MODERN TRENDS IN THE SOLUTION OF FORWARD AND INVERSE 3D ELECTROMAGNETIC INDUCTION PROBLEMS

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Abstract. The main results obtained during the last 5–8 yr in the solution of forward and inverse problems of 3D induction studies are summarized. The up-to-date status of 3D modelling is presented and prospective improvements in the formulation and numerical solution of forward problems are discussed. Approximate techniques and practical aspects of 3D modelling are specially considered.

The general scheme of 3D interpretation of electromagnetic geophysical observations is outlined and realistic formalized approaches to solving 3D inverse problems, namely direct inversion and formalized model fitting, are studied.

0. Introduction

The significant progress in the development of electromagnetic (EM) array observations achieved during the last ten years made it necessary to elaborate the system of EM data interpretation in 3D environments. Theoretical studies of 3D problems have been carried out during the last twenty years or more, but not until a few years ago did these works become of practical importance and serious applied researches were begun. Now many publications concerned with 3D induction studies are well known. Still, we have no generalized reviews on this subject (except brief ones like Ward, 1980), but in many special papers meaningful discussions of the problems and outlines of 3D techniques have been presented (Weidelt, 1975a; Vasseur and Weidelt, 1977; Hohmann, 1978; Banks, 1979; Weaver, 1979; Dawson and Weaver, 1979, Fainberg, 1980; Zhdanov, 1980; Pridmore *et al.*, 1981; Lee *et al.*, 1981; Stodt *et al.*, 1981; Berdichevsky *et al.*, 1982; etc).

It looks quite unrealistic to consider in detail the significant achievements in all of the numerous branches of 3D induction studies in a single review. Therefore the subject of this paper is restricted to the study of interpretation techniques of *time-harmonic* EM field anomalies caused by local and regional 3D conductivity structures. Moreover, attention is paid mainly to the theoretical and computational aspects of the rational formulation and effective solution of 3D forward and inverse problems. These problems form the basis of the interpretation theory and, as it can be seen from the experience of 2D induction studies, their intensive development produces the evident progress in elaboration of the whole interpretation system. It is not surprising that these problems are prevalent in the majority of publications of recent years.

It should be noted that general ideas of inhomogeneous media interpretation theory, well illustrated in 2D applications, can be expressed in the main in 3D formulations. This has been shown clearly in the last monographs of Berdichevsky and Zhdanov (1981, 1983). However, we have usually enough difficulties to overcome in the theoretical analysis and at the stage of numerical considerations. Really, in most 3D

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algorithms, due to the vectorial description of EM fields and the increased dimension of coordinate space, the computational expenses are much greater than for their 2D analogues. So it is practically impossible for our limited computer resources to use a number of techniques standard in 2D interpretation. Therefore, to construct the methods of 3D interpretation, we really have to optimize theoretical formulations and computational algorithms in the search for effective approximate techniques.

The most rapid progress has been seen currently in the field of 3D modelling and the last 2–3 yr have witnessed many important achievements. So the first and most detailed part of this review contains the discussion of the main advances and problems in the solution of 3D forward problems. In the second part, the first results of the formalized solution of the 3D inverse problems are considered.

1. Modelling of EM Anomalies Caused by 3D Conductivity Structures

In the sixties, 3D modelling capabilities in induction studies were practically limited only to the use of analytical representations derived for the simplest situations (e.g. a spheroid or an ellipsoid in a conducting space or half-space). In the early seventies, the development of general approaches for the solution of 3D forward problems was begun (Dmitriev and Zakharov, 1970; Zakharov and Ilyin, 1970; Tabarovsky, 1972b, 1975; Jones and Pascoe, 1972; Raiche, 1974; Lines and Jones, 1973; Weidelt, 1975a; Hohmann, 1975). First numerical results of that kind appeared in the last three papers. These experiments demonstrated the possibility of the accurate study of 3D models; however, it became clear that such computations needed the best computer resources. Therefore, the main task for the next years was to improve considerably the effectiveness of modelling techniques to make them a common practical tool.

The further development of the mathematical foundations of 3D modelling was carried out in two main directions: (i) the integral equation approach, (ii) the direct solution of differential boundary value problems. To synthesize the advantages of these techniques, a third approach of hybrid modelling schemes was originated.

Though we are not very far now from the mass use of 3D models in the practice of EM investigations, enough theoretical, computational and technological problems still exist in the field of numerical modelling.

These problems will be discussed below in Sections 1.1 to 1.5 of the paper. The progress observed in mathematical modelling was supplemented by the advances in analogue scale modelling, discussed in Section 1.6.

1.1. Integral Equation Method (IE)

While the first results in this field were obtained by means of the surface integral equations (Dmitriev and Zakharov, 1970; Tabarovsky, 1972b, 1975), the volume integral equation technique was later found to be more suitable for the description of complicated geoelectric structures and became the main subject of further studies. Probably the clearest and most complete formulations of this technique were presented by Weidelt (1975a), Hohmann (1975), Dmitriev and Farsan (1980), Hvozdara (1981a, b), Das and Verma (1981a, 1982), Wannamaker and Hohmann (1982).

Though IE algorithms were usually presented in a rather general form, practically all the numerical calculations published were performed for the simplest model of a conducting brick submerged in a homogeneous half-space (Weidelt, 1975a; Hohmann, 1975, 1978; Cauterman *et al.*, 1979; Ting and Hohmann, 1981; etc). Only during the last 2 yr did the first results for the brick in a two-layered earth appear (Hvozdara, 1981b; Wannamaker and Hohmann, 1982; Das and Verma, 1982).

