EM-FIELD TRANSFORMATIONS AND THEIR USE IN INTERPRETATION

V. V. SPICHAK

Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation (IZMIRAN), 142092 Troitsk, Moscow Region, U.S.S.R.

Abstract. Two large groups of methods used to transform electromagnetic fields are discussed. The first of them includes methods based on the linear relations between the components of the MT-fields measured at a single or several points at the earth's surface. In this context special attention is paid to the consideration of impedance transformations, apparent resistivity and inductive operators, most frequently used in interpretation.

The second group of methods concerns integral transformations of data. Firstly techniques are considered which are used for the division of the EM-fields into external and internal, normal and anomalous, surface and deep parts. Finally electromagnetic analogs of seismic methods are mentioned briefly.

All transformations discussed are accompanied by examples of their use in data interpretation.

Introduction

The non-uniqueness and mathematical incorrectness of the solution of the inverse geoelectrical problem, which are attributed to the insufficient quality and quantity of underlying electromagnetic data, seem to be the main cause of the advent of a large group of methods, techniques, approaches and, finally, of heuristic procedures used to interpret measured electromagnetic data. One of the main means employed to this end is the transformation of data into forms more convenient to analyze. It is evident from the above that within the framework of a single review paper having a limited volume it is hardly possible to achieve a complete coverage of all the practical methods of electromagnetic field transformation and, in addition, to cite numerous examples of data interpretation relying thereon. Many of them can be found in the monographs by Rokityansky (1982), Kaufman and Keller (1983), Vanyan and Shilovsky (1983), Berdichevsky and Zhdanov (1984), Zinger and Fainberg (1985), Zhdanov (1988), etc.

In this review paper we will confine ourselves to two large groups of methods used to transform electromagnetic fields which enjoy most favor in data interpretation:

– methods based on the linear relations between the components of the magnetotelluric field measured at a single or several points at the Earth's surface (Section 1) and

– methods within which we transform data taken, as a rule, simultaneously and over a regular grid pattern (Section 2).

In doing so, only a few examples of data interpretation will be given, restricted primarily to the works that were published after the VIIIth Workshop on Electromagnetic Induction in the Earth and Moon (1986) or in less accessible journals.

Surveys in Geophysics **11**: 271–301, 1990.

It is also worth noting that, unless otherwise stated, the electromagnetic fields in the Earth are assumed to be excited by a plane electromagnetic wave, displacement currents are taken to be negligible, the permeability is equal to permeability of a vacuum ($\mu \equiv \mu_0$), while the time dependence of the field is fitted by the factor exp $(-i\omega t)$.

1. Linear Transformations of Electromagnetic Field Components

According to the theory of linear relations between components of the magnetotelluric field, developed by Berdichevsky and Zhdanov (1984), for a fairly large class of geomagnetic variations (pulsations, bay-like disturbances, solar quiet diurnal changes (S_q), and world-wide magnetic storms (D_{st})) we can write

$$\bar{E}(\bar{r}) = \hat{Z}\bar{H}(\bar{r}) \tag{1}$$

$$\bar{H}(\bar{r}) = \hat{Y}\bar{E}(\bar{r}) \tag{2}$$

$$\bar{E}(\bar{r}) = \hat{T}\bar{E}(\bar{r}_0) \tag{3}$$

$$\bar{H}(\bar{r}) = \hat{M}\bar{H}(\bar{r}_0), \tag{4}$$

where

$$\hat{Z} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}, \quad \hat{Y} = \begin{bmatrix} Y_{xx} & Y_{xy} \\ Y_{yx} & Y_{yy} \end{bmatrix},$$

$$\hat{T} = \begin{bmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{bmatrix}, \quad \hat{M} = \begin{bmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{bmatrix}$$

are the magnetotelluric operators (impedance, admittance, telluric and magnetic, respectively) which do a linear transformation of the field, while r and r_0 are the radius vectors of observation points at the surface.

Berdichevsky and Zhdanov (1984) also consider the so called inductive operators J effecting transformations of the magnetic field as a whole and its parts into each other:

$$\bar{H}^u(\bar{r}) = \hat{J}^{uv}\bar{H}^v(\bar{r}), \tag{5}$$

where

$$\hat{J}^{uv} = \begin{bmatrix} J^{uv}_{xx} & J^{uv}_{yx} \\ J^{uv}_{yx} & J^{uv}_{yy} \\ J^{uv}_{zx} & J^{uv}_{zy} \end{bmatrix},$$

u, $v = t$, n, a, e, i (t corresponds to the total field, n–to the normal part, a–to the anomalous one, e–to the external field, i–to the internal field).

Within these designations, we arrive at the inductive vector (Parkinson, 1959; Wiese, 1962; Schmucker, 1970):

$$\bar{W}^{an} = J^{an}_{zx}\bar{d}_x + J^{an}_{z}y\bar{d}_y, \tag{6}$$

where $\bar{d}_x$ and $\bar{d}_y$ are the unit vectors in the x and y directions, correspondingly, and perturbation vectors (Schmucker, 1970)

$$\begin{aligned}\bar{p} &= J_{xx}^{an}\bar{d}_x + J_{yx}^{an}\bar{d}_y \\ \bar{q} &= J_{xy}^{an}\bar{d}_x + J_{yy}^{an}\bar{d}_y.\end{aligned} \tag{7}$$

Relations (1) through (7) are also known as transfer functions.

The conditions under which the above linear transforms hold are outlined in the monograph by Berdichevsky and Zhdanov (1984) and in the review paper of Gough and Ingham (1983). Variations of these relations in time are treated in review papers by Beamish (1982) and Kharin (1982). Menvielle and Szarka (1986) expose various causes of errors in their calculation, while Chave and Booker (1987) review the methods used to estimate transfer functions reducing the systematic and random noise.

The application of linear magnetotelluric transforms to electromagnetic data interpretation has been recently developed along two main lines:

(1) determination of apparent parameters of the geoelectric section at each point, which is based on frequency (time) characteristics of transforms, and

(2) studies of structural features (size, boundaries and even conductivity of an anomalous zone), which are based on theoretical examination of the properties of particular transforms, or most frequently on heuristic formulas.

Transforms of electromagnetic field components taken at a single point on the surface make it possible to establish the parameters of the conductivity distribution corresponding to a certain specific vicinity of this point. In the following these field transforms will be called point transforms. It should be noted that the method suggested by Goubau *et al.* (1978) for carrying out measurements at an additional reference point enhances the quality of estimation of the transforms, but this question goes beyond the framework of our consideration.

Obviously, the above definition of the considered class of transforms is rather conventional, since it includes the notion of the 'point vicinity', which has no clear physical meaning. Indeed, the electromagnetic field measured at a certain point may convey information about more or less distant regions of the Earth's interior. The problem of deriving this information from the field characteristics or from its point transforms (at least, defining the degree of their 'locality') calls for specific investigations (see, for example, the work by Dmitriev and Berdichevsky, 1979). At this place should be just mentioned that the interpretation of measured data from point transforms is carried out according to a scheme consisting of the following steps:

(a) calculation of transforms at individual points of the surface at various periods (or over some interval of time);

(b) graphic representation in the form of maps of isolines, vectors or pseudo-sections;

(c) analysis (data interpretation itself).

The following subsections will describe the point transformations most commonly used in practice (impedance, apparent resistivity, inductive operators) and give examples of their application to data interpretation.

1.1. Impedance transformations

In regions where the distribution of conductivity in the Earth is approximated fairly well by the function $\sigma(z)$, a horizontal electric field at any point on the surface can be found, as is known, from the magnetic field by simply multiplying the latter by a frequency dependent coefficient Z:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & Z \\ -Z & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}, \tag{8}$$

Here $Z = Z_{xy} = Z_{yx}$ is the Tikhonov–Cagniard impedance (Tikhonov, 1950; Cagniard, 1953).

Tikhonov (1953) proved the uniqueness of determination of parameters of a one-dimensional horizontally layered section from the dependence $Z(\omega)$, but it is evident that when the conductivity at a depth z is determined from a finite set of values $Z_i(\omega_i)$, the result is ambiguous. Weidelt (1985) constructed sets of extremal models (as a sequence of thin layers of different conductivities), setting upper and lower limits on $\sigma(z)$ which depend on the quantity and quality of available impedance data.

Sometimes, it is reasonable to employ other forms of sea impedance information instead of (8). For example, Fiskina, Zinger and Fainberg (1985) interpreted most advantageously deep electromagnetic data using the magnetic field transform

$$Z = -i\omega\mu_0[H_z/\mathrm{div}_s\bar{H}] = i\omega\mu_0\left[H_z/\frac{\partial H_z}{\partial z}\right]$$

(where $\mathrm{div}_s\bar{H}$ is the surface divergence), suggested earlier by Berdichevsky, Vanyan and Fainberg (1969). Berdichevsky, Zhdanova and Yakovlev (1983) have established by model calculations that the transform allowing for the exponential nature of magnetic field variations in a layer of sea water (Berdichevsky and Vanyan, 1969) yields better results, compared to those obtained conventionally.

If the geoelectric section at a specified point differs from a one-dimensional one, transform (8) can give, nevertheless, an approximate idea of the function $\sigma(z)$, which is basic to the so called formal interpretation (see, for instance, the monographs by Vanyan (1965) and Berdichevsky (1968)).

When the conductivity distribution in the vicinity of an observation point is two-dimensional, the impedance tensor is reduced, by turning the coordinate system about the vertical axis, to the form (Word *et al.*, 1970):

$$\hat{Z} = \begin{bmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{bmatrix}, \tag{9}$$

with $Z_{xy} + Z_{yx} \neq 0$ (unlike (8)).

The four degrees of freedom of transform (9) (two complex numbers Z_{xy} and Z_{yx}) are associated with four physically meaningful values: two principal values of impedance and, correspondingly, two preferred directions of a structure, one of which coincides with the local axis of two-dimensionality. This does not rule out, however, the possibility of interpreting such situations from other parameters. For instance, Kovtun and Vardanyants (1985) suggested that two-dimensional anomalies should be interpreted using the frequency characteristic of a relative value of the impedance modulus:

$$\alpha_{E,H} = \frac{|Z_1^{E,H}| - |Z_2^{E,H}|}{|Z_2^{E,H}|}$$

where $Z_1^{E,H}$ and $Z_2^{E,H}$ are the impedances for E- and H-polarizations of the field at two points above an anomaly (1–closer to the center). For the model concerned (Figure 1a), the position of a maximum ($T_{\max}$) of the frequency characteristic α_E proved to be independent of the normal section parameters or location of points 1 and 2 (Figure 1b) but instead is mainly governed by the integral conductivity of the insert cross-section: $G^{\alpha} \sim 1.2 \times 10^6\ T_{\max}(\mathrm{S} \cdot \mathrm{m})$ for the range of L, d_s and d_c values given by the inequality $0.3 \leqslant L/(d_s + d_c) \leqslant 3$. The value α_H reflects the degree of conductive coupling of the sedimentary section with a conducting body and permits us to define the setup of the conducting zone more closely. Figure 1c plots the behavior of α_H versus T with different values of parameter $T_s = d_s \cdot \varrho_s$ for the case where point 1 lies just above the center of the body, while point 2 is far off the structure. Variations in the transverse resistivity of the insert T_s make α_H change considerably but do not practically affect α_E. It is remarkable that α_H is related to the parameters of the section and insert within the DC-approximation, when the

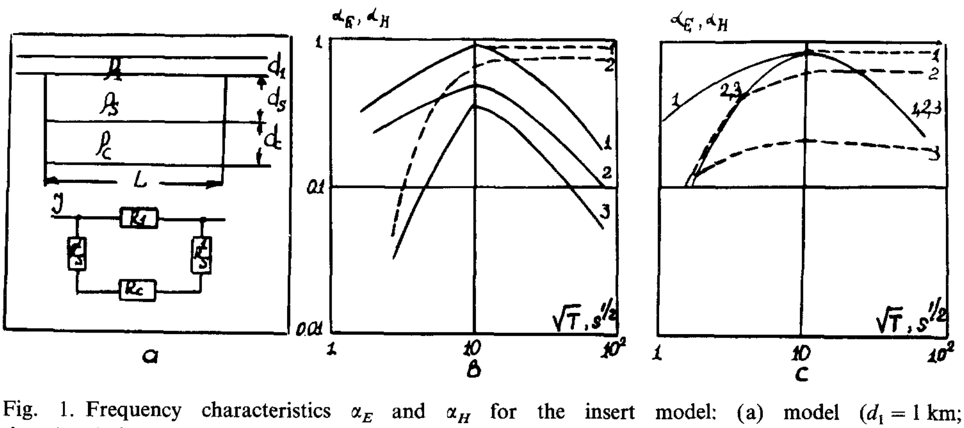

Fig. 1. Frequency characteristics α_E and α_H for the insert model: (a) model ($d_1 = 1$ km; $d_s = d_c = 10$ km; $L = 75$ km, $\varrho_1 = 7$ Ohm · m, $\varrho_c = 50\ \Omega$ m, $\varrho_s = 100\ \Omega$ m); (b) relationship of α_E (solid line) and α_H (broken line) to the chosen point of normalization z_2: (1) at infinity; (2) outside the insert (2,5 km from the edge); (3) above the insert (5 km from the edge); (c) T_s dependence of α_E (solid line) and α_H (broken line): (1) $5 \times 10^7\ \Omega\,\mathrm{m}^2$; (2) $5 \times 10^8\ \Omega\,\mathrm{m}^2$; (3) $5 \times 10^9\ \Omega\,\mathrm{m}^2$ (after Kovtun and Vardanyants, 1985).

insert can be replaced by equivalent parallel – connected resistors (Figure 1a):

$$\alpha_H = R_1/(R_1 + R_c + 2R_s^{\perp}).$$

where

$$R_1 = \varrho_1 L/d_1, \quad R_c = \varrho_c L/d_c, \quad R_s^{\perp} = 2\varrho_s d_s/L.$$

If the medium is locally three-dimensional, the impedance does not reduce to the form (9). In this situation, the interpretation may involve forms of the impedance transform, which are invariant under the coordinate system rotating about the vertical axis (Berdichevsky and Dmitriev, 1976):

$$Z^1_{\text{inv}} = 0.5(Z_{xy} - Z_{yx}) \tag{10}$$

$$Z^2_{\text{inv}} = (Z_{xx}Z_{yy} - Z_{xy}Z_{yx})^{1/2}. \tag{11}$$

Their application in practice provides 'averaged' parameters of a local one-dimensional section. Ingham (1988) shows, using a three-dimensional model as example, that the interpretation of a highly conducting anomaly (in particular, determination of the depth to its upper edge) by means of transform (10) reduces the extent of the error compared to conventional one-dimensional interpretation. Hermance (1982), Sule and Hutton (1986) and Ranganayaki (1984) employed transform (11). Particularly, in the latter work the pseudo-section $|\varphi_{\text{Det}}| \equiv |\arg Z^1_{\text{inv}}|$ produced along the profile concerned coincides virtually with the geologic section (Fig. 2).

