CORRECTION FOR DISTORTIONS OF MAGNETOTELLURIC FIELDS: LIMITS OF VALIDITY OF THE STATIC APPROACH

B. SH. SINGER

The Institute of the Physics of the Earth, US Academy of Sciences Troitsk, Moscow Region 142092, Russia

(Received 21 October 1990; accepted 14 August 1991)

Abstract. Subsurface inhomogeneities can be classified in three categories depending on the comparison of their sizes against the effective skin depth and "adjustment distance". These categories are geological noise, local and regional inhomogeneities.

Due to its random nature, geological noise leads to uncontrollable distortions of the magnetotelluric field. Such methods as curve shifting, distortion tensor techniques, decomposition and spatial filtering can effectively be used to correct for static shift caused by geological noise.

It is shown that the effects of local and regional subsurface inhomogeneities reduce to static shifts of MT-curves when rather rigid conditions are satisfied. The methods used to suppress the effect of geological noise have limited applicability to local and regional inhomogeneities, which can be accurately accounted for only by modelling.

1. Introduction

Due to its importance for magnetotellurics, the problem of local distortions of electromagnetic fields has a rather long history. Local distortions reduce the quality and resolving power of MT-soundings and create erroneous interpretations. There exist a number of approaches to the problem but none completely addresses all aspects of distortion. Further development of data acquisition, processing and interpretation tools will be unlikely to solve the problem in itself. This means that some special measures will always be necessary to remove the distortions before the data can be interpreted correctly.

Inductive and galvanic distortions taking place in 2-D media were first described by Berdichevsky and Dmitriev (1976a, b). Inductive distortions result from the excessive currents flowing in areas adjacent to the observation point. As a result, MT sounding in the vertical direction is often influenced by lateral conductivity structure as well. The distortions usually affect the ascending branch of the apparent resistivity curve \( \rho_a(T) \) and have negligible effect on the descending branch. Inductive distortion can introduce spurious inflections on the curve, leading to an erroneous impression that additional layers exist in the medium.

In 2-D media, the S-effect takes place for B-polarized fields observed over a surface inhomogeneity. It does not affect the ascending branch but vertically shifts the descending one. This leads to erroneous conclusions on depths of layers. Some distance from the anomaly the so called "edge effect" can be observed.
(Berdichevsky and Dmitriev, 1976b). It spoils the ascending branch of the curve and shifts the descending one. The value of the effect depends upon the distance from the anomaly. In 3-D media more complicated effects, such as flowing around, current channeling, etc. take place. The name “three-dimensional S-effect” is sometimes used in such cases. Instructive examples of modelling 3-D effects can be found in (Vasseur and Weidelt, 1977), (Singer et al., 1984) and (Park, 1985). As in 2-D media, the effect shifts that part of the $\rho_s(T)$ curve when inductive effects in the inhomogeneous layer become negligible. The apparent impedance phase is not affected.

The earth’s surface roughness also is a source of MT field distortions (Dmitriev and Tavartkiladze, 1975). Topographic distortions are frequency dependent at short periods and have static character at long periods. Significant distortions can originate from moderate surface roughness. The slope of the surface is often more important than the amplitude (Reddig and Jiracek, 1984).

The physical essence of the static effect has been described repeatedly. Current $j$ flowing through geoelectrical inhomogeneities creates electric charges whose density is $j \cdot \nabla (\varepsilon_0 / \sigma)$, where $\varepsilon_0 = 10^{-9}/36\pi F/m$, and $\sigma$ is the specific conductivity. While the magnetic field

$$H = \nabla \times A$$

depends only on the current distribution as

$$A(r) = \int_{R^3} \frac{j(r') \, dv'}{|r - r'| \, 4\pi},$$

the electric field

$$E = \omega \mu_0 A - \nabla \Phi,$$

is sensitive to the charge distribution ($\mu_0 = 4\pi \cdot 10^{-7} H/m$), as

$$\Phi(r) = \int_{R^3} \frac{j \cdot \nabla (\varepsilon_0 / \sigma) \, dv'}{|r - r'| \, 4\pi}.$$

To reveal geoelectrical structure at depth $z$, the frequency $\omega$ is chosen so that induced currents would flow at this depth. Although only a small part of the currents flow at the earth’s surface, they can produce significant charge if inhomogeneities exist there. It follows from Equations (1)–(4) that the galvanic part of the electric field prevails at low frequencies, while the major part of the magnetic field emerges from the currents flowing at depth $z$. If the medium is laterally uniform at this depth, then both the secondary and total magnetic fields will be uniform and any further decrease in frequency will affect only their amplitudes. This means that the surface current distribution can be found from the DC-problem solution. Thus the spatial distribution and the phase shift of the electric
field relative to the magnetic field will be conserved. This justifies the term “static distortions”. We shall terminate here the discussion on the physical essence of static distortion. More detailed consideration can be found in the previous excellent review (Jiracek, 1990) and two earlier ones (Jones, 1983; Menvielle, 1988).

2. Methods to Correct for Surface Inhomogeneities

The field configuration is usually chosen to represent basic peculiarities of the electromagnetic field. Therefore the distance between the neighboring points of observation is, as a rule, smaller than the depth of sounding. The same is valid for a numerical modelling mesh. Inhomogeneities whose size is smaller than or comparable with the distance between nodes of the experimental or numerical mesh cannot be adequately represented on the mesh. They are small also in comparison with the skin depth and will be referred to below as “geological noise”. Typical dimensions of such inhomogeneities do not usually exceed several kilometers. Inhomogeneities which are comparable with or greater than the skin depth but smaller than the “adjustment distance” will be referred to as “local inhomogeneities”. For inhomogeneities whose dimensions are comparable with the “adjustment distance” the term “regional inhomogeneities” will be used.

Methods used to correct for subsurface inhomogeneity distortions such as curve shifting, distortion tensor techniques, decomposition, spatial filtration and modelling are discussed below. Some of them can be used as a remedy against both geological noise and local inhomogeneities, others have only limited usage. For example the geological noise effect on practical MT sounding cannot be modeled numerically.