1.1.1. *3D Volume Integral Equation and Its Discrete Formulation*

Let us consider a geoelectrical model (Figure 1) consisting of a 1D normal conductivity distribution $\sigma_n(\mathbf{r}) = \sigma_n(z)$ and an arbitrary anomalous structure $\Delta\sigma$ which is zero outside the finite and, in general, multiple region V:

$$\sigma(\mathbf{r}') = \begin{cases} \sigma_n(\mathbf{r}') + \Delta\sigma(\mathbf{r}'), & \mathbf{r}' \varepsilon V, \\ \sigma_n(\mathbf{r}'), & \mathbf{r}' \not\varepsilon V. \end{cases}$$

The vector integral equation for the electric field in the region V can be presented as follows (Weidelt, 1975a):

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \mathbf{E}^n(\mathbf{r}) + \iiint_V \hat{G}^E(\mathbf{r},\mathbf{r}')\,\mathbf{J}_v(\mathbf{r}')\,\mathrm{d}V_{\mathbf{r}'}, \\ \mathbf{J}_v(\mathbf{r}') &= \Delta\sigma(\mathbf{r}')\,\mathbf{E}(\mathbf{r}'), \quad \mathbf{r}\,\varepsilon\,V, \quad \mathbf{r}'\,\varepsilon\,V; \end{aligned} \tag{1.1}$$

here $\mathbf{E}^n$ and $\hat{G}^E$ are the electric field and the electric Green's tensor for the normal model.

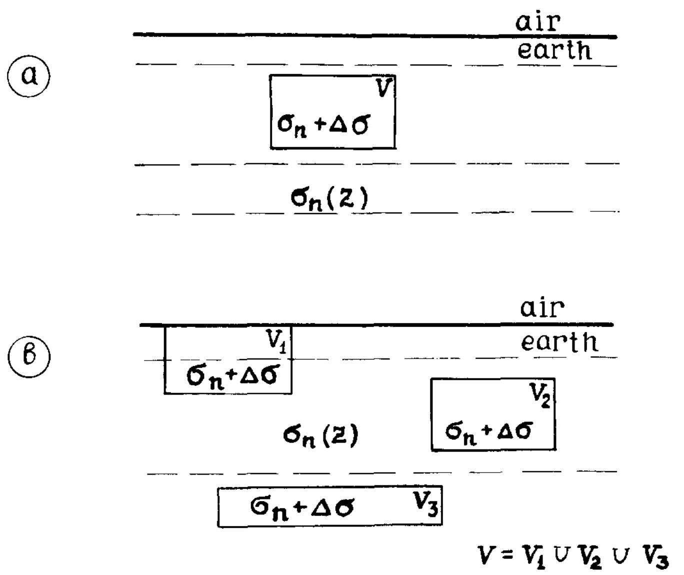

Fig. 1. Geoelectric structure in the formulation of IE modelling problems.

When Equation (1.1) is solved, the field outside V can be determined by means of the following integral formulae:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^n(\mathbf{r}) + \iiint_V \hat{G}^E(\mathbf{r},\mathbf{r}')\,\mathbf{J}_v(\mathbf{r}')\,\mathrm{d}V_{r'},$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^n(\mathbf{r}) + \iiint_V \hat{G}^H(\mathbf{r},\mathbf{r}')\,\mathbf{J}_v(\mathbf{r}')\,\mathrm{d}V_{r'}, \quad \mathbf{r} \notin V, \quad \mathbf{r}' \in V; \tag{1.2}$$

where in the second formula $\mathbf{H}^n$ and $\hat{G}^H$ are the magnetic field and the magnetic Green's tensor for the normal model.

The traditional approach to the numerical solution of Equation (1.1) is to subdivide the region V into a set of elementary cells $\{V_k, k = \overline{1, K}\}$, where some local parameterization of the electric field and anomalous conductivity should be constructed. Usually the Cartesian coordinate system is taken and rectangular elementary cells are chosen, while the functions $\Delta\sigma(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ are considered to be piece-wise constant:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_k = \mathbf{E}(\mathbf{r}_k),\ \mathbf{E}^n(\mathbf{r}) = \mathbf{E}^n_k = \mathbf{E}^n(\mathbf{r}_k),\ \Delta\sigma(\mathbf{r}) = \Delta\sigma_k,\ \mathbf{r} \in V_k,$$

where $\mathbf{r}_k$ is the centre of the cell V_k.

Under this simplest assumption, the problem (1.1) reduces to the vector linear system:

$$\mathbf{E}_i = \mathbf{E}^n_i + \sum_{k=1}^{K} \hat{\Gamma}_{ik}\mathbf{E}_k,\ i = \overline{1, K};$$

$$\hat{\Gamma}_{ik} = \Delta\sigma_k \cdot \iiint_V \hat{G}^E(\mathbf{r}_i, \mathbf{r}')\,\mathrm{d}V_{r'}. \tag{1.3}$$

It looks advisable to improve the discretization scheme in (1.1), though this involves very cumbersome representations of the linear system coefficients. This subject was discussed by Harrington (1968), Poggio and Miller (1973), Stenger (1978), Petrick (1978), Ting and Hohmann (1981).

1.1.2. *Numerical Solution of Discrete IE problems*

In determining the coefficients of system (1.3) we face the problem of the effective calculation of the volume integral $\hat{\Gamma}_{ik}$. Vector elements of the tensor $\hat{G}^E(\mathbf{r},\mathbf{r}')$ can be interpreted as electric field vectors at a point $\mathbf{r}$, in the normal conductivity model, due to the presence of elementary electric dipoles of unit moment located at a point $\mathbf{r}'$ and orented in the principal coordinate directions. The calculation of the EM fields of a dipole in a 1D medium is a standard problem which is usually solved by means of the Hankel transform. The corresponding representations of the tensor $\hat{G}^E(\mathbf{r},\mathbf{r}')$ for the multi-layered normal model were derived by Weidelt (1975a). Only the vertical integration of $\hat{G}^E$ in (1.3) can be done analytically in this case.