Several workers developed another approach to estimation of horizontal characteristics of a medium at an observation point, an approach in which physically meaningful characteristics of a local geoelectric section are selected from impedance tensor components in an amount equal to the number of its degrees of freedom (for instance, in a local three-dimensional medium this number is eight).

Thus, Eggers (1982) used, with this view in mind, biorthogonal analysis in a plane and found eigenvalues and eigenvectors of the matrix $\hat{Z}$ on the assumption that $E \cdot H = 0$. Since the actual electromagnetic fields in 3D-media do not always comply with this assumption, this formulation of the problem will not ensure at all times extreme values of impedance.

Using the factorization method, Spitz (1985) constructed two internal systems of coordinates for $\hat{Z}$ which are defined by two angles of rotation. However, as was noted by the author himself, it is not yet clear from the mathematical standpoint which of the two established coordinate systems should be preferred in data interpretation.

Counil *et al.* (1986) introduced, along with the starting basis in which all measured data are specified, two more real bases – 'electrical' and 'magnetic' – in which the electrical and magnetic fields are linearly polarized respectively. The linearly polarized electrical field is associated with electric-type impedance and the direction in which it attains its maximum value is called by the authors the direction of maximum current. Similarly, the linearly polarized magnetic field is in correspondence with the magnetic-type impedance, and the direction in which its maximum

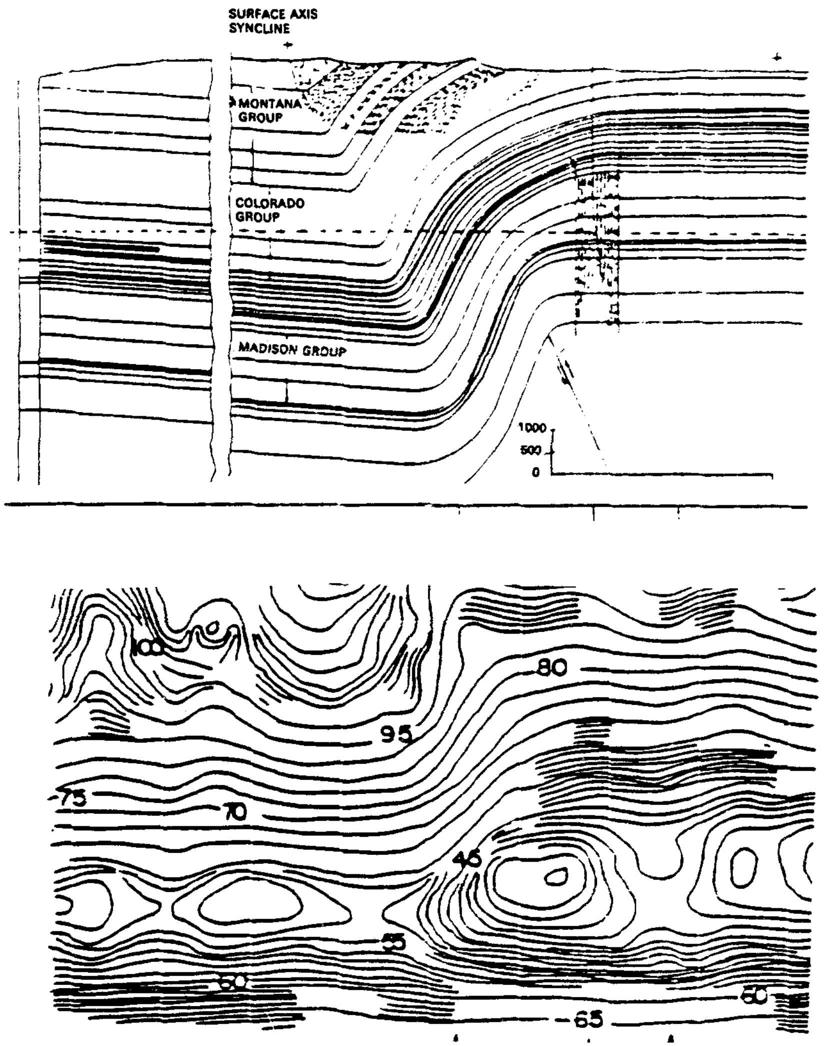

Fig. 2. A comparison of geologic structure-section with $/\varphi_{\mathrm{Det}}/$ pseudosection along the same line (after Ranganayaki, 1984).

is achieved has been given the name of the direction of maximum induction. These directions are shown in Figure 3 within a two-dimensional thin sheet model, as an example. The thin layer consists of two half-layers of differing conductivities ($\sigma_1 > \sigma_2$). At two points taken near the contrast interface the direction of maximum induction and maximum current are defined by a specified distribution of conductivity.

La Toracca *et al.* (1986) examined the eigenvalues of the impedance tensor by means of the SVD method. The authors arrived at eight parameters defining it uniquely (two complex eigenvalues and four angles).

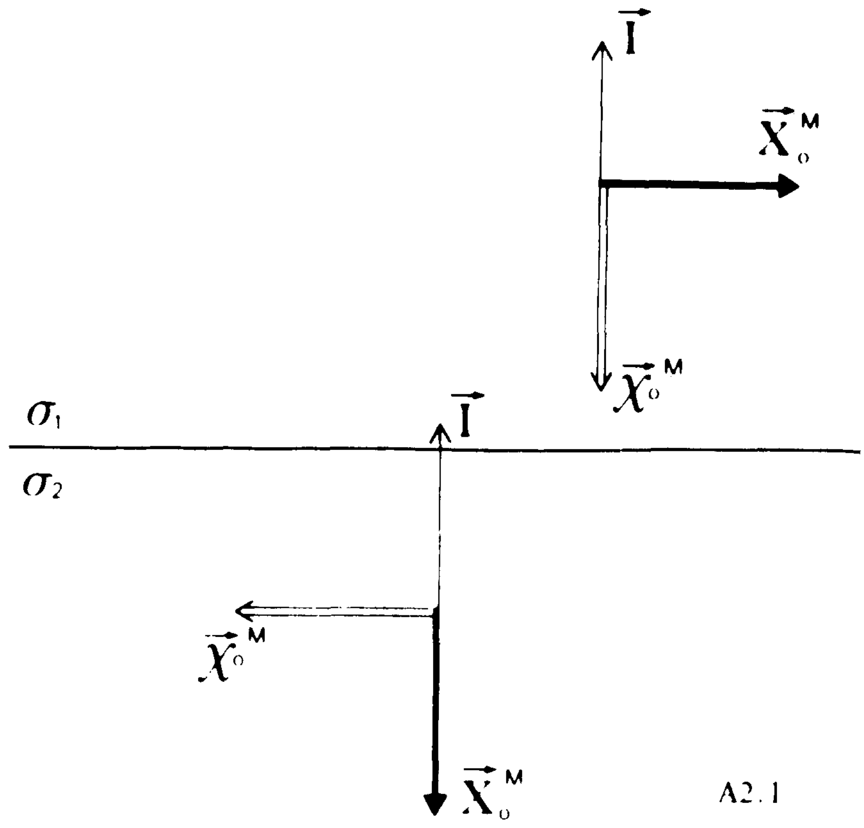

Fig. 3. Induction arrow I, maximum induction χ_0^M and maximum current X_0^M directions at stations close to the boundary between two half sheets of conductivities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) (after Counil *et al.*, 1986).

The transforms of $\hat{Z}$ as well as of $\hat{Y}$ in three-dimensional space were analyzed most generally by Yee and Paulson (1987a) and Tzanis (1988a). The former authors called it canonical decomposition, while the latter referred to it as a generalized rotation method. Despite certain differences in detail, the two methods are very close in the bulk of mathematics used to solve a particular problem. It is based on a group of rotations defined on a set of unitary matrices (2×2) in three-dimensional space. As a result, the initial coordinate system transforms into two internal systems defined by directions (angles of rotation) θ_E, φ_E; θ_H and φ_H (they are not necessarily mutually orthogonal and horizontal). In these systems, components of the fields E and H are interrelated and form two modes of the electromagnetic field inside the Earth. In the two above-mentioned studies, the authors compare in detail their own methods of analysis of the tensor to those developed by their precursors (Eggers, 1982; Spitz, 1985; Counil *et al.*, 1986; La Toracca *et al.*, 1986) and show that the recent findings are attainable, under certain specific assumptions, within the general 'canonical' approach.

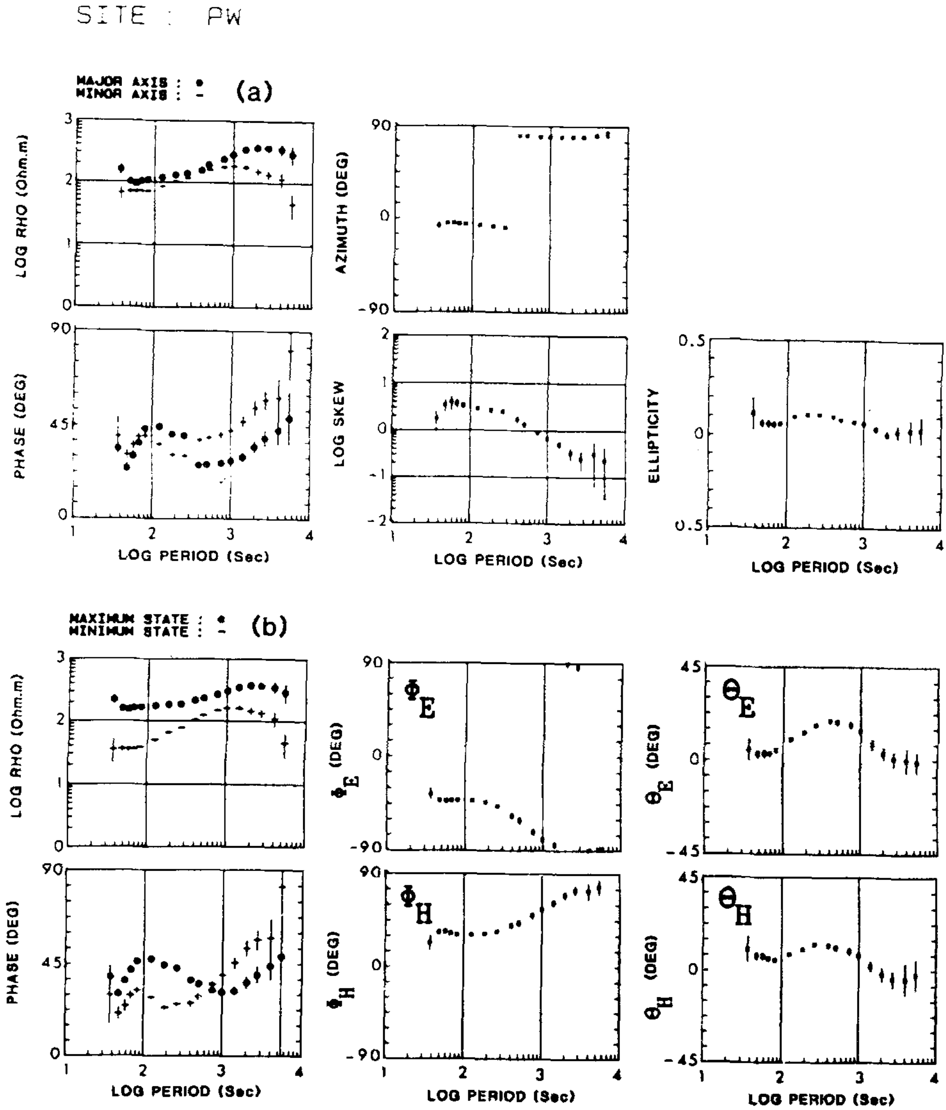

Fig. 4. (a) The conventional analysis results for site PW. (b) the UD results for site PW (after Tzanis, 1988b).