In 2-D situation and in the frequency band of deep MTS, the effects of the subsurface inhomogeneities are often negligible in the E-polarized field, which can be interpreted using one dimensional inversion. Exceptions are to be made for deep conductive fractures or sea shore areas, when 2-D interpretation is unavoidable. Sometimes B-polarization is more informative (Wannamaker et al., 1989). Local 2-D inhomogeneities can in both cases be incorporated in the model, but geological noise can not, as corresponding inhomogeneities are three-dimensional. The effects of geological noise should be taken into account for both E- and B-polarized fields.

Due to its uncontrollable character, geological noise leads to dangerous distortions of the MT field. Fortunately, the corresponding inhomogeneities are so small that even at high frequencies only a negligible part of the currents is induced in the inhomogeneities. In other words, the whole MT frequency band can be considered as the low frequency band with respect to geological noise. Thus, inductive distortions created by the geological noise inhomogeneities are negligible; the main effect is the static distortion of the electric field.
2.1. Curve shifting and distortion tensor techniques

Examples of how a small inhomogeneity affects the apparent resistivity curves are shown in Figure 1 (Sternberg et al., 1988) and Figure 2 (Chouteau and Bouchard, 1988). The effect of geological noise on the apparent resistivity curves is manifested as a frequency-independent vertical shift. The shift value can be evaluated at each field point if there exists some additional information on the conductivity distribution just under the point.

The information can be acquired from the global conductivity distribution (Rokityansky, 1982). The global curve in Figure 3 was provided by Fainberg (1983a, b) as a result of careful data selection. Corrections of Sq-harmonic frequency responses were made to compensate the effect of the oceans and the sedimentary cover. The short period bound of the curve is at the period $T = 8h$, which corresponds to the depth 400 km. It was assumed that the conductivity distribution becomes laterally uniform at such depth, and hence any undistorted MT curve should coincide with the global one at periods $T > 8h$. This gives a key to selection of undistorted MT curves. The procedure can be used only if the MT curves have a sufficiently long low-frequency branch.

The MT curves in Figure 3 were obtained on the East European Platform and selected as undistorted (Vladimirov and Dmitriev, 1972). If one continues the global curve along the envelope of the MT curves, the normal curve of the East
Fig. 1. Model example of the static shift effect (Sternberg et al., 1988). (a) The model and the acquisition geometry, (b) MT apparent resistivity curves for TE-polarization. Station distance from the center of the anomaly is noted on a plot, (c) MT apparent resistivity curves for TM-polarization, (d) TEM apparent resistivity curves.
Fig. 2. Topographic distortion of MT curve and its stripping by distortion tensor technique (Chouteau and Bouchard, 1988). (a) Geometry of initial model with topography over layered earth. (b) Geometry of flat earth model. (c) Initial model, terrain corrected (TCR), and flat earth model (FEM) TM apparent resistivity and impedance phase sounding curves at site A in (a).
European Platform with the bound at $T = 1h$ is obtained (Fainberg, 1983a). The approach has been used in (Vanyan et al., 1977; Vanyan et al., 1980; Avagimov et al., 1981; Berdichevsky et al., 1989).

In a number of regions the descending branches of MT curves either undershoot or overshoot the global curve. If the subsurface conductivity distribution is known then numerical modelling is the most natural way to obtain the regional reference curve. The MT curve undistorted by geological noise should have the descending branch which coincides with the normal regional curve. When using this approach it should be kept in mind that deep inhomogeneities can shift the low frequency branch (Berdichevsky et al., 1984) as well as a subsurface one.

An alternative approach was considered in (Berdichevsky et al., 1988). It was proposed to shift individual MT curves to a position corresponding to some "averaged" value of the surface conductance. Additional information on the section can also be acquired if there exist a simple parameterization of the conductivity distribution in one of the layers (Jones, 1988). Independently of which of the layers has been referred to for the static shift correction procedure, the resultant
curve contains adequate information on layers just below the inhomogeneous layer responsible for the shift down to the next inhomogeneous layer if the reference layer occupies the intermediate position.

It seems the most promising approach to a static shift correction is based on utilization of auxiliary magnetic field measurements. Unlike the telluric field the magnetic field is much less affected by surface inhomogeneities. Transient electromagnetic (TEM) sounding was used by Andrieux and Wightman (1984). TEM sounding curves for the model in Figure 1a are shown in Figure 1d (Sternberg et al., 1988). It can be seen that at $t = 0.1$ ms, TEM curves become insensitive to the surface inhomogeneity. Thus the joint interpretation of MT and TEM curves is a way to abolish the static shift. Naturally such interpretation is possible only if the maximum depth of TEM sounding is greater than the minimum depth of MT sounding and the medium at this depth interval is laterally uniform. The logical approach to find a correct reference curve is to carry out the TEM curve inversion followed by MT curve calculation for the revealed structure (Pellerin and Hohmann, 1990). The procedure was investigated numerically for a model consisting of a deep 3-D target and geological noise (Figure 4a). Distorted, undistorted, and calculated reference curves are shown in Figure 4b. It is curious that $\rho_{xy}$, $\rho_{yx}$ curves are so close at high frequencies that they could be erroneously accepted as undistorted. The method extracts the $\rho_{xy}$ curve which is close to the locally longitudinal one, and provides little information on the correct position of the $\rho_{yx}$ curve. Auxiliary sounding was also implied in Kaufman (1988) where the idea to use the ratio of apparent impedances $Z_a(\omega)/Z_a(\omega_0)$ instead of the impedance itself was discussed.

Natural magnetic fields can also be useful in the static correction procedure (Larsen, 1977). Joint interpretation of MT and magnetovariational soundings is also the essence of “Electrical Conductivity Reference Exploration (ECRE)” (Wollfgram and Scharberth, 1986).

The methods just described are designed to correct the apparent resistivity curves. Of even greater importance is restoring the telluric field itself. In 2-D media the undistorted E-polarized electric field can be found if one integrates the vertical magnetic field. In more complicated 3-D media the distortion tensor technique can be used (Schmucker, 1970; Larsen, 1977; Hempfling, 1977; Mozley, 1982; Berdichevsky and Zhdanov, 1984; Bahr, 1985; Junge, 1986; Junge, 1988; Jiracek et al., 1989).