Weidelt (1975a) and Hohmann (1975) introduced some simplifications which made it possible to express the whole of the integral $\hat{\Gamma}_{ik}$ as a Hankel transform. Usually the tensor $\hat{G}^E$ is decomposed into a singular part $\hat{G}^{EP}$, the Green's tensor of a homogeneous space, and a regular part $\hat{G}^{ES}$ taking into account the layered structure of the model. The integration of the primary tensor $\hat{G}^{EP}$ is performed analytically (Weidelt, 1975a; Das and Verma, 1981a). The integral of the secondary tensor $\hat{G}^{ES}$, in the form of Hankel transform, can be evaluated numerically by means of modern fast linear filtering techniques (Das and Verma, 1981a, b, 1982) or by using methods based on the deformation of the integration contour in the complex plane. (Tabarovsky and Morozova, 1981; Viurkov and Dreizin, 1982). An alternative idea is to carry out the whole integration numerically. Wannamaker and Hohmann (1982) achieved a significant saving by using a procedure which involved a 3D cubic interpolation scheme.

One more approach for the accurate and fast Greens tensor volume integration can be based on the following 2D Fourier transform representation:

$$\hat{G}^E(\mathbf{r}, \mathbf{r}') = F_{xy}^{-1}\left[\hat{g}^E(k_x, k_y, z, \mathbf{r}')\right] = \frac{1}{4\pi^2}\iint\limits_{-\infty}^{\infty} \hat{g}^E(k_x, k_y, z, \mathbf{r}')\, e^{-i(xk_x + yk_y)} \mathrm{d}k_x\, \mathrm{d}k_y. \tag{1.4}$$

Here the 'spectral' tensor $\hat{g}^E$ has a simple known exponential dependence on the Cartesian coordinates of the source vector $\mathbf{r}'$. After substituting (1.4) into (1.3) and transposing the Fourier and volume integrals, the latter one can be evaluated analytically. Therefore, the calculation of coefficients $\hat{\Gamma}_{ik}$ is reduced to a 2D numerical inverse Fourier transform. Representations like (1.4) seem also to be useful in the calculation of normal fields generated by finite-size sources, as used in existing controlled source systems.

1.1.3. *Linear System Solution*

The matrix of system (1.3) is filled and its dimension is rather high (in simple experiments of Ting and Hohmann (1981) up to 800 scalar complex equations were used and the model of Hvozdara (1981b) required 3000 equations). When the region V is subdivided into K cells, the number of operations for matrix computation is proportional to K^2, while the direct system solution leads to expenses proportional to K^3. So the latter expenses become dominant as K increases, though this effect can be reduced when the symmetry of the model is taken into account (Tripp, 1982).

Only the first steps have been made up till now in searching for optimal algorithms in this field. Weidelt (1975a) applied a Gauss-Seidel iterative method and found its convergence reliable only for conductivity contrasts less than 100. Hvozdara (1981b) achieved rapid convergence for contrasts of about 100 (with an improved initial approximation). There are several examples of the use of direct (elimination) methods (Ting and Hohmann, 1981; Das and Verma, 1982; Wannamaker and Hohmann, 1982). Also the approach of approximate reduction of the filled matrix to the banded form seems to be interesting (Poggio and Miller, 1973). From the physical point of view that means the neglect of mutual induction between distant elementary cells V_k.

1.1.4. *Approximate Solution of 3D Integral Equations*

Facing significant computational effort in the accurate solution of IE modelling problem, we need some fast modelling techniques producing qualitative or rough quantitative estimates of EM anomalies suitable for the different stages of 3D interpretation. The first possibility here is connected with the limitation of the linear system solution accuracy. Another opportunity consists of the use of the rough discretization of the integral operator in (1.1) (Dmitriev *et al.*, 1980). The simplest approach can be produced by the following assumption: $\mathbf{J}_v(\mathbf{r}) = \Delta\sigma(\mathbf{r})\mathbf{E}^n(\mathbf{r})$, $\mathbf{r} \varepsilon V$, so EM fields outside inhomogeneity V are directly determined by the use of operators (1.2). This technique was found to be useful in 2D formulations (Kaufman, 1974; Zakharov, 1979; Varentsov, 1981) and made possible the construction of fast formalized 2D model fitting procedures (Zhdanov and Varentsov, 1980, 1982; Varentsov, 1981, Berdichevsky and Zhdanov, 1981, 1983), while Le Mouel and Menvielle (1982) applied quite the same idea in quasi-3D modelling. Now a 3D algorithm is presented with the integration in (1.2) carried out in the spatial frequency domain. Using (1.4), the anomalous electric field in this domain can be expressed as follows:

$$\mathbf{e}^a(k_x, k_y, z) = F_{xy}[\mathbf{E}^a(x, y, z)] = \Delta\sigma \iiint_V \hat{g}^E(k_x, k_y, z, \mathbf{r}')\mathbf{E}^n(\mathbf{r}')\,\mathrm{d}V_{\mathbf{r}'}. \tag{1.5}$$

The expression for the magnetic anomaly $\mathbf{h}^a = F_{xy}[\mathbf{H}^a]$ takes just the same form. For the particular model of a brick-like structure in a layered medium with plane wave excitation the integral in (1.5) is analytical and so the modelling problem is reduced to the numerical 2D Fourier transform inversion. The anomalous field (1.5) is overestimated because the self induction in V is neglected. It may be very rough (close to the equivalent dipole response) in the case of an isometric inhomogeneity (Figure 2), but it seems to be useful for express-modelling of 3D conductivity structures which are elongated in the direction of the normal electric field.