Yee and Paulson (1987b) employ canonical decomposition to analyze the telluric operator (3), while Tzanis (1988b) applies the generalized rotation method to the interpretation of magnetotelluric data obtained in Northern England and Southern Scotland (Bands and Beamish, 1984). A glance at Figure 4 shows at the point PW estimated to display a local three-dimensional distribution of conductivity, the structure parameters established by Tzanis (1988a) (Figure 4b) differ substantially from those determined using the conventional rotation technique (Word *et al.*, 1970) (Figure 4a).

In Subsection 1.4 we will refer again to impedance transformation in the context of its application (along with other field transformations) to solve some methodological problems of data interpretation.

1.2. Apparent resistivity

As a rule, in the interpretation of realistic data one employs second- and even third-level transforms derived from the basic ones (say, (1)–(4)) through certain algebraic, differential or integral transformations. These transforms are intended for determining as accurately as possible the parameters of a local one-dimensional section at an observation point. It is also desirable that they enable higher depth resolution and be clear and transparent.

The first transform of this kind seems to have been suggested by Cagniard (1953):

$$\varrho_{a,|Z|} = \frac{1}{\mu_0 \omega} |Z|^2. \tag{12}$$

Applying (12) to realistic data at a single point, we evaluate the 'apparent resistivity' ϱ_a at a depth corresponding to the skin depth:

$$h_a = \left[\frac{2\varrho_a}{\mu_0 \omega}\right]^{1/2}.$$

Transform (12) is now basic to the interpretation carried out by the magnetotelluric sounding method.

Subsequently, modifications of transform (12) as well as of its transformation were suggested (Niblett and Sayn-Wittgenstein, 1960; Molochnov, 1968; Schmucker, 1970; Weidelt, 1972; Yakovlev *et al.*, 1975; Molochnov and Sekrieru, 1976; Bostick, 1977; Vanyan *et al.*, 1980; Le Vyet Zy Khyong and Berdichevsky, 1984; Murakami, 1985) to ensure enhanced resolution of the geoelectric section parameters in a particular period range. A comparative analysis of these transforms can be found in the works by Weidelt *et al.* (1980), Jones (1983), Schmucker (1987), Spies and Eggers (1986). The latter authors consider, in particular, the behavior of the apparent resistivity curves calculated according to formula (12) as well as by means of the following impedance transforms:

$$\rho_{a,\mathrm{Re}Z} = \frac{2}{\mu_0 \omega} (\mathrm{Re}Z)^2 \qquad \varrho_{a,\,\mathrm{Im}(Z^2)} = \frac{1}{\mu_0 \omega} \mathrm{Im}(Z^2)$$

$$\varrho_{a,\,\mathrm{Im}Z} = \frac{2}{\mu_0 \omega} (\mathrm{Im}Z)^2 \qquad \varrho_{a,|Z^2|} = \frac{1}{\mu_0 \omega} |Z^2|.$$

They show, using two two-layer models with resistivity contrasts 100 and 0.01 as an example, that the curve calculated from the real part of the impedance 'behaves' better than others do (fewer oscillations in the transition zone, maximum speed of convergence to the resistivity of the underlying rock), whereas the curve calculated from the imaginary part of the impedance 'behaves' worse than all the others. Nevertheless, the authors stress that one should not overrate the obtained results and draw conclusions about the resolution of particular techniques of Earth sounding just on the basis of apparent resistivity curves.

At the same time it is noteworthy that a joint analysis of the ϱ_a-curves and the behavior of the impedance phase conveying additional information about the local geoelectrical structure (see, for instance Fischer, 1985), improves the efficiency of local one-dimensional interpretation of two- and three-dimensional structures (Vaghin and Kovtun, 1981; Ranganayaki, 1984; Schnegg *et al.*, 1986; Fischer and Schnegg, 1986; Schnegg *et al.*, 1987; etc.). The general principles underlying this approach as well as multinumerous examples of interpretation are outlined in the work by Berdichevsky and Dmitriev (1976).

As noted earlier in Subsection 1.1, the most comprehensive information about the horizontal distribution of conductivity in the vicinity of an observation point is derived by examining all the elements of the impedance tensor (admittance or other linear transforms of the electromagnetic field components). If each component of the tensor Z is subject to transformation (12) we get an apparent resistivity tensor:

$$\hat{R}_a = \begin{bmatrix} \varrho_{a_{xx}} & \varrho_{a_{xy}} \\ \varrho_{a_{yx}} & \varrho_{a_{yy}} \end{bmatrix}. \tag{13}$$

The possibility of interpreting magnetotelluric data by means of transform (13) was already discussed by Vozoff (1972). A similar transform for the DC-methods can be found in the work by Bibby (1977). Methods of interpreting transient processes by various forms of apparent resistivity are treated at length in the monographs by Sidorov *et al.* (1977) and Kaufman and Keller (1983) (see the references cited therein). Various aspects of this interpretation are examined by Raiche and Spies (1981), Raiche (1983), Raiche and Gallagher (1985), Spies and Eggers (1986), Newman *et al.* (1987). Agheev (1986) employed the focusing transformation method suggested earlier by Svetov *et al.* (1983a, b) to the one-dimensional transformation of transient processes. The interpretation procedure lies in a succession of so-called focussing transformations of impedance curves, on the basis of contributions made by section parts to the values of impedance.

Note that by analogy with the 'apparent resistivity' transformation, we can use 'apparent conductivity' σ_a. Kryukova (1983) proposes an approximate transformation of the time dependence of the apparent conductivity σ_t into the relationship of the 'actual' conductivity σ_z to the corresponding depth. In Figure 5 the results of the author (solid line) are compared to the calculations (broken line) performed for a two-layered model ($\sigma_1/\sigma_2 > 1$). It is evident from the Figure that the $\bar{\sigma}_z$-curves are closer to the actual one-dimensional section than the $\bar{\sigma}_t$-curves are.

A procedure for interpreting apparent resistivity curves (13) plotted for a section having two- and three-dimensional conductivity anomalies is designed according to the purpose of the investigations. If it is necessary to establish an averaged one-dimensional section (say, in the global magnetotelluric sounding (Vanyan and

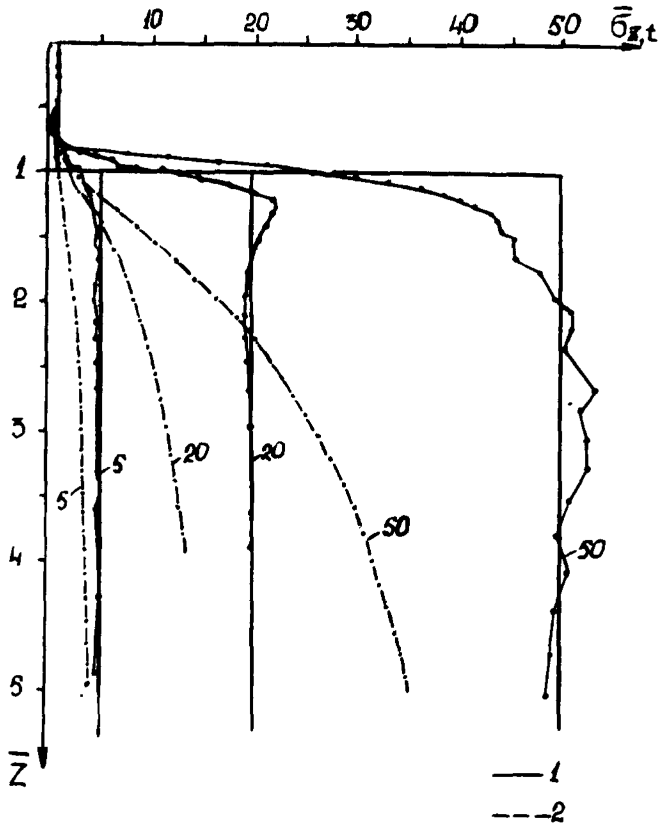

Fig. 5. Results of solution of the inverse problem for a two-layered medium (curves $\bar{\sigma}_z$) and initial curves of apparent resistivity $\bar{\sigma}_t$ (broken line) plotted versus the same depth z, where $\bar{\sigma}_{z,t} = \sigma_z/\sigma_1$ and $\bar{z} = z/z_1$ (σ_1 and z_1 are, correspondingly the conductivity and thickness of the 1st layer) (after Kryukova, 1983).

Shilovsky, 1983; Zinger and Fainberg, 1985), the differences between the actual ϱ_a-curves and those corresponding to a local one-dimensional section are interpreted as their distortions. The latter are analyzed by numerical and physical modelling of electromagnetic fields in two- and three-dimensional media in the works of Ting and Hohmann (1981), Wannamaker *et al.* (1984), Park (1984), Berdichevsky *et al.* (1984a, b, c, 1987), etc. An alternative way to interpret actual data may involve a transformation like (12) to the principal values of impedance obtained, say, via canonical decomposition (Yee, Paulson, 1987a). In local three-dimensional media this approach may help avoid errors possible in the case of one- and even two-dimensional interpretation of apparent resistivity curves.

1.3. Inductive operators

Horizontal conductivity gradients in the vicinity of an observation point can be evaluated by transforming the horizontal components of the magnetic field (5). Parkinson (1959), Wiese (1962), Schmucker (1970) introduced convenient graphic representations of operators, determined by relations (6) and (7), in the form of 'vectors', or 'arrows'. The review papers by Gregory and Lanzerotti (1980), Meyer (1982), Gough and Ingham (1982) offer a detailed treatment of various representations of inductive operators, their relationship and application to electromagnetic field interpretation.

Recent years have seen many publications devoted to this subject matter. For instance, Labson and Becker (1987) consider the behavior of induction arrows in two-dimensional models of contacts (over the VLF range), while Fischer and Weaver (1986) use them to compare the thickness of the continental and oceanic lithosphere. Ingham *et al.* (1987) studied the geoelectric pattern under the Cordilleras, Pajunpaa (1986) and Korja *et al.* (1986) in the Baltic shield, while Menvielle and Tarits (1986) investigated the Rhine-Graben conductivity anomaly.

Schmucker (1970) and Bailey *et al.* (1974) suggested hypothetical event analysis for data interpretation. The method comes about as follows. Employing transform (6) one can predict the vertical component of the magnetic field specifying a normal magnetic field of fixed polarization and intensity. Beamish and Banks (1983) employed this approach to produce isoline maps for three components of an anomaly magnetic field when they studied the conductivity anomaly in the north of Great Britain. Chamalaun *et al.* (1987) established a two-dimensional strike of a geoelectric structure in the north-west of India.

The interpretation of electromagnetic data can be made more efficient by studying the spatial-frequency characteristics of transformations (in particular, inductive operators) on typical two- and especially three-dimensional models. Lam *et al.* (1982) and Wannamaker *et al.* (1984) examined the behavior of induction arrows in the case where the horizontal layered section contains a three-dimensional conductivity anomaly. Nienaber *et al.* (1983) and Chen and Fung (1988a) were interested in the behavior of the real and imaginary induction arrows above the edge of a conducting plate. Chen and Fung (1985) studied imaginary arrows versus inducing field frequency by two-dimensional modeling. They discovered in particular a characteristic period T_c in which the phase difference between the components H_z and H_x is zero. The same conclusion has been reached by Beamish (1985) who analyzed the frequency dependence of an anomalous vertical field in the British Isles. The period dependence of inductive vector azimuths was also considered by Beamish and Banks (1984), Beamish (1987) and Chen and Fung (1988b).