The electric field at an observation point can be expressed via the undistorted electric field $E^U_\tau$ using the distortion tensor $\hat{D}$:

$$E_\tau = \hat{D}E^U_\tau.$$  

A similar expression is valid for distorted and undistorted impedances. If the normal impedance is known at one frequency then using the experimental fields it is possible to find the distortion tensor $\hat{D}$. In a frequency band where distortions
CORRECTIONS FOR DISTORTIONS OF MAGNETOTELLURIC FIELDS

Fig. 4. Removal of surficial distortion in case of 3-D target object (Pellerin and Hohmann, 1990). (a) Geometry of 3-D 5Ω·m, 5 m thick conductive surficial inhomogeneity and a large deeper 3-D 10Ω·m body in a layered earth, central loop TEM receiver. (b) Synthetic ρ_{xy} and ρ_{yx} together with undistorted and computed curves. 100 m crossed MT electrode array.

are static, the undistorted field can be found from $\tilde{D}^{-1}E_r$. Frequency dependent tensor stripping was discussed in (Chouteau and Bouchard, 1988; Jiracek et al., 1989). The distortion tensor can also be modeled for local inhomogeneities, otherwise it should be determined on the basis of experimental data. It seems that the most consistent approach should use the field calculated for a model taking into account the local inhomogeneities as the undistorted field. The distortion tensor technique should then be invoked to strip the effect of geological noise.

It was shown in (Groom and Bailey, 1989) that if the regional structure is two
2.2. Spatial filtering

Geological noise is a set of randomly distributed surface inhomogeneities whose deterministic separation is practically impossible. At the same time, geological noise causes enormous scatter in MT data. Apparent resistivity maps drawn for a single period look very irregular. Spatial filtering of such maps was used in (Berdichevsky and Nechaeva, 1975; Berdichevsky et al., 1989). Another approach consists of averaging of apparent resistivity curves. Berdichevsky et al. (1980) classified the Baikal Region into 10 zones with conformal effective \( \rho_0(T) \) curves. The curves belonging to each zone were averaged and only the averaged curve were used for the interpretation. Spatial curve averaging was used also in (Sternberg et al., 1982; Warner et al., 1983) and a number of other papers. Often it is the only tool available to correct for the distortions. Nevertheless, when using this approach one can never say with certainty that the averaged curve will not be shifted (Hermance, 1982).

A more consistent approach is based on the spatial filtration of the telluric field itself. This can be done in different ways. For example two smoothing procedures were proposed recently in (Kaufman, 1988). It should be noted also that a minimum level of intrinsic averaging is always involved in any MT experiment because of the finite length of electrical measurement dipoles. The observed magnetic field is measured at a point while the electric field is averaged along the dipole. If the dipole originates at \( r_1 \) and terminates at \( r_2 \), then the value measured is

\[
(r_2 - r_1)(E_r) = \int_{r_1}^{r_2} E_r(r) \, dr.
\]

This averaging provides an additional stabilization of the apparent impedance (Poll et al., 1989; Jones, 1988).

The E-polarized field contains information not only on the medium at depth \( 0 < z \ll |\Lambda_0| \) just below the observation point (where \( Z_0 = -\omega \mu_0 \Lambda_0 \) is the impedance and \( |\Lambda_0| \) is the skin depth) but on the lateral conductivity distribution at horizontal distances of order \( |\Lambda_0| \). The same is true for the B-polarized field if the observed electric field is smoothed with a spatial window whose width is \( |\Lambda_0| \) or greater. This is the basic concept of the "ElectroMagnetic Array Profiling (EMAP)" (Bostick, 1977; Bostick, 1984; Torres-Verdin, 1985; Word et al., 1986). It was stressed in (Bostick, 1986) that if a profile is approximately orthogonal to strike the averaging effectively takes place along both directions. This statement is valid for quasi-two-dimensional structure. In three-dimensional media, the situation is more complicated as the electromagnetic field does not decompose into
E- and B-polarization modes. Hence only areal averaging would guarantee the necessary effect.

The typical EMAP sensor disposition is shown in Figure 5 (Shoemaker et al., 1986). The dipole length is between 60 m and 600 m. At high frequencies when $|\Lambda_0|$ is smaller than the single dipole length, the signal is measured off every dipole. For lower frequencies, the weighted signals of several dipoles is combined. The window width ranges between $|\Lambda_0|$ and $3|\Lambda_0|$. It is not necessary to use very long electric arrays at low frequencies. Instead the transfer functions between the measured electric field and the magnetic field in a fixed reference point are calculated, and then the array can be moved to a new position. The important feature of EMAP is the employment of continuous arrays to avoid aliasing. Aliasing is practically unavoidable when sampling is used. The effective penetration depth is usually evaluated via the apparent impedance, which can be significantly scattered. Jones et al. (1989) proposed to use the electric field averaged along the profile to evaluate the cut off spatial frequency. The spatial averaging smooths out structures which are smaller than the skin depth. One dimensional inversion is usually used to interpret the EMAP acquired data. Some examples of 2-D interpretation also exist (Shoemaker et al., 1986).

Spatial filtering as well as the other approaches designed to suppress the static
shift are first of all a remedy against geological noise. The same methods can be used in the case of local inhomogeneities, but information on the depths corresponding to the maximum of the $\rho_a(T)$ curve is lost in this case. Regional distortions can not be suppressed by spatial filtering.

2.3. ELECTROMAGNETIC FIELD MODELLING

The approach based on physical or numerical modelling has now become a practical tool for taking into account local and regional scale distortions. At the same time its application for geological noise in practical MT soundings is inhibited by insufficiency of experimental information, and limits on the performance of computers.

The modern state of physical modelling (Moroz et al., 1975; Dosso et al., 1989) makes it possible to simulate rather complicated earth models. There exists a large number of papers devoted to 3-D numerical modelling (Raiche, 1974; Weidelt, 1975; Hohmann, 1975; Ting and Hohmann, 1981; Hvozdara, 1981; Zhdanov and Spichak, 1980; Velikhov et al., 1983; Park, 1985; Druskin and Knizhnerman, 1988; Wannamaker et al., 1984; Vanyan et al., 1984; Newmann, 1989).

