi\times10^4/\sqrt{T}$ when $\bar{\lambda}=10^9\,\Omega\,m^2$ and $0.8\pi/\sqrt{T}$ when $\bar{\lambda}=10^6\,\Omega\,m^2$. Thus the approximation (24) will be reasonable for $T\gtrsim3200$ s in the latter case and, since $\beta=2\times10^{-2}$ is very small, we expect to find $r\approx d$ for periods greater than 3200 s which is confirmed by the broken line curves in Fig. 5. We cannot use (24) for the larger value of $\bar{\lambda}$; direct calculation shows

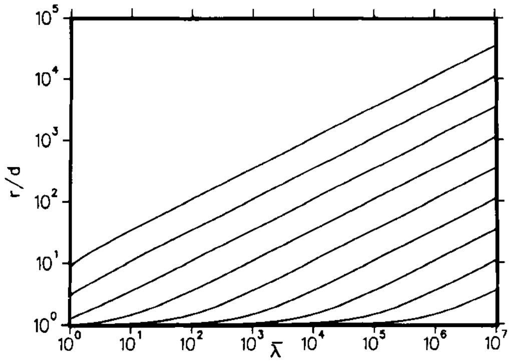

Figure 6. Graphs of r/d against the normalized integrated resistivity $\bar{\lambda}$ for various values of the ratio $\tau'/\lambda' \equiv 2\beta$. From bottom to top the values of 2β associated with the nine curves are 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1, 10 and 100 respectively.

that the left-hand side of (25) does not decrease much below 0.2 over the entire range of periods up to 10 000 s. Nevertheless the parameter of R–M is in excellent agreement with the frequency-dependent adjustment distance over the land mass for the whole range of typical MT periods.

In Fig. 6 we have plotted r/d against $\bar{\lambda}$ for various values of $2\beta \equiv \tau'/\lambda'$. The interpretation of these curves requires more effort but they contain a great deal of information because they are independent of σ and T. It is apparent in Fig. 6 that for a given set of parameter values there is a critical period, defined by the value of $\bar{\lambda}$ where the relevant curve begins to rise from the horizontal axis for $r/d = 1$, above which d is approximately equal to r and below which d becomes progressively smaller than r with decreasing period. In general, the trends that can be gleaned from the graphs are that d/r becomes smaller as the conductance τ increases and as the period T decreases.

5 CONCLUDING REMARKS

Investigation of the analytic solution of a highly idealized thin sheet model has revealed a frequency dependence of the adjustment distance that was not present in the parameter $r = (\tau\lambda)^{1/2}$ introduced by R–M. The frequency-dependent adjustment distance is always less than r for period ranges of interest and fortunately gives values significantly less than the enormous distances predicted by R–M for oceanic regions with a very resistive crust. Over continental regions where the conductance of the surface layer is smaller, r still represents an upper bound of possible values of d but the difference between the two parameters becomes insignificant for all practical purposes. These results suggest that one may take a less pessimistic view of the reliability of 1-D interpretations of sea-floor magnetotelluric measurements in the TM mode at locations reasonably distant from a coastline because they will be less affected by the coast effect than previously suggested, and that numerical grids of extravagant size are not necessarily required for accurate numerical modelling of anomalous regions.

The formula for adjustment distance developed in this paper is more complicated than the expression for r and its significance should not be unduly emphasized. It has been devised for a quite specific model under well-defined conditions and therefore represents a rather different approach to the problem from that taken by F–S who presented a more general treatment of the decay of anomalous magnetotelluric fields and who found somewhat different expressions for a frequency-dependent adjustment distance. In particular, we have required the integrated resistivity in the surface sheet to be laterally uniform. Our interpretations have been made for various combinations of conductance and integrated resistivity values without explicit mention of this fact; it should be borne in mind that the coast-effect variations presented here are based on the assumption that the oceanic and continental crust both have the same integrated resistivity which is probably not true in practice. Ideally one would like to be able to solve the problem analytically for a model in which the value of λ as well as τ is different for $y < 0$ and $y > 0$ but this does not appear possible. The importance of our result rests on its confirmation of a frequency dependence in the adjustment distance which is similar to the one obtained by F–S and in its suggestion that under naturally occurring conditions the simple expression obtained by R–M can give an overestimate of the range over which induced current systems re-adjust themselves to their 1-D patterns on the oceanic side of a coastline, but remains quite accurate on the landward side despite the absence of any dependence on frequency.

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