Jones (1986) studied the frequency dependence of magnetic transfer functions J_{zx} and J_{zy} (5) using two-dimensional modeling. The author showed that at sufficiently high frequencies, induction arrows composed of these components with $u = t$; $v = t$, n, may behave 'anomalously' with their heads showing the direction away from highly conducting zones. It is better, therefore, for the interpretation to involve the use of the transfer functions J_{zx}^{aa} and J_{zy}^{aa} calculated only from anomalous components of the magnetic field which confirms the conclusion of Summers (1981). The parameter $R = H_z^a/H_x^a$ was helpful in the interpretation of two-dimensional structures carried out by Ingham *et al.* (1983), Chen and Fung (1986), Jones (1986), and

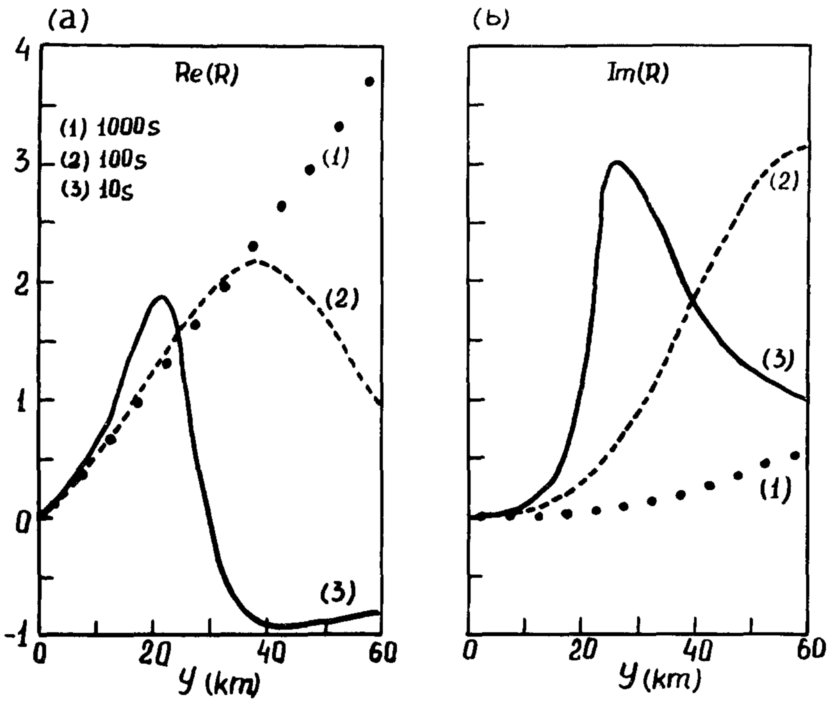

Fig. 6. Anomalous field ratio R for the symmetric 2-D-model. One-sided results plotted against distance (y) from the centre ($y = 0$) of the rectangular conducting prism, shown in Figure 7. Results at three periods are shown: (1) 1000 s, (2) 100 s and (3) 10 s. (a) Real part of R; (b) Imaginary part of R (after Beamish, 1987).

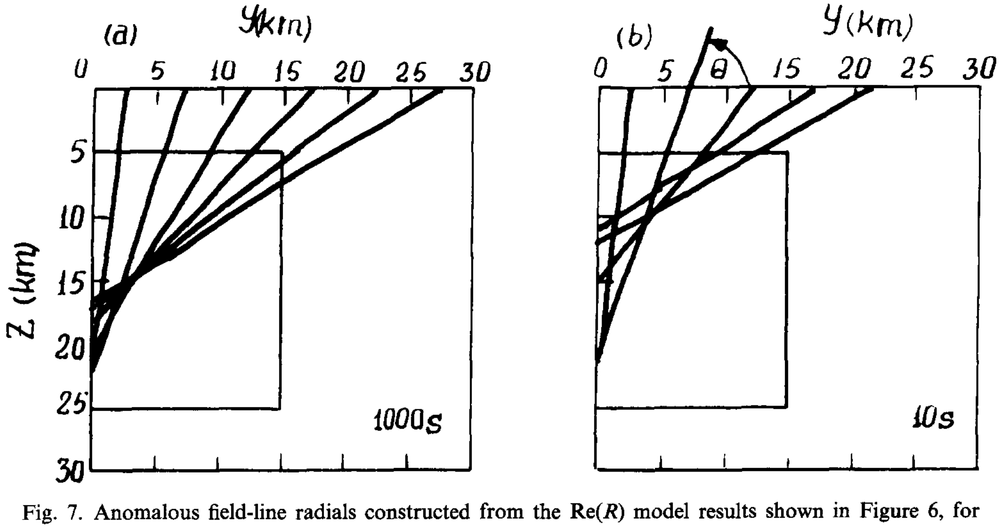

Fig. 7. Anomalous field-line radials constructed from the Re(R) model results shown in Figure 6, for two periods (a) 1000 s and (b) 10 s. The inner rectangle shows one-half of the buried conducting region (after Beamish, 1987).

Beamish (1987). In the latter work, model and practical examples demonstrate that the R-ratio may provide the location of horizontal boundaries of an anomaly (Figure 6) as well as its upper edge from the intersection of rays traced from each observation point downwards at an angle defined by the equality $\cot \theta = R$ (Figure 7).

As noted earlier, inductive operators are also useful in the interpretation of areal electromagnetic data (say, by means of hypothetical event analysis). In doing so, the accuracy of the estimation of a three-dimensional conductivity function is generally limited by the fact that here we deal with point transformations of the field. The transition to integral transforms naturally allowing for the relationship of electromagnetic field components measured at different points of the surface (see Section 2), however, calls for synchronous observations which are difficult to do in practice. To surmount this difficulty, Beamish and Banks (1983) suggested that a common reference point should be used for data recording. The limitation of this solution to the problem lies in the fact that the presence of anomalous horizontal fields at reference points shifts the results. This approach was subsequently refined by Banks (1986) who reduced vertical magnetic field components to a single instant of time by means of couplings between the components of the tensor M and J. Another solution to the problem has recently originated with Parkinson (1990). It involves step-by-step definition of the vertical magnetic field at the surface within an ever increasing accuracy via Gilbert transforms (with known magnetic transfer functions). Figure 8 presents magnetic field plots for a two-dimensional model (E-polarization) which have been obtained either by direct calculations or by the iterative procedure (6 iterations) suggested by the author. The figure displays a very good agreement of the plots for the vertical field component and a small discrepancy between the horizontal component plots (which may be attributed to the calculation error).

1.4. Some Aspects of Data Interpretation

While interpreting electromagnetic data measured at a single point or over an area, simultaneously or separately, an attempt is often made to estimate the dominant size of a studied conductivity anomaly as well as to divide an observed field into parts consistent with various mechanisms of their formation. Some of these questions can be answered by resorting to the point transforms of the field discussed in Subsections 1.1 through 1.3.

Beamish (1986) used the magnetotelluric sounding data obtained in Southern Scotland and in the north of Great Britain to study whether it is possible to divide the response observed at a single point into one-, two- and three-dimensional parts by means of the dimensionality indicators put forward by Kao and Orr (1982) or by the traditional ones (skew, ellipticity and eccentricity). An analysis has revealed that the parameter skew as well as the dimensionality indicators of Kao and Orr yield fairly reliable estimates. Ranganayaki (1984) investigated, with this aim in mind, along with skew some more parameters, and established that their isoline

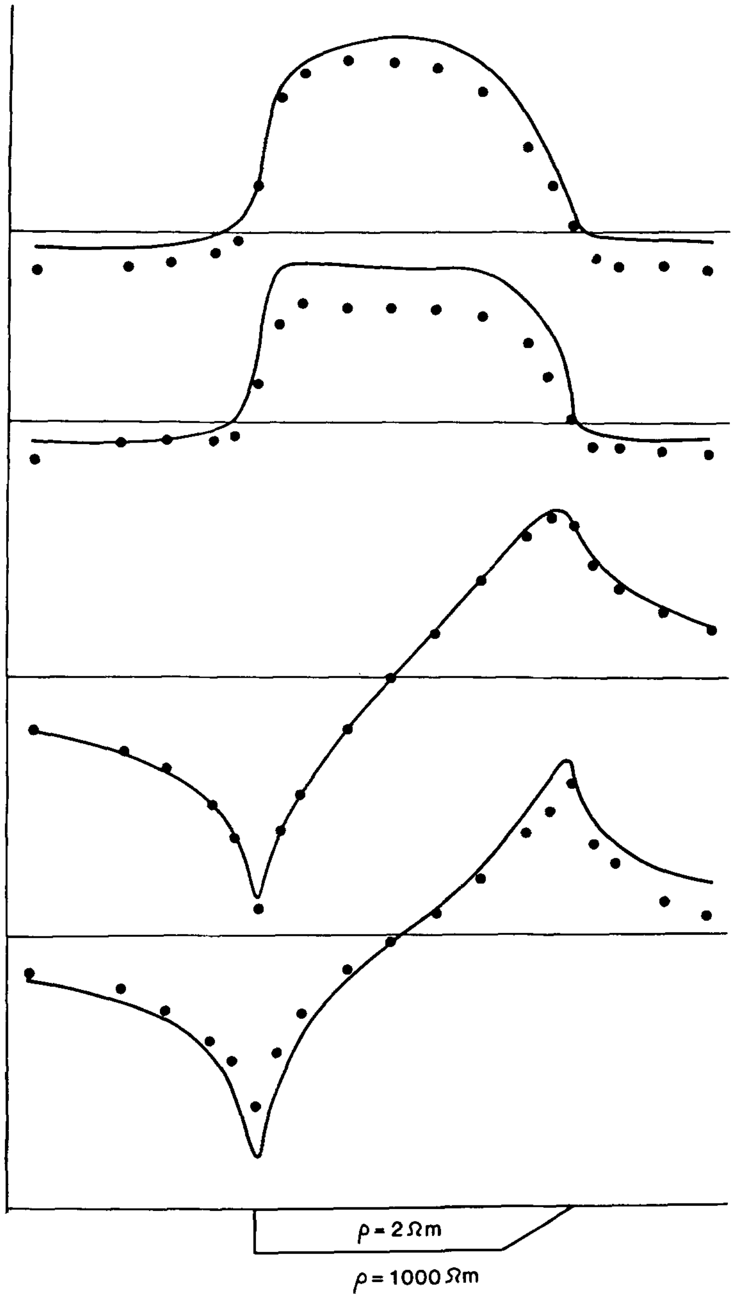

Fig. 8. Anomalous fields for the two-dimensional model shown at the bottom. The solid line has been taken from the Brewitt-Taylor and Weaver program, the dots have been calculated from the transfer functions. Reading from top to bottom: In-phase horizontal, quadrature horizontal, in-phase vertical, quadrature vertical (after Parkinson, 1990).

maps at the Earth's surface provide an estimate of the predominant size of a geoelectric structure at measurement points. Meanwhile Hermance (1982) had earlier shown by numerical calculations on a thin sheet model that electromagnetic field anomalies and the parameter skew are now always correlated quite clearly. Having applied canonical decomposition to the telluric operator $\hat{T}$, Yee and Paulson (1987b) showed that in two-dimensional geoelectric structures the information contained within this operator is fitted by five (rather than eight in a three-dimensional case) parameters. This fact is basic to the procedure suggested by the authors for separating 'two'- and 'three-dimensional' contributions to $\hat{T}$. Iliceto *et al.* (1986) proposed that an indicator of two- and three-dimensionality should be provided by the parameter

$$R = \frac{|t_{xy} + t_{yx}|}{|t_{xx} + t_{yy}|},$$

where $t_{\alpha\beta}$ $(\alpha,\beta = x,y)$ are the elements of the tensor $\hat{T}$.

For the sake of illustration, the authors performed numerical calculations for several typical two-dimensional models. Thus, Figure 9 depicts the pseudo-sections R produced for a graben model. It is evident from the figure that the pseudo-section of this parameter yields a fairly good approximation of the conductivity pattern.

Another important problem successfully solved in terms of point field transforms is the determination of the regional strike of a structure and identification of a local disturbance against its background. To this end, Banks and Beamish (1984) took the frequency dependence of the azimuths of real inductive vectors at various points of the surface. In this way, ranges of periods (and accordingly, of space coordinates) were established over which currents induced in the Earth are determined by the local and regional patterns of conductivity.

Menvielle and Tarits (1986) examining the Rhine–Graben conductivity model had virtually to decide upon one of the two explanations of the magnetic field anomaly – by local induction in a two-dimensional structure or by static deviation of telluric currents by poorly conducting crystalline masses (regional structure). To determine the answer to these questions, the authors resorted to the notion of adjustment distance of the inductive mechanism: for $\lambda^2 S > 20$ ($\lambda^2 = 2/\mu\omega\sigma$, S is the cross-sectional area of the anomaly) they decide on the second mechanism, while for $\lambda^2/S < 20$ they tend to the first mechanism. Their theoretical considerations have been confirmed experimentally: the curves for the moduli and the phases of inductive vectors at two different points at $T > 1000$ s coincide up to a constant factor.

Zhang *et al.* (1987), Bahr (1988), Groom and Bailey (1990) studied the properties of the impedance Z in a long-wave approximation, using for this purpose a model consisting of a near-surface local inhomogeneity and a regional structure.