A separate branch consists of thin sheet modelling programs (Vasseur and Weidelt, 1977; Singer et al., 1984; Vanyan et al., 1984; McKirdy et al., 1985; Robertson, 1988). Thin sheet modelling programs appear to be an adequate tool to analyze and correct local and regional distortions as they make available sufficiently large numerical meshes on moderate performance computers. Thin sheet modelling was used for interpretation of MT observations in the Carpathian Region (Zhdanov et al., 1987), the Ural Region (Diakonova et al., 1987), in Turkmenia (Avdeev et al., 1988) and many others. An example of how the thin sheet modelling and inversion work was presented in (Avdeev et al., 1990a), where the main features of the MT field distribution at the South Turanian Plate and South Caspian Depression was explained (Figure 6). These features include the edge effect observed at the South Kara-Kum Platform for the telluric field orthogonal with respect to Kopet-Dagh mountains, significant depression of the electric field at the South Caspian Depression, magnetovariational anomaly beside Kopet-Dagh and so on. As a result, a crustal conductor was revealed under the Kopet-Dagh mountains, and the hypothesis on the possible existence of an astenospheric layer was proved.

Thin sheets can also be used to simulate electromagnetic fields induced in a model consisting of both surface and deep inhomogeneities. For example a preliminary model for the region of the EMSLAB experiment was presented in (Fainberg

Fig. 6. Comparison of experimental and theoretical curves at the South Turanian Plate and South Caspian Depression (Avdeev et al., 1990a). (a) The South Slope of The Kara-Kum Platform, (b) Baihan and Western Kopetdagh zones, (c) Northern side of the South Caspian Depression, (d) South Caspian Depression, $\perp$ - transverse curves, $\|$ - longitudinal curves.
et al., 1988). The model consisted of two inhomogeneous layers embedded in
the otherwise layered structure. The program used was based on the “Iterative
Dissipative Method (IDM)” (Fainberg and Singer, 1980; Singer and Fainberg,
1985), and was able to carry out calculations on the mesh consisting of $10^4$ nodes.
At low frequencies simpler DC thin sheet programs can be useful to investigate
the effects of local inhomogeneities (Hermance, 1982; Vanyan et al., 1983; Barash-
kov A. S., 1988).

It should also be noted that problems similar to that of MT arise also in
soundings with powerful controlled sources (Singer et al., 1985; Kaikkonen et al.,
1987; Vanyan et al., 1989a; Vanyan et al., 1989b; Fainberg et al., 1989). Topo-
graphic distortions have been modeled in (Dmitriev and Tavartkiladze, 1975;
Tavartkiladze, 1975; Thayer, 1975; Jiracek and Holcombe, 1981; Holcombe, 1982;
Mozley, 1982; Reddig and Jiracek, 1984; Reddig, 1984; Wannamaker et al., 1986;
Jiracek et al., 1989).

3. The Validity Limits for Static Shift Correction Procedures and Spatial Filtering

This section is devoted to discussion of conditions which guarantee that the effect
of surface inhomogeneities on electromagnetic fields reduces to static shift. Some
analytical examples demonstrate the consequences which occur when the con-
straints are violated. The influence of spatial filtering on MT fields is also discussed.

3.1. The Adjustment Distance

The concept of adjustment distance is of great importance in any analysis of MT
field distortions. It was shown in (Dmitriev, 1969; Ranganayaki and Madden,
1980) that B-polarized anomalous electric fields decay outside the surface anomaly
in accordance with an exponential law, with the characteristic scale $\sqrt{ST}$. Here
$S$ is the surface layer conductance, and $T$ is the transverse resistance of the
intermediate high ohmic layer underlaid by a perfect conductor. The decay of the
anomalous electric field was studied analytically by Dawson et al., (1982) for a
model consisting of two homogeneous half-sheets, overlaying the thin high ohmic
layer underlaid by a uniform half space. It was shown in (Fainberg and Singer,
1987) that the toroidal part of the anomalous electric field decays outside the 3-
D surface anomaly as $1/r^3$. The poloidal part decays as $K_1(r/\lambda_L)$, where $K_1$ is the
McDonald's function and

$$\lambda_L = \sqrt{\frac{T}{S^{-1} + Z_0}}. \quad (5)$$

Here $Z_0 = -\omega \mu_0 \lambda_0$ is the Tikhonov–Cagniard impedance of the laterally uni-
form medium underlying the surface layer. Equation (5) is valid for a layered
underlying medium if $|\lambda_L| \gg |\lambda_0|$. The anomalous field amplitude depends on the
3.2. Static shift

We shall consider the model consisting of an inhomogeneous surface layer whose conductance is \( S(x, y) \), and a laterally uniform underlying medium (Figure 7). The surface current distribution excited by an external field \( j^e \) satisfies the integral equation (Vasseur and Weidelt, 1977; Singer and Fainberg, 1985; Fainberg et al., 1990a)

\[
j^e = j^0 - \hat{G}_0 \ast (S_0 R^* j^0),
\]

where "\( \ast \)" means the convolution operation,

\[
R^* = 1/S - 1/S_0,
\]

\( S_0 \) is some constant value and \( \hat{G}_0 \) is the tensor Green's function for a 1-D model consisting of the uniform surface layer with conductance \( S_0 \), and the same underlying structure as in the original model:

\[
\hat{G}_0 = -(n \times \nabla_x) \otimes (n \times \nabla_x) Q^T_0 - \nabla_x \otimes \nabla_x Q^R_0.
\]

Here \( \nabla_x \) is the spatial differentiation operator with respect to lateral coordinates, \( n \) is the unit vector orthogonal to the earth's surface and directed outward and "\( \otimes \)" means the tensor product operation. Functions \( Q^T_0 \) and \( Q^R_0 \) are determined by expressions
where $J_1$ is a first order Bessel function, $Z_k^T$ and $Z_k^P$ are the spectral impedances of underlying structure for toroidal and poloidal modes.