1.2. Differential Equation Method (DE)

The method is based on the direct numerical solution of boundary value problems derived form Maxwell's equations. The modelling region M is chosen with suitable boundary conditions and usually is much greater than the region V containing the conductivity anomalies (Figure 3a). Finite differences (FD) or finite elements (FE) are the most common discretization techniques here. Using these techniques the whole region M is covered by the system of cells (elements) and linear equations are written for

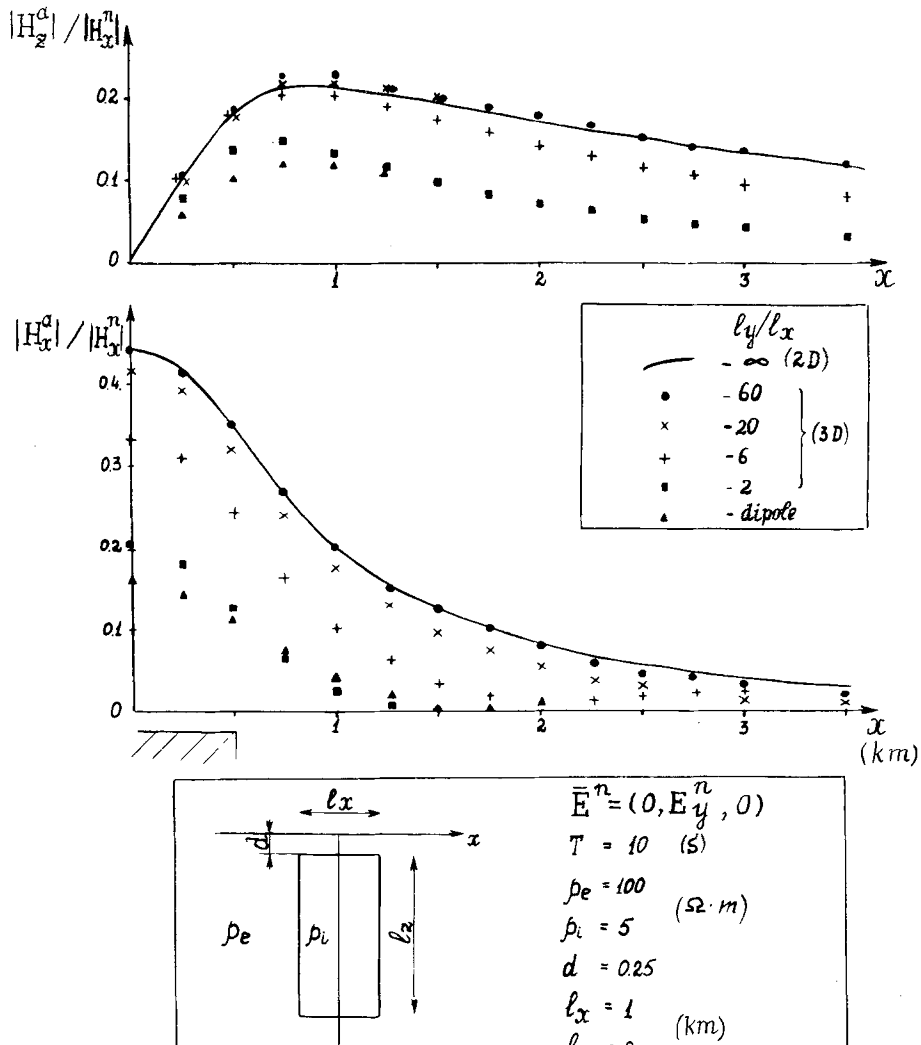

Fig. 2. 3D anomalous magnetic express-modelling response (produced by formula (1.5)) on profile ($x > 0$, $y = 0, z = 0$) for a conducting brick of variable strike extent placed in a homogeneous conducting halfspace.

the field values at grid nodes or for the coefficients of some local approximations of the field. The coefficients of these equations depend only on the local conductivity distribution and the local grid geometry and can be determined without significant computational effort.

The first application of FD technique in 3D modelling was made by Jones and his colleagues (Jones and Pascoe, 1972; Lines and Jones, 1973; Jones and Vozoff, 1978; Jones, 1978). Numerous calculations for the models of practical interest were