Zhang *et al.* (1987) claim that the regional strike is characterized by the direction at which the elements of the columns of Z are proportional and their ratios $\beta = Z_{xx}/Z_{yx}$ and $\gamma = Z_{yy}/Z_{xy}$ are real and independent of the period. The local strike is noted for the direction at which the impedance diagonal elements are

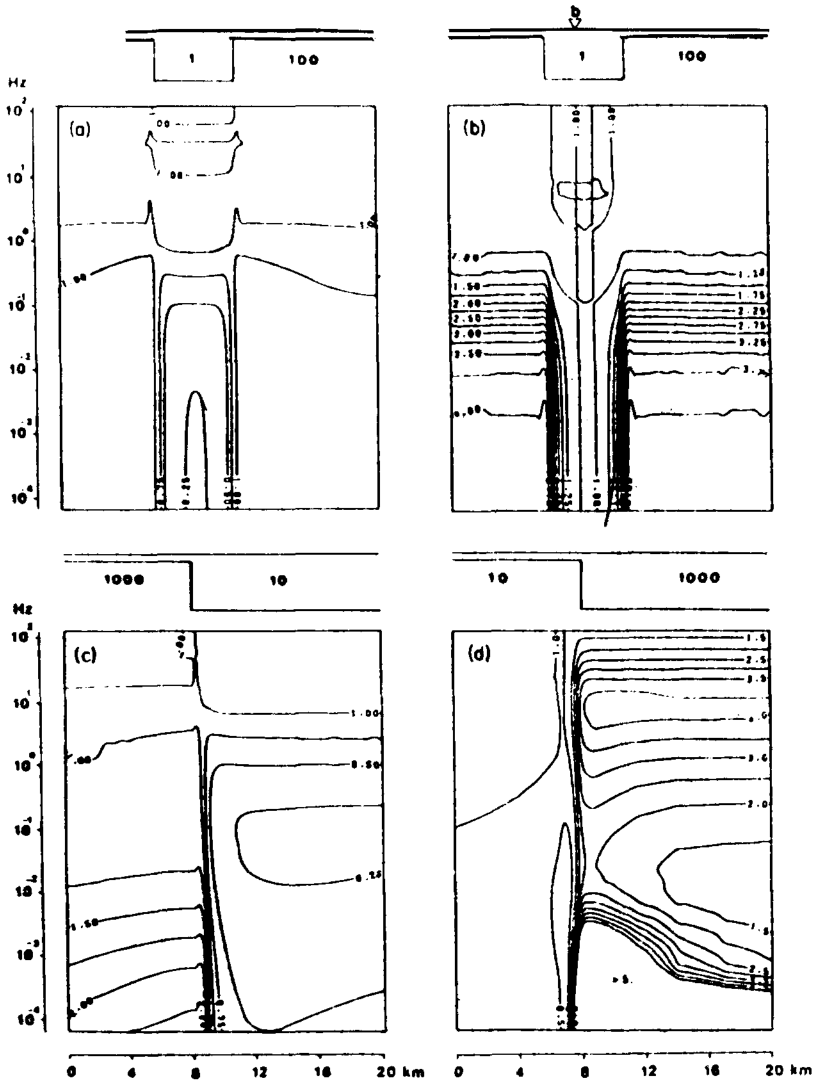

Fig. 9. Distance-frequency pseudo-sections of $R(\omega)$ for the graben model (1 : 100 resistivity ratio) with reference base at left infinity (a) and located at (b); (c) and (d) report pseudo-sections of the step model (4) with resistivity ratios 10 : 1000 and 1000 : 10 respectively (after Iliceto *et al.*, 1986).

proportional and the parameter $\alpha = Z_{xx}/Z_{yy}$ is real, negative and independent of the period T.

To separate the effects of local disturbance and regional induction Bahr (1988) has elaborated a method of telluric vectors. It relies on the information about the phase of all the impedance elements. Figure 10 plots phases of all the impedance tensor elements versus the coordinate system chosen. At $\alpha_2 = 47°$, the phases φ_{xy} and φ_{yy} corresponding to the unit vector $\bar{e}_y$ are close, whereas the other two are not. When $\alpha_1 = 59°$, the phases corresponding to $\bar{e}_x$ are identical. This circumstance underlies a method of determining the regional strike. Within this method, a system

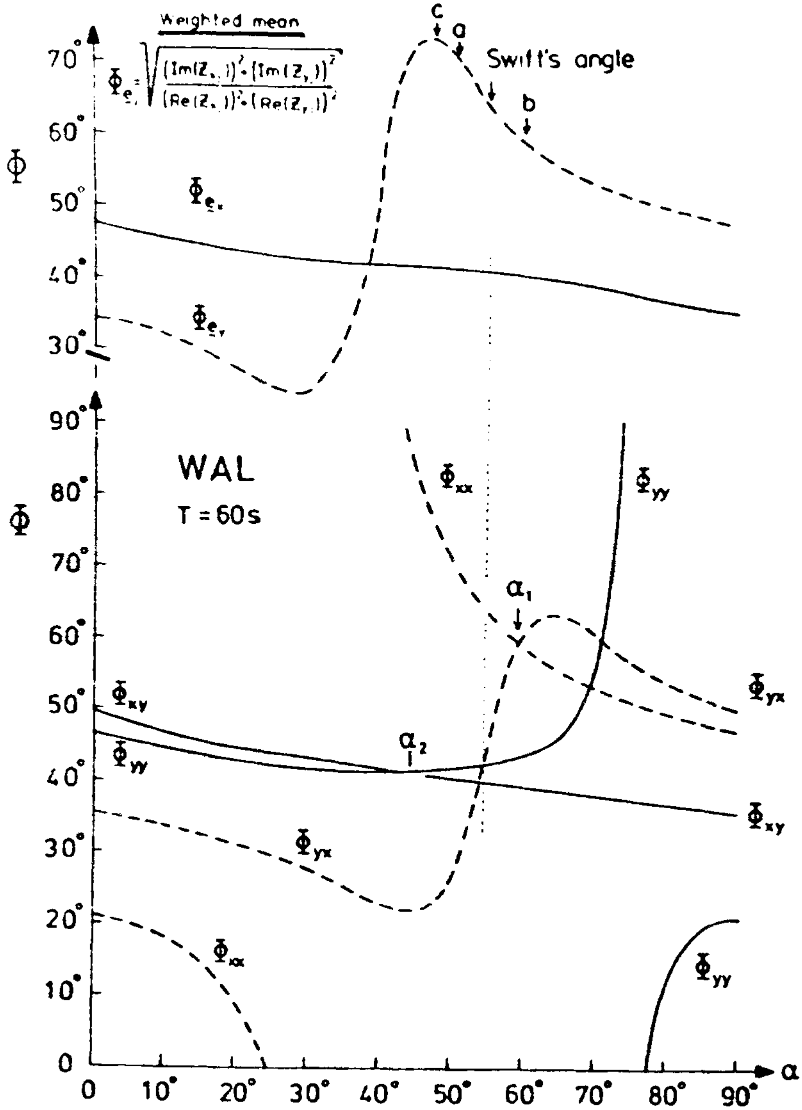

Fig. 10. Phases of the elements of the impedance tensor (bottom) and phases of the 'telluric vectors' of site WAL, $T = 1$ min, at a stepwise coordinate transformation (after Bahr, 1988).

of coordinates is chosen to correspond to α_1 and instead of four impedance phases one employs two phases of telluric currents

$$\tan \varphi_x = \left[\frac{(\mathrm{Im}Z_{xx})^2 + (\mathrm{Im}Z_{yx})^2}{(\mathrm{Re}Z_{xx})^2 + (\mathrm{Re}Z_{yx})^2}\right]^{1/2}$$

$$\tan \varphi_y = \left[\frac{(\mathrm{Im}Z_{xy})^2 + (\mathrm{Im}Z_{yy})^2}{(\mathrm{Re}Z_{xy})^2 + (\mathrm{Re}Z_{yy})^2}\right]^{1/2}$$

which are subsequently examined.

Under the same model, when the frequency is low enough that the inductive response can be neglected, Groom and Bailey (1988) decompose the data to obtain 7 parameters per frequency: regional strike, two parameters describing the effects of the local electric field distortion (twist and shear) and two complex regional impedances.

It should be noted that the approaches to the decomposition of the impedance tensor mentioned above are in mathematical sense less general than the ones developed by Iee and Paulson (1987) and Tzanis (1990a, b). Nevertheless, in a number of cases they may occur more useful from the point of view of geological interpretation.

In this Section we have considered only some of the point transformations of the electromagnetic field and their applications to the interpretation of actual data. While point transformations are indispensable in the analysis of nonsynchronous field records, in the presence of electromagnetic data recorded synchronously at several points (or even masses of points) we can employ interpretation methods based on accurate integral field transformations.

2. Integral Transformations

The solution of many geoelectrical problems involves integral transformations of the field (direct problems, signal processing, etc.). In this Section, however, we will confine ourselves only to the electromagnetic field transformations directly related to data interpretation.

To increase the effectiveness of this process the field recorded synchronously at a certain quantity of points is worth to divide in advance into parts in accordance with their origin. Another fruitful idea based on the integral transformation of the field, is concerned with the analytic continuation of it down from the surface, where the observations are made.

2.1. Division of the Field into Parts and Analytic Continuation

In their monograph Berdichevsky and Zhdanov (1984) expose comprehensively the transformation methods in which the division and continuation operators can be regarded as linear filters performing the following integral transformations:

(a) division of the magnetic field into external and internal parts;
(b) division of the magnetic field into normal and anomalous parts;
(c) division of the magnetic field into surface and deep parts;
(d) separation of the major part of a deep anomaly;
(e) analytic continuation of the magnetic field;
(f) analytic continuation of a deep magnetic anomaly;
(g) analytic continuation of the major part of a deep magnetic anomaly.

In a two-dimensional case, for instance, this set of operators includes two types of the matrix operators $\hat{B}$ and $\hat{C}$.

(1) the operator $\hat{B}$ affects the magnetic field:

$$\bar{H}^T(x',z') = \hat{B}\bar{H}(x,0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{b}(k_x, z')\bar{h}(k_x)e^{-ik_x x'}\,\mathrm{d}k_x = \\ = \int_{-\infty}^{\infty} \bar{G}^b(x'-x,z')\bar{H}(x,0)\,\mathrm{d}x, \tag{14}$$

where H and h are the magnetic field and its spectrum at the surface, respectively:

$$\bar{H} = \begin{bmatrix} H_x \\ H_z \end{bmatrix}, \quad \bar{h} = \begin{bmatrix} h_x \\ h_z \end{bmatrix} = \int_{-\infty}^{\infty} \begin{bmatrix} H_x \\ H_z \end{bmatrix} e^{ik_x x}\,\mathrm{d}x;$$

$\hat{G}^b$ and $\hat{b}$ stand for the kernel of integral transformation and its characteristic, respectively:

$$\hat{G}^b = \begin{bmatrix} G^b_{xx} & G^b_{xz} \\ G^b_{zx} & G^b_{zz} \end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \begin{bmatrix} b_{xx} & b_{xz} \\ b_{zx} & b_{zz} \end{bmatrix} e^{-ik_x x}\,\mathrm{d}k_x;$$

$\bar{H}^T$ is the transformed field:

$$\bar{H}^T = \begin{bmatrix} H^T_x \\ H^T_z \end{bmatrix}.$$

(2) The operator $\hat{C}$ affects the sum vector of the magnetic field and excess current and the transformation has the form:

$$\bar{H}^T(x',z') = \hat{C}\bar{F}(x,0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{c}(k_x,z')\bar{f}(k_x)e^{-ik_x x}\,\mathrm{d}k_x = \\ = \int_{-\infty}^{\infty} \hat{G}^c(x'-x,z')\bar{F}(x,0)\,\mathrm{d}x,$$

where $\bar{F}$ and $\bar{f}$ are the sum vector and its spectrum, respectively:

$$\bar{F} = \begin{bmatrix} H_x \\ I^s_y \\ H_z \end{bmatrix}, \quad \bar{f} = \begin{bmatrix} h_x \\ i^s_y \\ h_z \end{bmatrix} = \int_{-\infty}^{\infty} \begin{bmatrix} H_x \\ I^s_y \\ H_z \end{bmatrix} e^{ik_x x}\,\mathrm{d}x$$

and $\hat{G}^c$, $\hat{c}$ are the kernel of integral transformation and its spectral characteristic, respectively:

$$\hat{G}^c = \begin{bmatrix} G^c_{xx} & G^c_{xy} & G^c_{xz} \\ G^c_{zx} & G^c_{zy} & G^c_{zz} \end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \begin{bmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{zx} & C_{zy} & C_{zz} \end{bmatrix} e^{-ik_x x}\,\mathrm{d}k_x$$

Thus, the division and continuation operators act as linear filters with spectral characteristics b, c and spatial characteristics $\hat{G}^b$, $\hat{G}^c$.

For example, basing on relation (14) we obtain

$$\bar{H}^{e,i} = \hat{B}^{e,i}\bar{H}$$

$$\bar{H}^{a,n} = \hat{B}^{a,n}\bar{H} \qquad \text{etc.,}$$

where $\bar{H}^e, \bar{H}^i$ are, correspondingly, external and internal fields; $\bar{H}^a, \bar{H}^n$ anomalous and normal fields.