It was shown by Singer and Fainberg (1985) that the Green’s tensor possesses an important property. For an arbitrary vector $\mathbf{u} \in \mathbb{L}_2$ the norm

$$\|G_0 \ast \mathbf{u}\| \leq \|\mathbf{u}\|. \quad (10)$$

This is valid for any value of $S_0$, but below the value

$$\frac{2}{S_0} = \frac{1}{S_{\min}} + \frac{1}{S_{\max}},$$

is assumed. At low frequencies when

$$S_{\max} \cdot |Z_0| \ll 1, \quad (11)$$

the value $|\zeta_k| \approx S_0 \cdot |Z_0|$, and it can be shown that $S_0(Z_k^T - Z_k^P)$ is real and frequency independent. Hence the Green’s function has a form

$$\hat{G}_0 = \hat{G}_0' + \hat{G}_0'',$$

where $\hat{G}_0'$ is real and frequency independent and $\|\hat{G}_0''\| \leq S_0 \cdot |Z_0|$. Designating $\hat{L}_0 \hat{j}^* = \hat{j}^* + \hat{G}_0''(S_0 R^* \hat{j}^*)$ one gets

$$\|\hat{L}_0\| \gg 1 - \frac{S_{\min}^{-1} - S_{\max}^{-1}}{S_{\min}^{-1} + S_{\max}^{-1}} = \frac{S_0}{S_{\max}}.$$  

Hence Equation (6) reduces to

$$\hat{j}_r = \hat{L}_0^{-1} \hat{j}_0'' - \hat{L}_0^{-1} \hat{G}_0''(S_0 R^* \hat{j}^*). \quad (12)$$

Taking into account that $\hat{L}_0$ does not depend on frequency and

$$\|\hat{L}_0^{-1} \hat{G}_0''(S_0 R^* \hat{j}^*)\| \leq \frac{S_{\max}}{S_0} \cdot S_0 |Z_0| \cdot |\hat{j}^*| = S_{\max} |Z_0| \cdot |\hat{j}^*|,$$

one can easily see that the only frequency-dependent term in Equation (12) has a relative value smaller than $S_{\max} |Z_0|$, and therefore the frequency dependence of $j^*$ is determined by $j_0''$, which in its turn is expressed via the normal field. Thus in the frequency band where $S_{\max} |Z_0| \ll 1$ the phase shift of the anomalous electric field with respect to the normal one is negligible. Further decrease in frequency does not affect the spatial distribution of the field. This means that the electric field measured at any point of the earth’s surface differs from a normal one by a
CORRECTIONS FOR DISTORTIONS OF MAGNETOTELLURIC FIELDS

325

frequency independent factor. This corresponds to a vertical shift of apparent resistivity curves. The phase curves are unaffected.

The previous discussion shows that under certain conditions, the effect of local inhomogeneities as well as of geological noise can be suppressed by a plain vertical shift of the \( \rho_d(T) \) curves on a logarithmic scale. The next question is how significantly can these conditions be softened. It is quite obvious that the 1-D character of the underlying structure as well as the restriction (11) on the frequency band are sufficient if they take place in a region exceeding the adjustment distance \( |\lambda_L| \). The following examples show that violation of these conditions leads to the serious consequences.

3.2.1. The Edge Effect

Consider the case when the surface layer (Figure 7) consists of two uniform half planes, i.e.

\[
S(x) = \begin{cases} 
S_0^L & x < 0 \\
S_0^R & x > 0
\end{cases}
\]

Then the apparent impedance of the B-polarized mode is distributed along the earth surface as

\[
Z_a(x) = \begin{cases} 
Z_0^L \cdot \left(1 + \frac{\sqrt{(S_0^L Z_0^L)/(S_0^L Z_0^R)}}{\lambda_L^L} - 1\right) \cdot \exp\left(\frac{x}{\lambda_L^L}\right), & x < 0 \\
Z_0^R \cdot \left(1 + \frac{\sqrt{(S_0^R Z_0^R)/(S_0^R Z_0^L)}}{\lambda_L^R} - 1\right) \cdot \exp\left(-\frac{x}{\lambda_L^R}\right), & x > 0
\end{cases}
\]

where \( Z_0^L, Z_0^R \) are the left and right section impedances, \( \lambda_L^L, \lambda_L^R \) are the corresponding values of the "adjustment distance" (5). The apparent resistivity curves at different points of the profile are shown in Figure 8. It can be seen that the contact affects not only the vertical positions of the curves but also changes their form. The vertical shift is the only effect when condition (11) is satisfied for both conductive and resistive parts of the model. Thus Equation (11) should indeed be valid along the distance comparable with the "adjustment distance" to make sure that the \( \rho_d(T) \) curve has undistorted form.

3.2.2. The Deep S-effect

The next point is now to show that the requirement of lateral uniformity for underlying structure is also necessary. If the underlying structure is laterally non-uniform, then the magnetic field which induces current in the surface layer has a frequency-dependent distribution. This can invalidate the static character of the surface effects in case of local inhomogeneities. The deep inhomogeneity can also cause nonstatic effects via galvanic interaction with the surface layer. A deep inhomogeneity can be the object of the survey, or it can also be an obstacle for the exploration of deeper layers. One can expect that the influence of small deep
Fig. 8. The edge effect example. The model (Figure 7) parameters: 

\[ S_0(x) = \begin{cases} 100S, & x < 0; \\ 1000S, & x > 0 \end{cases} \]

\[ \sigma(z) = \begin{cases} 10^{-4} \text{S/m}, & 0 < z < h_1 = 10^4 \text{m}; \\ 10^{-1} \text{S/m}, & 10^4 \text{m} < z < 1.5 \cdot 10^4 \text{m}; \\ 10^{-3} \text{S/m}, & 1.5 \cdot 10^4 \text{m} < z < 7.5 \cdot 10^4 \text{m}; \\ 1 \text{S/m}, & 7.5 \cdot 10^4 \text{m} < z < \infty. \end{cases} \]

\[ \rho_a |_{x = \infty} \] — the apparent resistivity curve at point \( x \).

inhomogeneities will not be noticeable at the earth’s surface (Berdichevsky et al. 1982).

Against which parameter should the size of deep inhomogeneities be compared? The parameter should be bound with the distance along which “normalization” of the horizontal electric field (or decaying of vertical currents) takes place. From point of view it should be analogous to \( \lambda_L \).