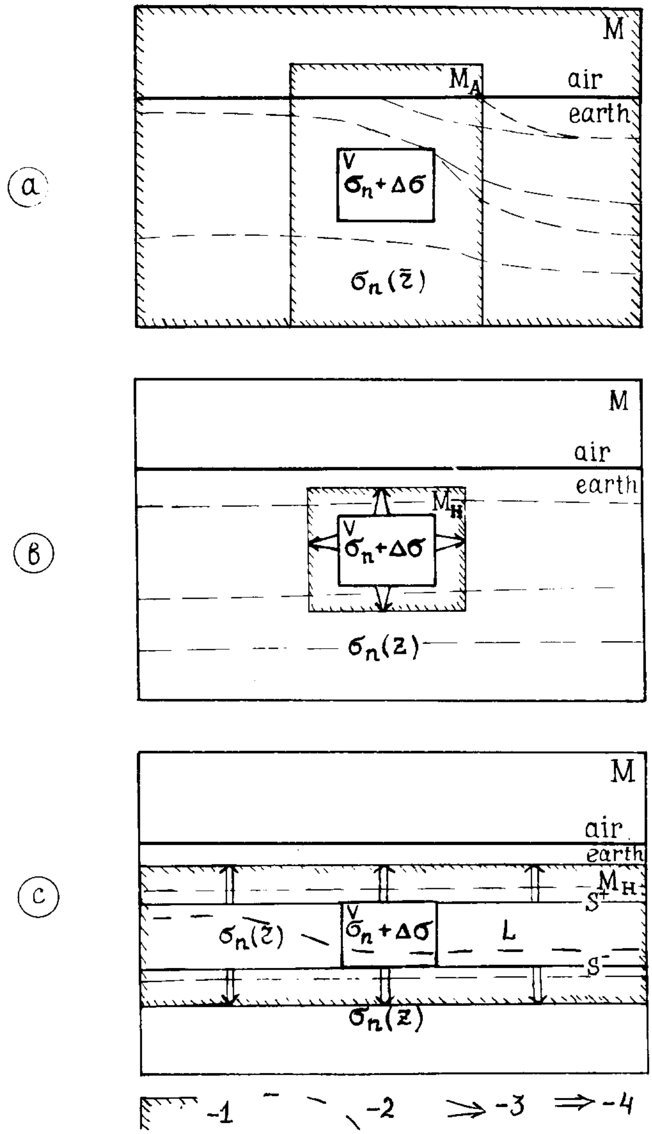

Fig. 3. Geoelectric structure in formulation of DE modelling problem (a) and hybrid modelling schemes (b, c): (1) modelling region boundary, (2) interfaces of the normal conductivity structure, (3) action of the integral operator (1.2) to determine boundary field, (4) boundary field dynamic correction using the analytical continuation technique.

performed using their programs (see references in: Ramaswamy *et al.*, 1980; Lam *et al.*, 1982). Further developments were produced by Yudin (1980, 1981, 1982), Zhdanov and Spichak (1980), Varentsov and Golubev (1780), Zhdanov *et al.* (1982). The FE approach was elaborated in 3D induction studies firstly by Reddy *et al.* (1977) and then by Pridmore (1978), Pridmore *et al.* (1981).

Now, by means of these techniques, we have an opportunity to study more complicated models. However the accuracy of such computations should not be overestimated while the corresponding costs are extremely high. The necessary improvements in the DE approach were specially considered by Pridmore and Sill (1978), Pridmore *et al.* (1981), Zhdanov *et al.* (1982). The most critical aspects of these problems will be discussed below.

1.2.1. *Boundary Value Problem Formulation*

At this stage we have to chose the field in terms of which the DE problem is to be posed (electric or magnetic, total or anomalous) and to obtain the differential equation as well as to determine the modelling region and the conditions on its boundary. When the whole set of EM field components should be modelled the DE problem formulation for the electric field looks more advisable (Jones, 1978; Yudin, 1980; Pridmore *et al.*, 1981; Zhdanov *et al.*, 1982), though if only the magnetic field is to be studied, the opposite choice is natural (Sheen, 1978). It is useful to decompose the geoelectric model, as in the IE method, into normal and anomalous structures and here we have justification to consider general 2D and even 3D normal models $\sigma_n(\mathbf{r})$ (Figure 3a).

The anomalous field DE problem formulation produces some advantages at the stage of discretization because the anomalous field structure is more simple and smooth in the regions with sharp spatial variations of normal field and normal conductivity. Although the supplementary task of the normal field calculation arises, the improvements of discrete approximation may compensate (Sheen, 1978; Yudin, 1980). In this formulation there is no need to assume 1D conductivity structure near modelling region boundaries. It is especially effective to study a set of models with the same normal conductivity distribution (for example, in model fitting procedures).

The traditional boundary conditions used in modelling problems are those appropriate to the normal field (Jones, 1978; Yudin, 1980). The most common are the Dirichlet type conditions in which the corresponding normal field values are assigned to the solution on the modelling region boundary (total field formulation) or set to zero there (anomalous field formulation). In both cases the boundary should be placed sufficiently far from conductivity anomalies for the anomalous field values on it to be neglected.

To diminish the size of the modelling region, the boundary conditions may take into account the anomalous field structure and thus be valid when the boundary is close to geoelectric inhomogeneities. Integral operators can be applied for this purpose and ideas of that kind will be discussed in the next section. Another way is to study the asymptotic behaviour of the EM field far from geoelectric inhomogeneities. This approach proved to be very effective in 2D modelling (Weaver and Brewitt-Taylor, 1978; Varentsov and Golubev, 1980; Varentsov, 1981; Zhdanov *et al.*, 1982).

A 3D analogue of the Weaver-Brewitt-Taylor condition, valid in a nonconducting halfspace (air, model basement), where the field attenuation is the most slow, was suggested by Varentsov and Golubev (1980), and applied to a FD modelling scheme (Zhdanov *et al.*, 1982). Formulated for the anomalous electric field in the atmosphere, this condition takes the form of the following first-order differential equation:

$$D_1[\mathbf{E}^a](\mathbf{r}) = \mathbf{O}(1/r^2), \quad D_1 = 1 + x\frac{\mathrm{d}}{\mathrm{d}x} + y\frac{\mathrm{d}}{\mathrm{d}y} + z\frac{\mathrm{d}}{\mathrm{d}z} = 1 + \mathbf{r} \cdot \nabla; \qquad (1.6)$$

where the origin of the coordinate system is taken on the earth's surface closest to the centre of anomalous region V (Figure 3a). The use of Equation (1.6) on the boundaries in the atmosphere could greatly decrease the modelling region (compare regions M and M_A in Figure 3a), while the numerical solution of the DE problem does not differ appeciably from the case when the 'normal field' boundary conditions are chosen.

1.2.2. *Numerical Solution of Boundary Value Problems*

The discrete formulation of 3D DE modelling problems is studied in detail by Pridmore *et al.*, (1981), Yudin (1981) and Zhdanov *et al.* (1982). In both FE and FD approaches the regular subdivision of the modelling region is considered and linear equations are constructed for the field values at the nodes of this grid. The linear system coefficients in FE schemes are derived by means of the Galerkin method (Reddy *et al.*, 1977) or by using the variational formulation of the problem (Pridmore *et al.*, 1981). The linear equations produced by FD methods can be obtained in terms of an integro-interpolating (balance) technique (Zhdanov and Spichak, 1980; Zhdanov *et al.*, 1982) or by using the variational approach (Yudin, 1981).