Another type of field integral transformations relies on the body of Stratton–Chu-type integrals developed in the monograph of Zhdanov (1988). Within this approach, formulas for the division of the field, recorded over arbitrary surface, may be obtained (Zhdanov and Spichak, 1983). For instance,

$$\bar{E}^a(\bar{r}_0) = 1/2\bar{E}(\bar{r}_0) + \bar{K}_0^e(\bar{r}_0) + \Delta\bar{K}^e(\bar{r}_0)$$

$$\bar{E}^n(\bar{r}_0) = 1/2\bar{E}(\bar{r}_0) - \bar{K}_0^e(\bar{r}_0) - \Delta\bar{K}^e(\bar{r}_0)$$

$$\bar{H}^a(\bar{r}_0) = 1/2\bar{H}(\bar{r}_0) + \bar{K}_0^m(\bar{r}_0) + \Delta\bar{K}^m(\bar{r}_0)$$

$$\bar{H}^n(\bar{r}_0) = 1/2\bar{H}(\bar{r}_0) - \bar{K}_0^m(\bar{r}_0) - \Delta\bar{K}^m(\bar{r}_0),$$

where

$$\bar{K}_0^{e,m}(\bar{r}_0) =$$

$$= \iint_s \left\{ i\omega\mu_0 G_0^{e,m}\left(\frac{r_0}{r'}\right)\left[\bar{n} \times \bar{H}\right] + \left[\operatorname{curl} G_0^{e,m}\left(\frac{r_0}{r'}\right)\right]\left[\bar{i} \times \bar{E}\right]\right\} ds',$$

$$\Delta\bar{K}_0^{e,m}(\bar{r}_0) =$$

$$= \iint_s \left\{ i\omega\mu_0\, \Delta G_0^{e,m}\left(\frac{r_0}{r'}\right)\left[\bar{n} \times \bar{H}\right] + \left[\operatorname{curl} \Delta G^{m,e}\left(\frac{r_0}{r'}\right)\right]\left[\bar{n} \times \bar{E}\right]\right\} ds',$$

(G_0^e and G_0^m are the tensor Green functions of an inhomogeneous medium of the electric and magnetic types, respectively; $\Delta\hat{G}^{e,m} = \hat{G}^{e,m} - \hat{G}_0^{e,m}$; $\hat{G}^{e,m}(\bar{r}_0/\bar{r}')$-Green's function of the homogeneous medium with conductivity $\sigma = \sigma(\bar{r}_0)$.

Using linear transforms of spatial spectra of the field, Shabelyansky (1985) constructed operators performing these transformations in a three-dimensional medium. Zhdanov *et al.* (1987) employed such operators to divide the magnetic field observed on the Voronezh crystalline massif into a normal and anomalous, surface and deep one (Figure 11). In a similar manner, Zhdanova (1986) divided fields measured at the bottom of the sea (within two-dimensional ocean models excited by an H-polarized field). Zhdanov and Shabelyansky (1988) solved the problem of dividing the electromagnetic field taken at the sea bottom into a normal and anomalous parts in the presence of an extraneous field source for a plane model of the Earth. Aslibekyan *et al.* (1986) extended the field division methods (spectral and spatial) developed for natural electromagnetic fields to a transient case, while Frenkel (1983) suggested an algorithm for interpolation of transient fields on the basis of solution of the Stratton–Chu integral equation. Berdichevsky and Yakovlev (1984) derived a pair of integral transforms relating the electric and magnetic components of an anomalous field observed at the surface. The authors discussed the applicability of these transforms to the solution of several interpretation problems.

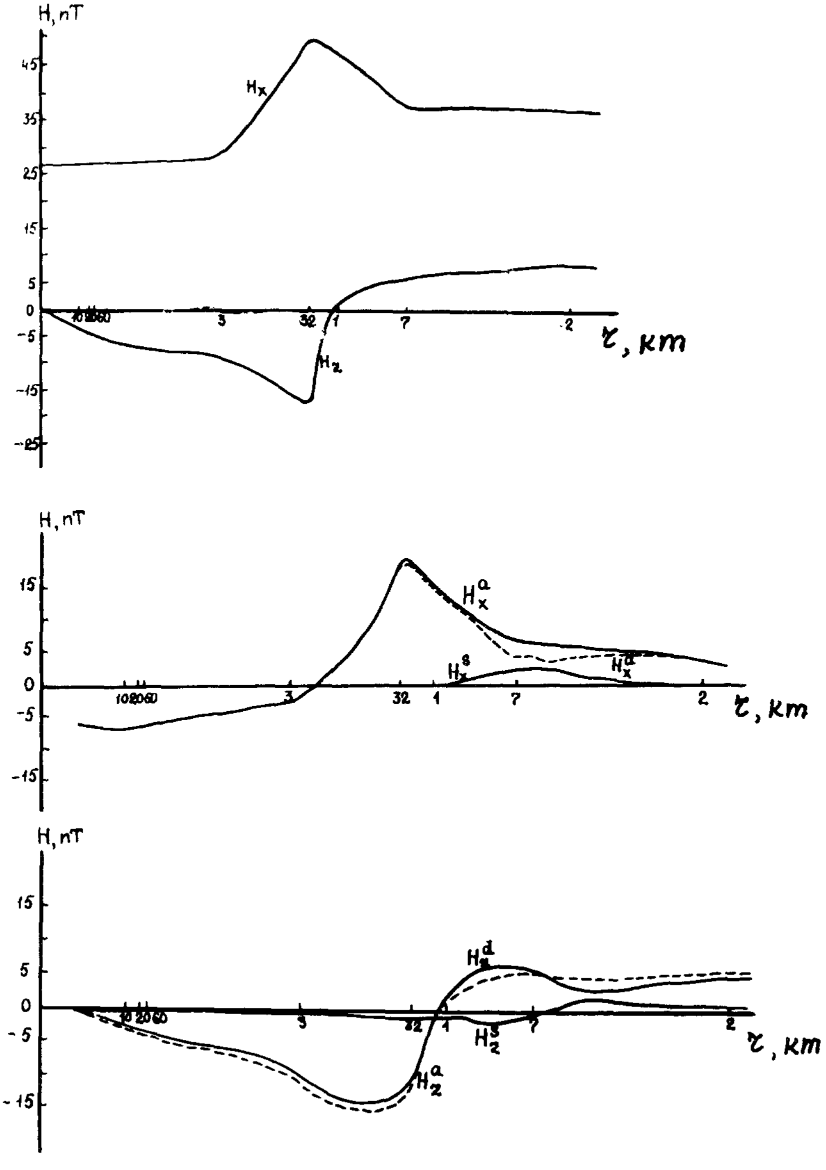

Fig. 11. Geomagnetic fields of VCM anomaly (E-polarization) for $T = 1800$ s: (a) observed magnetic fields H_x and H_z, (b) results of H_x separation: a-anomalous part, s-surface, d-deep; (c) results of H_z separation (after Zhdanov *et al.*, 1987).

2.2. Electromagnetic analogs of seismic methods

The approach to seismic data interpretation known under the name of 'seismic holography' relies on the idea of reversed continuation of wave fields into a lower half-space. A similar approach to electromagnetic data interpretation aimed at geoelectrical imaging of a medium has been proposed by Zhdanov (1981). Then,

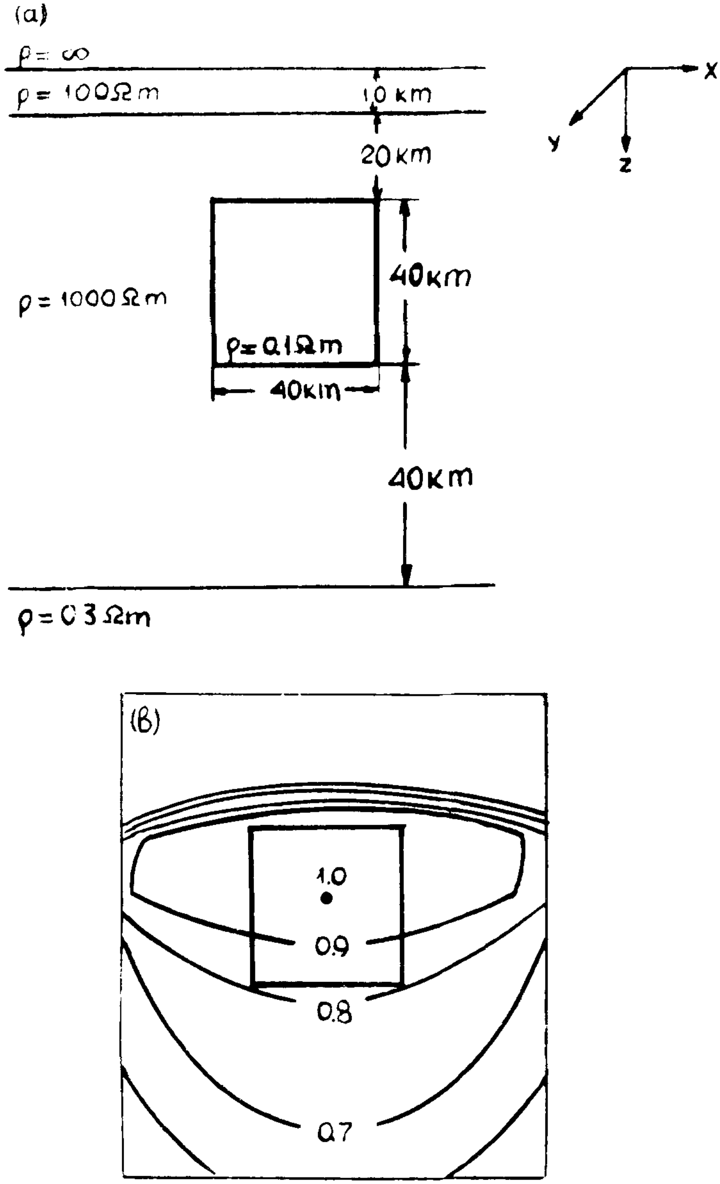

Fig. 12. (a) Two-dimensional model of a horizontally layered medium with a rectangular insert; (b) map of E_y^{m1} isolines. Values of E_y^{m1} are normalized by the value of the field at the point of local extremum (after Velikhov *et al.*, 1987).

Zhdanov and Frenkel (1984) elaborated a method of migration of transient electromagnetic fields and studied the properties of the migration field. In the work of Zhdanov and Frenkel (1985), this method was extended to the frequency domain, while Velikhov *et al.* (1987) put forward a set of migration transformations of transient electromagnetic fields based on the Stratton–Chu-type integrals

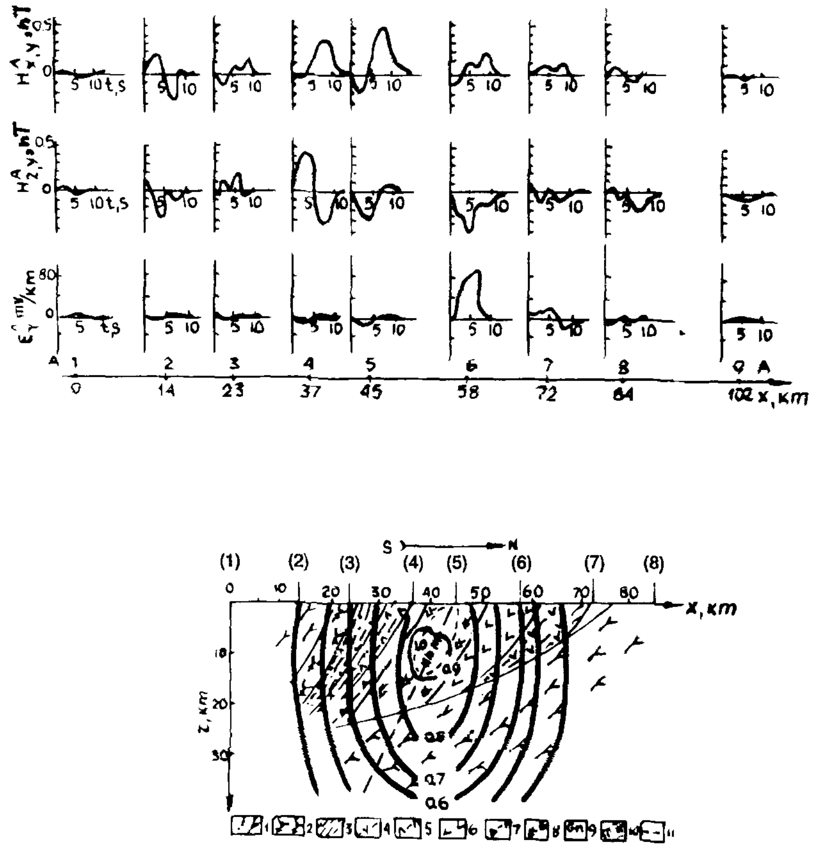

Fig. 13. (a) Graphs of the main part of the anomalous field along profile A-A; (b) map of H_x^{m1} isolines constructed at the moment $t' = 1.2$ s and geological section in the Imandra-Varzuga structure region (after Velikhov *et al.*, 1987).

represented in reversed time. The authors of the latter work studied the properties of these transformations in a two-dimensional case, and in particular, established optimal values of the 'migration constants'. The authors also demonstrated the capabilities of the method using a model example (Figure 12) and interpreting MHD-sounding data on the Imandro–Varzuga zone of the Kola Peninsula (Figure 13).

It should be noted here that the electromagnetic migration method is not the only analog for the approaches employed in seismic surveying and prospecting. Lee *et al.* (1987) suggested a finite difference algorithm for analytic continuation of electromagnetic fields, or the wave equation migration method. Its application to data interpretation is restricted, as indicated by the authors themselves, by gradual variations in conductivity.