The model (Figure 9) we shall examine consists of two conductive thin sheets surrounding a resistive layer. The deep conductive layer is underlaid by another resistive layer and a laterally uniform conductive half space. If the surface conductive layer is uniform, i.e. \( S_0(x) = \text{Constant} \), and the deep layer has the conductance

\[ S_1(x) = \begin{cases} S_1^f, & |x| < L; \\ S_1^f, & |x| > L'. \end{cases} \]

and the condition
Fig. 9. The model with two conductive thin sheets.

\[ 2L \ll |\Lambda_L|, \quad \Lambda_L = \sqrt{\frac{Z_1}{S_1^{-1} + Z_1}}, \]

\((Z_1)\) is the impedance of the medium underlying the deep conductive layer) is satisfied, then the apparent impedance of the B-polarized mode is

\[
Z_a(x) = \begin{cases} 
\xi Z_0' + (Z_0(L) - \xi Z_0') \cdot \frac{\cosh(x/\lambda'_L)}{\cosh(L/\lambda'_L)}, & |x| < L \\
Z_0' + (Z_0(L) - Z_0') \cdot \exp(-|x| - L/\lambda'_L), & |x| > L
\end{cases}
\]

where \(Z_0'\) is the impedance of the normal section and \(Z_0\) is the normal impedance of the section at point \(x = 0\),

\[
\lambda'_L = \sqrt{\frac{T_1}{1/S_0 + 1/S_1' - \omega \mu_0 h_1}}, \quad \lambda'_L = \sqrt{\frac{T_1}{1/S_0 + 1/S_1' - \omega \mu_0 h_1}},
\]

\[
Z_a(L) = \frac{Z_0'/\lambda'_L + Z_0'/\lambda'_L \cdot \tanh(L/\lambda'_L)}{1/\lambda'_L + 1/\lambda'_L \cdot \tanh(L/\lambda'_L)},
\]

\[(13)\]
Fig. 10. The deep S-effect example. The earth model (Figure 9) parameters: $S_0 = 100$ S, $h_1 = 10^4$ m, $T_1 = 10^8 \cdot m^2$, $S'_1 = 2000$ S, $S'_4 = 500$ S, $L = 10^7 m$, $h_2 = 3 \cdot 10^4$ m, $T_2 = 3 \cdot 10^5 \cdot m^2$, $z = \infty$, $z > h_1 + h_2$. The apparent resistivity curves: $\rho_0|_{x = \infty} = \rho_{\text{norm}}/|_{x = 0}$ - the normal curve, $\rho_0|_{x = 0}$ - the locally normal curve at point $x = 0$, $\rho_0|_{0, |\lambda| = |L| = |\lambda|}$ - the curve at $x = 0$ in case of wide deep anomaly.

$$\xi = \frac{S'_1 Z'_0 (1/S_0 + 1/S'_1 - \omega \mu_0 h_1) - \omega \mu_0 h_1 (S'_4 - S'_4)/S_0}{S'_1 Z'_0 (1/S_0 + 1/S'_1 - \omega \mu_0 h_1)}$$

and $h_1$ is the first resistive layer thickness. It should be noted that parameter $\xi$ is equal to unity at frequencies higher than and including the second ascending branch.

It follows from Equation (14) that the effect of a small deep anomaly on the surface electric field is of order $L/|\lambda|$. From this point of view, deep geological noise does not exist. An example of the deep S-effect is shown in Figure 10. The shift of the second descending branch of the curve $\rho_0|_{x = 0, L \gg |\lambda|}$ with respect to a normal curve is controlled by parameter $\xi$. As can be seen from Figure 10 the distortion of the apparent resistivity curve becomes static only when the low frequency band with respect to the deep inhomogeneity has been reached.

The previous examples make it possible to conclude that constraint (11) on the
frequency band as well as the laterally uniform character of the underlying section should hold along a distance of the same order as the “adjustment distance”. In this case the effect of local surface inhomogeneities on the telluric field reduces to the static one. It should be stressed here that this statement is valid for the local inhomogeneities. Analogous conditions for geological noise are much softer.

3.2.3. 2-D and 3-D Inhomogeneities

In accordance with their definition, geological noise inhomogeneities are small in comparison with the effective penetration depth and with the “adjustment distance”. Larger inhomogeneities were referred to as local and regional inhomogeneities. Field distributions along the earth’s surface substantially differ for two and three dimensional inhomogeneities. Two following examples illustrate this circumstance.

In case of a 2-D model (Figure 7) with the surface layer conductance

\[ S_0(x) = \begin{cases} S_0^I, & |x| < \alpha \\ S_0^R, & |x| > \alpha \end{cases} \]

the apparent impedance of the B-polarized mode is distributed along the surface in accordance with the law

\[ Z_a(x) = \begin{cases} Z_0^I + \{Z_a(\alpha - 0) - Z_0^I\} \cdot \frac{\cosh(x/\lambda_L^I)}{\cosh(\alpha/\lambda_L^I)}, & |x| < \alpha \\ Z_0^R + \{Z_a(\alpha + 0) - Z_0^R\} \cdot \exp(-|x|\alpha/\lambda_L^R), & |x| > \alpha \end{cases}, \quad (16) \]

where \( Z_0^I \) is the normal impedance, \( Z_0^R \) is the locally normal impedance for the area above the inhomogeneity, \( \lambda_L^I \) and \( \lambda_L^R \) are determined by Equation (5) after substitution of \( S_0 \) with \( S_0^I \) and \( S_0^R \),

\[ \frac{S_0^I}{S_0^R} \cdot Z_a(\alpha - 0) = Z_a(\alpha + 0) = Z_0^I \cdot \frac{1 + \lambda_L^I/\lambda_L^I \cdot \tanh(\alpha/\lambda_L^I)}{1 + \lambda_L^I/\lambda_L^I \cdot \tanh(\alpha/\lambda_L^I)}, \quad (17) \]

The following features of electric field distribution in the presence of 2-D surface inhomogeneities are displayed by these expressions.