All the approaches generate linear systems with quite similar structures and high dimensions (up to tens of thousands of scalar complex equations). So the problem of the optimal choice of linear system solution algorithm is of great importance. The system matrix has a banded structure and, due to the regular grid geometry, the band is also regular with a dominance of completely zero diagonals. However, the bandwidth is relatively larger than in the 2D case and the solution by direct methods is realistic now only in the case of rather small dimensions (less than thousands of equations). Significant progress could be achieved with the help of specialized elimination algorithms taking into account the specific structure of the matrix band. Another very interesting approach contains the use of semi-iterative procedures. For example, combining approximate but fast and compact matrix decomposition with the conjugate gradient iterative method can cause a significant increase in effectiveness (in comparison with pure iterative or direct methods) and has been achieved in DE problems similar to those in 3D induction studies (Kershaw, 1978). However, valuable improvements can be introduced by traditional iterative techniques, such as the S.O.R. method (Pridmore *et al.*, 1981, Zhdanov *et al.*, 1982; Varentsov and Golubev, 1982). Here we face two main problems: how to optimize the convergence of iterations and how to control the accuracy of the solution (i.e. where to stop the process). Several algorithms, of the prediction-correction type, to select iterative parameters were designed for 2D modelling (see references in the last two cited papers) and can be modified for 3D applications. To accelerate convergence, it is useful to improve the accuracy of the initial approximation, for example, by using a set of 2D solutions.

The problem of internal accuracy control is studied specially in Zhdanov *et al.*, 1982.

The best approach to estimate the total modelling error consists of examining the validity of some general relations attributed to the EM field in the model. For example, the accuracy of the formulae (1.2), when applied to the grid solution, can be studied (Zhdanov *et al.*, 1982) and the two following criteria to halt the iterative process are derived ($\mathbf{E}^a$ is evaluated iteratively and a 1D normal model is considered). The first of them controls the absolute value of the modelling error:

$$\delta^{(k)} = \|\mathbf{E}^a(k) - \iiint_V \hat{G}^E \, \Delta\sigma(\mathbf{E}^n + \mathbf{E}^a(k)) \, \mathrm{d}V\| / \|\mathbf{E}^a(k)\| \leqslant \varepsilon_0 \tag{1.7}$$

here k is an index of iteration and the vector norm is defined in the observation region. The second criterion takes into account the change of the first estimate during the iterative process:

$$(1/\Delta k) \cdot |\delta^{(k-\Delta k)}| / \delta^{(k)} \leqslant \varepsilon_1 \tag{1.8}$$

which indicates the situation when the DE problem discretization errors are much greater than the errors of the linear system solution.

Usually after the solution of the linear system it is necessary to determine the grid values of all other EM field components. The direct FD approximation of the appropriate Maxwell's equation leads to significant errors on rough grids. The accuracy may be improved if anomalous fields are considered and a realistic solution approximation is substituted into the differential operator of Maxwell's equation. However, in models with a 1D normal structure and finite conductivity anomalies, it seems preferable to obtain the EM fields in the observation domain by means of integral operators (1.2) applied to the electric current distribution modelled inside the inhomogeneities (Sheen, 1978; Pridmore *et al.*, 1981; Yudin, 1982).

1.3. Hybrid Modelling Schemes

Let us consider now the class of modelling algorithms in which the combination of the IE formalism with DE techniques produces a new kind of computational scheme called hybrid.

An obvious approach to construct a hybrid scheme is based on the use of integral boundary conditions. It is natural to apply here integral expressions (1.2) which present the solution outside the inhomogeneous region in terms of the internal field. So the modelling region M can be reduced to a rather small domain M_H containing all conductivity anomalies (the normal model having a 1D structure, as in Figure 3b). This idea was elaborated by several authors (Sheen, 1978; Petrick, 1978; Petrick *et al.*, 1981; Lee *et al.*, 1981). The first numerical results were published by Lee *et al.* (1981).

By employing such an approach, a linear system with a specific structure is generated (Sheen, 1978; Lee et al, 1981). The discretization at boundary nodes is typical for the IE method, while at internal nodes either the FD or FE technique is used most frequently. The problem posed for the anomalous electric field produces the following system:

$$\begin{bmatrix} A_{ii} & A_{ib} \\ A_{bi} & A_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{E}_i^a \\ \mathbf{E}_b^a \end{bmatrix} = \begin{bmatrix} \mathbf{R}_i \\ \mathbf{R}_b \end{bmatrix}, \tag{1.9}$$

where the subscripts i and b indicate internal and boundary equations, A_{ii} is a square banded matrix (as in DE problems), A_{bb} is the unit matrix, A_{ib} is sparse and A_{bi} is filled rectangular.

When solving (1.9) for the internal field, $\mathbf{E}_i^a$, the following system was adopted (Lee *et al.*, 1981):

$$(A_{ii} - A_{ib}A_{bi})\,\mathbf{E}_i^a = \mathbf{R}_i - \mathbf{A}_{ib}\mathbf{R}_b. \tag{1.10}$$

Though the modelling region is now substantially diminished, the number of internal unknowns is still of the order of thousands. The application of direct algorithms to this system with a filled matrix is difficult and a semi-iterative approach seems more suitable. Lee *et al.* (1981) proposed inversion of the banded matrix A_{ii} and the following iterative scheme:

$$\begin{aligned} &\mathbf{E}_b^a(o) = 0,\ \mathbf{E}_i^a(o) = A_{ii}^{-1}\,\mathbf{R}_i; \\ &\mathbf{E}_b^a(k) = (1 - w)\,\mathbf{E}_b^a(k-1) - w[A_{bi}\mathbf{E}_i^a(k-1) - \mathbf{R}_b] \\ &\mathbf{E}_i^a(k) = -A_{ii}^{-1}[A_{ib}\mathbf{E}_b^a(k) - \mathbf{R}_i],\ k = 1, 2, \ldots, \end{aligned} \tag{1.11}$$

where the coefficient w, which promotes the stabilization and acceleration of convergence, is in inverse proportion to the conductivity contrast in the model. Satisfactory convergence (after tens of iterations) with the simplest conducting brick model was achieved only with conductivity contrasts less than several hundreds.