The linkage between electromagnetic and seismic methods is not restricted by only analogs. Thus, for example, a direct transfer from the parabolic equation to hyperbolic (using well known integral transforms) turns out to be as fruitful for solution of the direct and inverse problems (Kabanikhin and Martakov, 1988), as for the interpretation of electromagnetic data (Shuman, 1986, 1987). In the latter two works, in particular, a method of 'effective trajectories' was developed, the idea of which is to transform the initial diffusion equation to the wave equation followed by the involvement of propagating discontinuities of its solutions. A method is proposed for numerical realization of this approach and it is illustrated by model examples.

Thus, in synchronous electromagnetic observations we have at our disposal a fairly large arsenal of techniques permitting effective data interpretation (both with a natural and controlled source excitation).

Summary

An analysis of various field transformations and corresponding methods of interpretation has indicated certain trends in this field of investigation. Firstly, there is a trend to hybridization and combination of transformation methods used for interpretation; secondly, data interpretation tends more and more clearly to rely on mathematical methods and procedures adequate to a problem (particularly, this is true of point transformations); thirdly, on transition from one- to two- and three-dimensional interpretations we obviously need a better insight into the behavior of electromagnetic fields and their transformations in typical two-dimensional and particularly in three-dimensional media; and finally, with the advent of computer-assisted analysis the concept of data interpretation undergoes changes – it becomes more Science than Art.

Acknowledgements

The author is thankful to Prof. M. S. Zhdanov whose books have exerted great influence on him and facilitated the preparation of the present review paper. Thanks are also to Prof. U. Schmucker for helpful discussion of the manuscript.

The author acknowledges all colleagues who have sent him the reprints of their papers and preprints. Valuable suggestions made by Prof. G. Fischer and two referees helped to improve the initial version of the paper. The reviewer expresses his deep gratitude to these scientists.

References

Agheev, V. V.: 1986, 'Interpretation of One-Dimensional Impedance Transfer Characteristics in the Time Domain', in M. Zhdanov (ed.), *Fundamental Problems of Marine Electromagnetic Investigations*, ISMIRAN, Moscow, pp. 57–71.

Aslibekyan, S. C., Berdichevsky, M. N., Zhdanov, M. S., and Frenkel, M. A.: 1986, 'Analysis of Electromagnetic Fields in the Time Domain', Preprint 44 (658), IZMIRAN, Moscow.

Bahr, K.: 1988, 'Interpretation of the Magnetotelluric Impedance Tensor: Regional Induction and Local Telluric Distortion', *J. Geophys.* **62**, 119–127.

Bailey, R. C., Edwards, R. N., Garland, G. D., Kurtz, R., and Pitcher, D.: 1974, 'Electrical Conductivity Studies Over a Tectonically Active Area in Eastern Canada', *J. Geomagn. Geoelectr.* **26**, 125–146.

Banks, R. J.: 1986, 'The Interpretation of the Northumberland Trough Geomagnetic Variation Anomaly Using Two-Dimensional Current Models', *Geophys. J. R. Astr. Soc.* **87**, 595–616.

Banks, R. J. and Beamish, D.: 1984, 'Local and Regional Induction in the British Isles', *Geophys. J. R. Astr. Soc.* **79**, 539–553.

Beamish D.: 1982, 'The Time-Dependence of Electromagnetic Response Functions', *Geophys. Surveys* **4**, 405–434.

Beamish, D.: 1985, 'The Frequency Charcteristics of Anomalous Vertical Fields Observed in the British Isles', *J. Geophys.* **57**, 207–216.

Beamish, D.: 1986, 'Geoelectric Structural Dimensions From Magnetotelluric Data: Methods of Estimation, Old and New', *Geophysics* **51**, 1298–1309.

Beamish, D.: 1987, 'A Simple Function for Mapping Induced Currents', *Geophys. J. R. Astr. Soc.* **90**, 485–494.

Beamish, D. and Banks, R. J.: 1983, 'Geomagnetic Variation Anomalies in Northern England: Processing and Presentation of Data from a Non-Simultaneous Array', *Geophys. J. R. Astr. Soc.* **75**, 513–539.

Berdichevsky, M. N.: 1968, 'Electrical Prospecting by the Magnetotelluric Profiling Technique', Nedra, Moscow.

Berdichevsky, M. N. and Dmitriev, V. I.: 1976, 'Basic Principles of Interpretation of Magnetotelluric Sounding Curves', in A. Adam (ed.), *Geoelectric and Geothermal Studies*, KAPG Geophysical Monograph, Acad. Kiado, Budapest, pp. 165–221.

Berdichevsky, M. N., Dmitriev, V. I., Lebedeva, N. A., and Barashkov, I. S.: 1984a, 'Magnetotelluric Soundings of the Crust Conducting Zones, *IAN SSSR F Z* **9**, 50–56.

Berdichevsky, M. N., Golubtsova, N. S., and Debabov, A. S.: 1984b, 'Quasi-Two-Dimensionality of Near-Surface Structures', *IAN SSSR F Z* **2**, 95–102.

Berdichevsky, M. N., Golubtsova, N. S., and Debabov, A. S.: 1984c, 'Magnetotelluric Soundings above a Three-Dimensional Depression', *IAN SSSR F Z* **12**, 80–88.

Berdichevsky, M. N., Moroz, I. P., Kobzova, V. M., and Bilinsky, A. I.: 1987, *Physical Modeling in Geoelectrics*, Naukova Dumka, Kiev.

Berdichevsky, M. N. and Vanyan, L. L.: 'Perspectives of the Deep Marine MT-Soundings', *IAN SSSR F Z* **11**, 51–56.

Berdichevsky, M. N., Vanyan, L. L., and Fainberg, E. B.: 1969, 'Magnetovariational Sounding using Spatial Field Derivatives', *Geomag. Aer.* **9**, 369–371.

Berdichevsky, M. N. and Yakovlev, A. G.: 1984, 'Magnetotelluric Analog of the Kertz Formulas', *Geomag. Aer.* **24**, 805–811.

Berdichevsky, M. N. and Zhdanov, M. S.: 1984, *Advanced Theory of Deep Geomagnetic Sounding*, Elsevier, Amsterdam.

Berdichevsky, M. N., Zhdanova, ON. N. and Yakovlev, A. G.: 1983, 'Electromagnetic Anomalies at the Ocean Bottom', in M. Zhdanov (ed.), *Problems of Investigations of Electromagnetic Fields in Aquatic Media*, IZMIRAN, Moscow, pp. 16–25.

Bibby, H. M.: 1977, 'The Apparent Resistivity Tensor', *Geophysics* **42**, 1258–1261.

Bostic, F. X.: 1977, 'A Simple Almost Exact Method of MT-analysis', in *Workshop on Electrical Methods in Geothermal Research*, U.S. Geological Survey, Contr. No. 14080001-8-359.

Cagniard, L.: 1953, 'Basic Theory of the Magnetotelluric Method of Geophysical Prospecting', *Geophysics* **18** 605–635.

Chamalaun, F. H., Prasad, S. N., Lilley, F. E. M., Srivastava, B. J., Singh, B. P., and Arora, B. R.: 1987, 'On the Interpretation of the Distinctive Pattern of Geomagnetic Induction Observed in Northwest India', *Tectonophysics* **140**, 247–255.

Chave, A. D. and Booker, J. R.: 1987, 'Electromagnetic Induction Studies', *Rev. Geophys.* **25**, 989–1003.

Chen, P. E. and Fung, P. C. W.: 1985, 'Significance of the Sing Changing of the Imaginary Arrows in Geomagnetic Induction Investigation', *Geophys. J. R. Astr. Soc.* **80**, 257–263.

Chen, P. E. and Fung, P. C. W.: 1986, 'The Frequency Response of Two-Dimensional Induction Anomalies Revisited', *J. Geopmagn. Geoelectr.* **38**, 873–881.

Chen, P. E. and Fung, P. C.: 1988a, 'On the Behaviour of the Imaginary Parkinson Arrows near the Anomalous Conductor-Host Medium Interface', *Phys. Earth Planet. Inter.* **50**, 195–198.

Chen, P. F. and Fung, P. C. W.: 1988b, 'Frequency Response of the Transfer Functions of the Current Channelling between Two Oceans', *Geomag. Geoelectr.* **40**, 335–355.

Counil, J. L., Le Mouel, J. L., and Menvielle, M.: 1986, 'Associate and Conjugate Directions Concepts in Magnetotellurics', *Annales Geophysicae* **4B**, 115–130.

Counil, J. L., Menvielle, M., and Le Mouel, J. L.: 1987, 'Upper Mantle Lateral Heterogeneities and Magnetotelluric Daily Variation Data', *PAGEOPH* **125**, 319–340.

Dmitriev, V. I. and Berdichevsky, M. N.: 1979, 'The Fundamental Model of Magnetotelluric Sounding', *Proc. IEEE* **67**, 1034–1044.

Eggers, D. E.: 1982, 'An Eigenstate Formulation of the Magnetotelluric Impedance Tensor', *Geophysics* **47**, 1204–1214.

Fischer, G.: 1985, 'Some Remarks on the Behavior of the Magnetotelluric Phase', *Geophys. Prosp.* **33**, 716–722.

Fischer, G. and Schnegg, P.-A.: 1986, 'Modelling Active Audio-Magnetotelluric Data', *J. Geophys.* **59**, 49–55.

Fischer, G. and Weaver, J. T.: 1986, 'Theoretical Investigation of the Ocean-Coast Effect at a Passive Continental Margin', *Phys. Earth Planet. Inter.* **42**, 246–254.

Fiskina, M. V., Zinger, B.Sh., and Fainberg, E. B.: 1985, 'Applicability of the Spatial Derivation Method to Deep Electromagnetic Investigations with Controllable Field Sources', in M. Zhdanov (ed.), *Electromagnetic Soundings of the Earth*, IZMIRAN, Moscow, pp. 105–112.

Frenkel, M. A.: 1983, 'One Algorithm of Interpolation of a Transient Electromagnetic Field', in M. Zhdanov (ed.), *Mathematical Modeling of Electromagnetic Fields*, IZMIRAN, Moscow, pp. 135–142.

Goubau, W. M., Gamble, T. D. and Clarke, J.: 1978, 'Magnetotelluric Data Analysis Removal of Bias', *Geophysics* **43**, 1157–1166.

Gough, D. I. and Ingham, M. R.: 1983, 'Interpretation Methods for Magnetometer Arrays', *Rev. Geoph. and Space Phys.* **21**, 805–827.

Gregori, G. P. and Lanzerotti, L. J.: 1980, 'Geomagnetic Depth Sounding by Induction Arrow Representation: a Review', *Rev. Geophy. and Space Phys.* **18**, 203–209.

Groom, R. W. and Bailey, R. C.: 1990, 'Decomposition of magnetotelluric impedance tensors in the presence of local three-dimensional galvanic distortion', *J. Geophys. Res.* (in press).

Hermance, J. F.: 1982, 'The Asymptotic Response of Three-Dimensional Basin Offsets to Magnetotelluric Fields at Long Periods: the Effect of Current Channeling', *Geophysics* **47**, 1562–1573.

Iliceto, V., Malaguti, R., and Santarato, G.: 1986, 'A Model Study of Telluric Fields in Two-Dimensional Structures', *Geophys. J. R. Astr. Soc.* **86**, 399–412.

Ingham, M. R.: 1988, 'The Use of Invariant Impedances in Magnetotelluric Interpretation', *Geophys. J.* **92**, 165–169.

Ingham, M. R., Bingham, D. K., and Gough, D. I.: 1983, 'A Magnetovariational Study of a Geothermal Anomaly', *Geophys. J. R. Astr. Soc.* **72**, 597–618.

Ingham, M. R., Gough, D. I., and Parkinson, W. D.: 1987, 'Models of Conductive Structure Under the Canadian Cordillera', *Geophys. J. R. Astr. Soc.* **88**, 477–485.

Jones, A. G.: 1983, 'On the Equivalence of the Niblett and Bostick Transformations in the Magnetotelluric Method', *J. Geophys.* **53**, 72–73.

Jones, A. G.: 1986a, 'Parkinson's Pointers Potential Perfidy!', *Geophys. J. R. Astr. Soc.* **87**, 1215–1224.

Jones, A. G.: 1986b, 'On the Use of Line Current Analogues in Geomagnetic Deep Sounding', *J. Geophys.* **60**, 56–62.

Kabanikhin, S. I. and Martakov, S. V.: 1988, 'Investigation of the Projection-Difference Method of Solution of the Direct and Inverse Geoelectrical Problems', Preprint 13, Inst. Math. SO AN SSSR, Novosibirsk.

Kao, D. and Orr, D.: 1982, 'Magnetotelluric Studies in the Market Weighton Area of Eastern England', *Geophys. J. R. Astr. Soc.* **70**, 323–337.

Kaufman, A. A. and Keller, G. V.: 1983, *Frequency and Transient Soundings*, Elsevier, Amsterdam.

Kharin, E. P.: 1982, 'Changes in Transfer Functions with Time', *Geophys. Surveys* **4**, 455–466.

Korja, T., Zhang, P., and Pajunpaa, K.: 1986, 'Magnetovariational and Magnetotelluric Studies of the Oulu Anomaly on the Baltic Shield in Finland', *J. Geophys.* **59**, 32–41.