1. The anomalous field amplitude over the inhomogeneity is determined by the ratio \( S_0^I/S_0^R \). It can reach enormous values.

2. Apart from the inhomogeneity the anomalous field amplitude is dependent on the ratio \( \alpha/\lambda_L^I \) in case of the conductive anomaly and on \( \alpha \cdot \lambda_L^I/(\lambda_L^I)^2 \) in case of the resistive one. The anomalous field decays in accordance with an exponential law with characteristic distance \( \lambda_L^I (\alpha \ll |\lambda_L^I|) \).

The picture is quite different in the case of an isometric surface inhomogeneity.
\[ S_0(r, \phi) = \begin{cases} S_0^i, & |r| < \alpha \\ S_0^s, & |r| > \alpha \end{cases} \]

In the low frequency band the apparent impedance is

\[ Z_{\alpha'}^{\gamma'}(r, \phi) = Z_0^s \cdot \left\{ 1 - \frac{df}{dr} \cdot \cos^2 \phi - \frac{f(r)}{r} \cdot \sin^2 \phi \right\}, \tag{18} \]

\[ Z_{\alpha'}^{\gamma'}(r, \phi) = Z_0^s \cdot \left\{ \frac{f(r)}{r} - \frac{df}{dr} \right\} \cdot \sin \phi \cdot \cos \phi, \]

where

\[ f(r) = \begin{cases} f_a I_1(r/\lambda_0^i) / I_1(\alpha/\lambda_0^i), & 0 < r < \alpha \\ f_a K_1(r/\lambda_0^i) / K_1(\alpha/\lambda_0^s), & r > \alpha \end{cases} \tag{19} \]

\[ f_a = \left[ S_0^i - S_0^s \right] \frac{S_0^i}{\lambda_0^i} \frac{I_1(\alpha/\lambda_0^i)}{I_1(\alpha/\lambda_0^s)} - \frac{S_0^s}{\lambda_0^s} \frac{K_1(\alpha/\lambda_0^i)}{K_1(\alpha/\lambda_0^s)}. \]

\( I_1 \) is the first order modified Bessel function. In case of a small inhomogeneity, whose size \( a \ll \min\{|\lambda_0^i|, |\lambda_0^s|\} \) these expressions reduce to

\[ f(r) = \begin{cases} f_a \cdot \frac{r}{\alpha}, & 0 < r < \alpha \\ f_a \cdot \frac{\alpha}{r}, & r > \alpha \end{cases}, \quad f_a = \alpha \cdot \frac{S_0^i - S_0^s}{S_0^i + S_0^s}. \]

Thus in case of a small surface inhomogeneity the anomalous electric field:

1. Is restricted for arbitrary values of \( S_0^i \) and \( S_0^s \) and
2. It decays outside the anomaly as \((\alpha/r)^2\).

This means that in order to find the electric field in the presence of small surface inhomogeneities one can restrict oneself to the problem of electromagnetic induction by a magnetic field specified by the undisturbed model (the model without the inhomogeneities mentioned above). Zero boundary condition for the anomalous field can be used at distances much greater than the anomaly size. This is valid in the frequency band where the anomaly does not affect the magnetic field, i.e. when the inductive effects originating from the excessive currents in the anomaly have terminated \((a \cdot \max\{\alpha \mu_0 S, |\lambda_0^i|\} \ll 1)\). This is almost always valid for geological noise inhomogeneities in the MT frequency band. The static shift takes place independently of whether the undisturbed structure is one, two or three dimensional.
3.3. Spatial filtering

Due to surface inhomogeneities the electric field is nonuniform even when the primary field and an underlying medium are uniform. In such circumstances the spatial filtering method seems to be a natural remedy against distortions. How does the spatial filtering process affect MT data? This question is discussed below for the model shown in Figure 7.

The tangential components of the electric \( E_r \) and magnetic \( H_r \) fields at the surface of a laterally uniform medium (just under the surface layer) are related by the equation

\[
\mathbf{n} \times \mathbf{H}_r^- = \hat{Y} \ast \mathbf{E}_r = \mathbf{E}_r/Z_0^- + \int_{\mathbb{R}^2} \hat{Y}(\mathbf{r} - \mathbf{r}')\{\mathbf{E}_r(\mathbf{r}) - \mathbf{E}_r(\mathbf{r}')\} \, d\mathbf{s}',
\]

which can be deduced from a well known results of Dawson and Weaver (1979) and McKirdy et al. (1985). The admittance kernel \( \hat{Y} \) was considered in (Avdeev et al., 1989). It was shown that the toroidal part of the kernel decays exponentially with the characteristic distance determined by the effective penetration depth \( |\lambda_0| \) (\( Z_0^- = -\omega \mu_0 \lambda_0 \) is the Tikhonov–Cagniard impedance of the underlying section). The poloidal part also decays exponentially but along the distance \( |T| \), where \( T \) is the transverse resistance of the section.

If the electromagnetic field changes significantly only along distances greater than the adjustment distance, the second term in Equation (20) can be omitted, and Equation (20) reduces to the Leontovich's expression. If the field changes are more rapid, then high spatial harmonics can be suppressed using the spatial filtering process, so that the filtered field would satisfy the equation

\[
\mathbf{n} \times \langle \mathbf{H}_r^- \rangle_r = \langle \mathbf{E}_r \rangle_r/Z_0^-.
\]

Taking into account that the tangential magnetic fields at the earth's surface and just under the surface layer satisfy the equation

\[
\mathbf{n} \times (\mathbf{H}_r - \mathbf{H}_r^-) = S \cdot \mathbf{E}_r,
\]

one can easily find the expression for the filtered field

\[
\mathbf{n} \times \langle \mathbf{H}_r \rangle_r = \langle \mathbf{E}_r \rangle_r \cdot \langle 1/Z_0 \rangle_r + \mathbf{Q}(\mathbf{r}).
\]

Here the averaged admittance and the bias are defined as

\[
\langle 1/Z_0 \rangle_r = 1/Z_0^- + \langle S \rangle_r,
\]

\[
\mathbf{Q}(\mathbf{r}) = \langle S \cdot \mathbf{E}_r \rangle_r - \langle S \rangle_r \cdot \langle \mathbf{E}_r \rangle_r.
\]

Thus the bias is determined by the correlation between \( S(\mathbf{r}) \) and \( \mathbf{E}_r(\mathbf{r}) \), and can be evaluated using experimental data. If the bias is small in comparison with \( \langle \mathbf{E}_r \rangle_r \cdot \langle 1/Z_0 \rangle_r \) then Equation (22) reduces to the Leontovich's expression. It follows
from Equation (24) that $Q$ is restricted by the relative deposit of surface currents into the magnetic field. Hence the bias should disappear in the low frequency branch of the MT band.