It seems preferable, following Sheen (1978), to reformulate the system (1.9) in terms of the boundary field:

$$(I - A_{bi}A_{ii}^{-1}A_{ib})\,\mathbf{E}_b^a = \mathbf{R}_b - A_{bi}A_{ii}^{-1}\mathbf{R}_i, \tag{1.12}$$

I is the unit matrix.

The dimension of this system is determined by the number of boundary unknowns and is relatively small (several hundreds). The effective application of a direct method of solution is quite possible here. Having solved the system and stored the inverse matrix A_{ii}^{-1}, it is easy to expand the solution inside the modelling region:

$$\mathbf{E}_i^a = -A_{ii}^{-1}(A_{ib}\mathbf{E}_b^a - \mathbf{R}_i). \tag{1.13}$$

The main computational costs in this approach are concentrated at the stages of: (i) inversion of the banded matrix A_{ii} (the matrix dimension is significantly smaller than that of the DE problem) and (ii) direct solution of the linear system (1.12) with a filled matrix, (with a much smaller dimension than the corresponding IE problem). So the scheme (1.12-1.13) looks advantageous when compared with the standard application of IE and DE techniques, though still we have no numerical results here.

Another kind of hybrid scheme was introduced by Yudin (1982) which was based on the iterative solution (using Seidel's technique) of a standard anomalous field DE modelling problem with inhomogeneous Dirichlet type boundary conditions corrected periodically during the iterative process. Initially the boundary field values are set to zero. The dynamic correction is performed by means of analytical continuation of the grid solution away from the surfaces, $S^{\pm}$, of the inhomogeneous layer, L, containing all conductivity anomalies (Figure 3c). The continuation of 3D EM fields in 1D media is carried out using a spatial frequency domain algorithm (Berdichevsky and Zhdanov, 1981). In this approach the conductivity distribution should be one-dimensional only outside the inhomogeneous layer L. This is an advantage over the previous hybrid schemes. However, the convergence of the iterative process described has not been proved and needs a special study.

1.4. Practical problems of 3D modelling

We have considered above the main approaches to the solution of general 3D modelling problems. Each of them has its own advantages and difficulties and the suitable applications, where these approaches could be the most effective, should be outlined.

The IE technique is becoming a standard tool to study models with local inhomogeneities of a rather simple structure. This class is of great importance in theoretical and methodical 3D investigations (Cauterman *et al.*, 1979; Stodt *et al.*, 1981; Sandberg and Hohmann, 1982; Das and Verma, 1982). A new generation of cost effective computer programs dealing with multi-layered models containing several inclusions with an arbitrary conductivity distribution is now appearing (Wannamaker and Hohmann, 1982; Das and Verma, 1982).

The DE approach is still the only possibility to analyse models with complicated 2D normal and 3D anomalous conductivity structure. The improvements summarized in Section 1.2 could produce substantial progress in DE modelling, without diminishing its simplicity and universality. We are waiting to obtain advanced DE programs which implement these improvements (primarily, the asymptotic boundary conditions and internal accuracy control). However, the development of controlled source DE modelling may be substantially limited by the lack of effective direct algorithms to solve the corresponding linear systems with a different set of right-hand side vectors.

The hybrid technique looks to be the most suitable for models of middle range complexity, but its effectiveness should be better established by numerical experiments. Moreover, an important aid to the progress of 3D modelling would be the comparison of results from existing computer programs, with emphasis on computational effectiveness and universality. As a result of this comparison, the optimal applications and the actual directions for further development of 3D modelling approaches could be conceived more clearly and concretely. The first set of tests models was adopted by international conferences in Australia (Sydney, 1977) and U.S.A. (Berkeley, 1978) and the results of several comparative calculations were published recently (Pridmore *et al.*, 1981, Das and Verma, 1982; etc).

As we approach the stage of mass scale 3D modelling applications, we have to pay much more attention to the problems of rational computer implementation of modelling algorithms (Pridmore and Sill, 1978.) Really, the formal optimization of programming code, as well as the optimal use of hardware options, could be nearly as effective as theoretical or computational improvements. To succeed in the solution of these problems, the development of specialized computer systems is desirable. Organization of data banks of modelling results would be possible by the use of such systems and investigation, comparison and distribution of modelling programs could be performed more effectively.

Many important practical problems appear at the stages of model design (automation of model description, formal generation of interface parameters for modelling programs, etc.) and presentation of modelling results (choice of standard and optional sets of modelled EM data, elaboration of formats of data presentation, computer storage of modelling results, etc). However, we are now only beginning to understand these problems.

1.5. Quasi-3D modelling

In many important geoelectric situations, 3D conductivity anomalies can be approximated using inhomogeneous thin layers or axially symmetric structures, i.e. by objects with 2D anomalous conductivity distributions. In such classes of models we still deal with 3D EM anomalies while the dimension of the modelling problem is diminished. Therefore, it becomes possible (i) to study much more detailed models than in general 3D cases with the same computational effort or (ii) to reduce significantly the computational cost in comparison with 3D modelling, without the loss of accuracy.