Kovtun, A. A. and Vardanyants, I. L.: 1985, 'Models of Conducting Form', in M. Zhdanov (ed.), *Electromagnetic Soundings of the Earth*, IZMIRAN, Moscow, pp. 133–141.

Kryukova, L. G.: 1983, 'Approximate Method of Sounding Data Interpretation by Field Establishment in the Near Zone', in M. Zhdanov (ed.), *Mathematical Modelling of Electromagnetic Fields*, IZMIRAN, Moscow, pp. 214–222.

Labson, V. F. and Becker, A.: 1987, 'Natural-Field and Very Low-Frequency Tipper Profile Inetrpretation of Contacts', *Geophysics* **52**, 1697–1707.

Lam, H. L., Jones, F. W., and Hibbs, R. D.: 1982, 'The Response of Perturbation and Induction Arrows to a Three-Dimensional Buried Anomaly', *Geophysics* **47**, 51–59.

La Torraca, G. A., Madden, T. R., and Korringa, J.: 1986, 'An Analysis of the Magnetotelluric Impedance for Three-Dimensional Conductivity Structures', *Geophysics* **51**, 1819–1829.

Lee, S., McMechan, G. A., and Aiken, C. L. V.: 'Phase-Field Imaging: the Electromagnetic Equivalent of Seismic Migration', *Geophysics* **52**, 678–693.

Le Vyet Zy Khyong, and Berdichevsky, M. N.: 1984, 'The generalization of the Schmucker method for the MTS interpretation', *IAN F Z* **2**, 108–114.

Menvielle, M. and Szarka, L.: 1986, 'Distortions of Electromagnetic Fields: Topographic and Man-Made (Review paper)', in *Proc. of the VIII IAGA Workshop on Electromagnetic Induction*, Neuchâtel, Switzerland, 24–31 August.

Menvielle, M. and Tarits, P.: 1986, '2-D or 3-D Interpretation of Conductivity Anomalies: Example of the Rhine-Graben Conductivity Anomaly', *Geophys. J. R. Astr. Soc.* **84**, 213–226.

Meyer, J.: 1982, 'Induktions-Pfeile in Reellen und Komplexen', in *Elektromagnetische Tiefenforschung*, Neuestadt, West Germany, pp. 95–114.

Molochnov, G. V.: 1968, 'Magnetotelluric Sounding Data Interpretation by Means of Effective Depth of Electromagnetic Field Penetration', *IAN SSSR F Z* **9**, 88–94.

Molochnov, G. V. and Sekrieru, V. G.: 1976, 'Determination of Parameters of the Geoelectric Section from Effective Resistivity and Field Penetration Depth', *IAN SSSR F Z* **2**, 64–71.

Murakami, Y.: 1985, 'Two Representations of the Magnetotelluric Sounding Survey', *Geophysics* **50**, 161–164.

Niblett, E. R. and Sayn-Wittgenstein, C.: 1960, 'Variation of Electrical Conductivity with Depth by the Magnetotelluric Method', *Geophysics* **25**, 998–1008.

Nienaber, W., Hebert, D., and Dosso, H. W.: 1983, 'Induction Arrows for a Buried Conducting Plate', *Phys. Earth Planet. Inter.* **32**, 306–311.

Newmann, G. A., Anderson, W. L., and Hohmann, G. W.: 1987, 'Interpretation of Transient Electromagnetic Soundings Over Three-Dimensional Structures for the Central-Loop Configuration', *Geophys. J. R. Astr. Soc.* **89**, 889–914.

Pajunpaa, K.: 1986, 'Magnetometer Array Studies in Southeastern Finland on the Baltic Shield', *J. Geophys.* **59**, 23–31.

Park, S. K.: 1985, 'Distortion of Magnetotelluric Sounding Curves by Three-Dimensional Structures', *Geophysics* **50**, 785–797.

Parkinson, W. D.: 1959, 'Direction of Rapid Geomagnetic Fluctuations', *Geophys. J. R. Astr. Soc.* **2**, 1–14.

Parkinson, W. D.: 1990, 'The Analysis of Single Site Induction Data', *Phys. Earth Planet. Inter.* (in press).

Porstendorfer, G.: 1987, 'Perspectives to Extract Information on Electrical Conductivity, Induced Polarization and Magnetic Susceptibility of Geological Media from Variations of the Natural Electromagnetic Field of the Earth', HHI Report 21, 115–121.

Raiche, A. P.: 1983, 'Comparison of Apparent Resistivity Functions of Transient Electromagnetic Methods', *Geophysics* **48**, 787–789.

Raiche, A. P. and Gallagher, R. G.: 1985, 'Apparent Resistivity and Diffusion Velocity', *Geophysics* **50**, 1628–1633.

Raiche, A. P. and Spies, B. R.: 1981, 'TEM Master Curves for Interpretation of Two Layered Earths', *Geophysics* **46**, 53–64.

Ranganayaki, R. P.: 1984, 'An Interpretive Analysis of Magnetotelluric Data', *Geophysics* **49**, 1730–1748.

Rokityansky, I. I.: 1982, *Geoelectromagnetic Investigation of the Earth's Crust and Mantle*, Springer Verlag, Berlin-Heidelberg-N.Y.

Schmucker, U.: 1970, 'Anomalies of Geomagnetic Variations in the South-Western United States', *Bull. Scripps. Inst. Ocean.* **13**, 1–165.

Schmucker, U.: 1987, 'Substitute Conductors for Electromagnetic Response Estimates', *PAGEOPH* **125**, 341–367.

Schnegg, P.-A., Fischer G., Le Quang, B. V., and Ranieri, G.: 1987, 'A Magnetotelluric Study in the Campidano Graben of Sardinia', *J. Geophys.* **61**, 30–38.

Schnegg, P.-A., Fischer, G., Le Quang, B. V., and Weaver, J. T.: 1986, 'Investigation of a Buried Vertical Fault with Natural and Controlled Source AMT', *Annal. Geophys.* **4B**, 139–144.

Sekrieru, V. G., Molochnov, G. V., and Kozulin, Yu. N.: 1988, 'Possibilities of Magnetotelluric Sounding Data Interpretation from Phase Curves', *IAN SSSR F Z* **1**, 100–103.

Shabelyansky, S. V.: 1985, 'Spatial Analysis of Three-Dimensional Electromagnetic Fields', in M. Zhdanov (ed.), *Electromagnetic Soundings of the Earth*, IZMIRAN, Moscow, pp. 88–95.

Shuman, V. N.: 1986, 'Effective Trajectories in Electromagnetic Techniques Involving Artificial Field Excitation. Frequency Domain. Theoretical issues', *Geophyzicheskiy zhurnal* **8**, 49–55.

Shuman, V. N.: 1987, 'Electromagnetic Soundings in the Transient Regime: Geometrical Approach', *Geophyzicheskiy zhurnal* **4**, 56–65.

Sidorov, V. A., Gubatenko, V. P., and Glechikov, V. A.: 1977, *Establishment of the Electromagnetic Field in Inhomogeneous Media (as Applied to Geophysical Investigations)*, Saratov University, Saratov.

Spies, B. R. and Eggers, D. E.: 1986, 'The Use and Misuse of Apparent Resistivity in Electromagnetic Methods', *Geophysics* **51**, 1462–1471.

Spitz, S.: 1985, 'The Magnetotelluric Impedance Tensor Properties with Respect to Rotations', *Geophysics* **50**, 1610–1617.

Sule, P. O. and Hutton, V. R. S.: 1986, 'A Broad-Band Magnetotelluric Study in Southeastern Scotland. Data acquisition, Analysis and One-Dimensional Modelling', *Annal. Geophys.* **2**, 145–156.

Summers, D. M.: 1981, 'Interpreting the Magnetic Fields Associated with Two-Dimensional Induction Anomalies', *Geophys. J. R. Astr. Soc.* **65**, 535–552.

Svetov, B. S., Karinsky, S. D. and Agheev, V. V.: 1983, 'Application of Focusing Transformations to the Solution of a One-Dimensional Inverse Problem of Magnetotelluric Sounding', in M. Zhdanov (ed.), *Mathematical Modelling of Electromagnetic Fields*, IZMIRAN, Moscow, pp. 122–134.

Svetov, B. S., Karinsky, S. D., Agheev, V. V., and Sorokin, M. O.: 1983, 'Focussing Transformations of Magnetotelluric Sounding Data', in M. Zhdanov (ed.), *Problems of Investigation of Electromagnetic Fields in Aquatic Media*, IZMIRAN, Moscow, pp. 161–167.

Tikhonov, A. N.: 1950, 'Determination of Electrical Characteristics of Deep Layers of the Earth's Crust', *DAN SSSR* **73**, 295.

Tikhonov, A. N.: 1965, 'Mathematical Substantiation of the Theory of Electromagnetic Soundings', *Zh. Vych. Mat. Fiz.* **5**, 545–548.

Ting, S. C. and Hohmann, G. W.: 1981, 'Integral Equation Modelling of Three-Dimensional Magnetotelluric Response', *Geophysics* **46**, 182–197.

Tzanis, A.: 1990a, 'A Characteristic State Formulation of the Magnetotelluric Tensors in Three Dimensions. Part. I: A Generalized Rotation Analysis Based on a Group Theoretical Approach', *Geophys. J.* (in press).

Tzanis, A.: 1990b, 'A Characteristic State Formulation of the Magnetotelluric Tensors in Three Dimensions. Part 2: Application of the Generalized Rotation Analysis', *Geophys. J.* (in press).

Vaghin, S. A. and Kovtun, A. A.: 1981, 'Features of the Amplitude and Phase Anomalies of Magnetotelluric Parameters Over Three-Dimensional Inhomogeneities', in M. Zhdanov (ed.), *Study of the Deep Structure of the Earth's Crust and Upper Mantle in Seas and Oceans Using Electromagnetic Techniques*, IZMIRAN, Moscow, pp. 155–160.

Vanyan, L. L.: 1965, *Fundamentals of Electromagnetic Soundings*, Nedra, Moscow.

Vanyan, L. L., Berdichevsky, M. N., Vasin, N. D., Okulyesskiy, B. A., and Shilovskiy P. P.: 1980, 'On the Normal Geoelectric Profile', *Phys. Solid Earth* **16**, 131–133.

Vanyan, L. L. and Shilovsky, P. P.: 1983, *Deep Electrical Conductivity of Oceans and Continents*, Nauka, Moscow.

Velikhov, Ye. P., Zhdanov, M. S. and Frenkel, M. A.: 1987, 'Interpretation of MHD-Sounding Data from the Kola Peninsula by the Electromagnetic Migration Method', *Phys. Earth and Planet. Inter.* **45**, 149–160.

Vozoff, K.: 1972, 'The Magnetotelluric Method in the Exploration of Sedimentary Basins', *Geophysics* **37**, 98–141.

Wannamaker, P. E., Hohmann, G. W., and Ward, S. H.: 1984, 'Magnetotelluric Responses of Three-Dimensional Bodies in Layered Earths', *Geophysics* **49**, 1517–1533.

Weidelt, P.: 1972, 'The Inverse Problem of Geomagnetic Induction', *J. Geophys.* **38**, 257–289.

Weidelt, P.: 1985, 'Construction of Conductance Bounds from Magnetotelluric Impedances', *J. Geophys.* **57**, 191–206.

Weidelt, P., Muller, W., Losecke, W., and Knodel, K.: 1980, 'Die Bostick Transformation', in V. Haak and J. Homilius (eds.), *Protokoll uber das Kolloqium Electromagnetische Tiefenforschung*, Berlin-Hannover, pp. 227–230.

Wiese, H.: 1962, 'Geomagnetische Tiefenetellurik', *Geoph. Pura e Appl.* **52**, 83–103.

Word, D. R., Smith, H. W., and Bostick, F. X.: 1970, 'An Investigation of the Magnetotelluric Tensor Impedance Method', Tech. rep. 82, Electr. Geophys. Res. Lab., Univ. of Texas at Austin.

Zhdanov, M. S., Maksimov, V. M., Gruzdev, V. N., and Shabelyansky, S. V.: 1987, 'Results of Separation of the Field of Bay Disturbances of the Voronezh Cristalline Massifs', in M. Zhdanov (ed.), *Methods of Solution of Direct and Inverse Geoelectrical Problems*, IZMIRAN, Moscow, pp. 100–107.

Zhdanov, M. S. and Spichak, V. V.: 1983, 'Stratton–Chu-Type Integrals for Inhomogeneous Media and Some of Their Applications to Geoelectrical Problems', in M. Zhdanov (ed.), *Mathematical Modelling of Electromagnetic Fields*, IZMIRAN, Moscow, pp. 4–25.

Zhdanov, M. S. and Shabelyansky, S. V.: 1988, 'Separation of the Electromagnetic Field at the Bottom of Seas and Oceans into Normal and Anomalous Ones with due Refence to a Hydrodynamic Source', Preprint 25 (779), IZMIRAN, Moskow.

Zinger, B.Sh. and Fainberg, E. B.: 1985, 'Electromagnetic Induction in Inhomogeneous Thin Layers', IZMIRAN, Moscow.