The 2-D model displayed in Figure 11a includes a surface layer consisting of stripes with width 48 and 32 km and conductances 250 and 1500 S. The resistive layer has thickness 40 km and specific conductivity $10^{-3}$ S/m, the deeper half space is occupied by a perfect conductor. The B-polarized electric field distribution shown in Figure 11b displays the "classical" S-effect. The apparent resistivity curve calculated using the averaged field is shown in Figure 12 (Avdeev et al., 1990b) together with the curve corresponding to the averaged value $\langle S \rangle = 750$ S. The descending branches of both curves coincide. The most significant discrepancy takes place at the ascending branch and near the maximum of the normal curve. In the example both curves are conformal but this is not a general rule.

The bias depends on the definition of the averaged impedance. It was proposed in (Dmitriev and Berdichevsky, 1988) to assume
In this case the bias is equal to

$$Q(r) = \langle S \cdot E_r \rangle_r - \langle 1/S \rangle_r^{-1} \cdot \langle E_r \rangle_r.$$  \hspace{1cm} (26)

This value can be neglected if the surface current is constant along the area covered by the filter. This takes place when such effects as current channeling, flowing around as well as leakage of currents can be omitted. As to geological noise, any bias originating from filtering is usually negligible as the MT frequency band belongs to the low frequency band with respect to the geological noise inhomogeneities.

### 4. Conclusion

Before terminating the review, it seems reasonable to clarify what one has in mind when speaking about distortions.

#### 4.1. Geological noise

First of all we had in mind the effects due to small uncontrollable surface inhomogeneities, which were referred to as geological noise. Therefore the electromag-
netic fields which would be observed in the absence of geological noise were considered as undistorted fields. Commonly speaking these fields are three dimensional, and they display information on deep and large scale (local and regional) surface inhomogeneities.

Nowadays the problem of geological noise looks even more significant than ever before. There exist now tools for data acquisition and processing which guarantee several percent accuracy in data. At the same time the data are often contaminated by the noise on several hundred or even thousand percents. Due to the ill posed character of the inverse problem, results of interpretation can appear much more contaminated or even erroneous. Fortunately the static effects are the main effects originating from the geological noise.

The static shift correction procedure based on auxiliary TEM soundings seems to be very promising, as it should not be too expensive. It can be used for correction of apparent resistivity curves as well as in distortion tensor techniques. The most consistent approach should employ the field affected by large scale inhomogeneities (local and regional) as the undistorted fields.

Another approach based on the random nature of geological noise is spatial filtering of the electric field. The areal modification of EMAP would indeed be able to suppress the geological noise effects (Jiracek, 1990; Andrieux, pers. comm., 1990), but it is important to preserve such extremely important features of the EMAP approach as antialiasing. One of the important results of the approach is designing acquisition equipment which promises to bring synchronous areal measurements into reality. The importance of this point can not be overestimated. Even if the data acquisition, processing and distortion correction were carried out with no errors, it would not mean that the resultant data are sufficient for interpretation of 3-D electromagnetic fields. On the other hand, if synchronous measurements are available, the magnetic field measurements will often provide the same information free of geological noise effects. Even such parameters as the transverse resistance can be inferred from magnetic observations (Fainberg et al., 1990b).

In 2-D situations contaminated by geological noise inhomogeneities the decomposition approach of Groom and Bailey (1989), combined with static corrections of the mean impedances, also looks very attractive.

It should be noted that the most attractive feature of magnetotellurics is a chance to make inferences from single point observations. The method of spatial derivatives (Berdichevsky et al., 1969; Kuckes, 1973; Fainberg and Berdichevsky, 1977) has the same feature. The apparent impedance determined by the method is much less affected by surface inhomogeneities than the MT impedance and may provide correct results even when significant galvanic distortions exist (Fainberg and Singer, 1987). Therefore designing of experimental equipment for measurements of the magnetic field derivatives would have rich future.
4.2. LOCAL INHOMOGENEITIES

The term "distortions" has quite another meaning for local inhomogeneities. In this case distortion means deflection of the curve from the locally normal curve. The problem to restore the locally normal curve from the distorted one is strictly speaking insoluble. It can be solved approximately and use only for rough preliminary interpretation. As it was shown in Section 3.2, local distortions reduce to static shift only when rather hard conditions are satisfied in the vicinity of the observation point. The size of this vicinity is controlled by the effective penetration depth in case of E-polarized field and by the "adjustment distance" in case of the B-polarized or 3-D field. The period should correspond to the low frequency band and the underlying medium should be one dimensional in this vicinity.

In other words the methods used as remedies against geological noise have only limited application against local inhomogeneities. Results of their application are of limited accuracy. The only tool is modelling. Although an enormous amount of information is necessary to carry out the modelling, this corresponds to physical essence of the phenomenon and can not be bypassed on the final stage of interpretation at least. Concerning local distortions the synchronous measurements could also provide the information necessary for more sophisticated methods of interpretation (Berdichevsky and Zhdanov, 1984; Avdeev et al., 1986).

Acknowledgements

The author is grateful to all colleagues who responded to his call for references and papers. My greatest gratitude is addressed to E. Fainberg whose influence on my scientific position was always very significant. I deeply appreciate the remarks and recommendations of A. Jones, A. Schultz, P. Andrieux and two referees who helped me to improve the paper.

References


Fainberg, E. B.: 1983a, 'Global and Regional Magnetovariational Sounding of the Earth (in Russian)', Dr.Sc. Thesis, Moscow, IZMIRAN.


CORRECTIONS FOR DISTORTIONS OF MAGNETOTELLURIC FIELDS