The most investigated quasi-3D structure is the fundamental Price-Sheinmann model consisting of an infinitesimally thin conducting sheet, with a surface conductivity distribution $S(\mathbf{r}_s) = S_n + \Delta S(\mathbf{r}_s)$, $\mathbf{r}_s = (x, y)$, placed between the non-conducting atmosphere and underlying 1D conductivity structure $\sigma_n(z)$. When the anomaly $\Delta S(\mathbf{r}_s)$ is finite (nonzero only in the region V) we can easily reformulate the problem in terms of the general IE approach (Section 1.1). So the Equation (1.1) takes the form (Weidelt, 1977; Vasseur and Weidelt, 1977);

$$\begin{aligned}
&\mathbf{E}_s(\mathbf{r}) = \mathbf{E}_s^n(\mathbf{r}) + \iiint\limits_V \hat{G}_S^E(\mathbf{r},\mathbf{r}')\mathbf{J}_s(\mathbf{r}')\,\mathrm{d}V_{\mathbf{r}'}, \quad \mathbf{r},\mathbf{r}' \varepsilon V;\\
&\mathbf{J}_s(\mathbf{r}') = \Delta S(\mathbf{r}')\mathbf{E}_s(\mathbf{r}'), \quad \mathbf{E}_s = (E_x, E_y, 0),\\
&\mathbf{r} = (x, y, 0), \quad \mathbf{r}' = (x', y', 0),
\end{aligned} \tag{1.14}$$

The 'surface' tensor $\hat{G}_S^E$, is related to the 3D tensor $\hat{G}^E$, of the corresponding normal section $\{S_n, \sigma_n(z)\}$. The practical application of this technique was discussed by Weidelt (1977), Vasseur and Weidelt (1977), Menvielle *et al.* (1982).

The main limitations in the formulation (1.14) is the finite size of the inhomogeneous domain V. This difficulty was overcome in the method presented by Dawson and Weaver (1979), Weaver (1979, 1982). They derived a 2D integral equation for the total electric field $\mathbf{E}_s$, and considered models in which the derivative of the conductivity normal to the edges of the model vanished outside V. Besides Fainberg and Zinger (1980) formulated the integro-differential problem in terms of the anomalous electric field, assuming a 2D normal conductivity distribution $S_n(\mathbf{r}_s)$ and using for a normal field the results obtained in the global modelling scheme (Fainberg, 1980; Fainberg and Zinger, 1980). These authors elaborated an original iterative solution of the problem which was convergent for an arbitrary distribution $S(\mathbf{r}_s)$. We should mention also the developments of the FE solution of the Price equation (Egorov and Palshin, 1981) as well as the two-sheet modification of the Price analysis (Ranganayaki and Madden, 1980).

A significant improvement to a thin sheet model consists in the replacement of classic Price-Sheinmann boundary conditions by more accurate analogues. The corresponding generalization was made by Dmitriev (Dmitriev and Gushchina, 1979) and then by Berdichevsky and Zhdanov (1981, 1983). So a new class of quasi-3D models called 'inhomogeneous thin layers' (with finite thickness and 2D conductivity distribution $\Delta\sigma_s(\mathbf{r}_s)$) was originated. The first modelling technique here (Debabov, 1976) was constructed on the basis of Dmitriev's conditions. The problem was reduced to a system of scalar integro-differential equations, posed for TE and TM field mode potentials, and solved iteratively. The significant acceleration of computation was achieved by the transition to the spatial frequency domain. Using this approach, a variety of models was investigated (Golubtsova, 1981). The same boundary conditions were applied in the scheme with two inhomogeneous thin layers (Dmitriev *et al.*, 1981). More general conditions were employed in the modelling problem by Zhdanov and Tikhomirova (1982). For the case of a 3D field and 2D anomalous conductivity structure, the integral equation for the magnetic field was formulated directly in the spatial frequency domain and solved iteratively.

The advantages attributable to quasi-3D thin sheet and axially symmetric models were presented by Dmitriev and Barashkov (1980): see also (Berdichevsky *et al.*, 1982). The study of general axially symmetric models, in terms of the IE approach, was established by Dmitriev, Ilyin, Kaufman, Tabarovsky, and Zakharov. The last results here were presented by Zakharov and Nesmeyanova (1977), Barashkov (1981).

Now the quasi-3D modelling seems to be quite a usual tool of 3D interpretation, at least for the problems of deep induction studies (Ádám *et al.*, 1982; Weaver, 1982). One of the main tasks for the near future is to conduct the comparative analysis of the variety of known techniques and to choose the most effective of them. Another important problem is to improve the criteria of validity of quasi-3D approximations of natural geoelectric structures.

1.6. Analogue Scale Modelling

This method is free of some of the difficulties inherent in numerical modelling. In the main the costs here do not depend so much on the complexity of the modelled conductivity structures and excitation-observation systems. Analogue modelling techniques have been used in induction studies for a long time, but rapid progress has been seen in this field during the last 5–10 yr. The sensitivity of EM measurements was improved, a high level of automation achieved, the minicomputer control of modelling process introduced and new materials appeared which could be used to model middle range conductivity contrasts. The up-to-date means of 3D scale modelling facilitated various investigations of complicated geoelectric structures in the fields of prospecting and deep EM studies.

The most detailed investigation of models with subsurface conductivity anomalies can be found in the work of Dosso and his colleagues. These studies contain a set of theoretical models describing 3D coast and island EM effects (Nienaber *et al.*, 1977; Chan *et al.*, 1981; etc.) as well as more complicated regional models of real coastal conductivity structures (Dosso *et al.*, 1980a, b; Ramaswamy *et al.*, 1980; Nienaber *et al.*, 1981). Several types of field excitation were considered (Hibbs *et al.*, 1978; Ramaswamy and Dosso, 1978; Miles and Dosso, 1979). The problem of modelling accuracy was specially studied using some simple analytical estimates (Ramaswamy and Dosso, 1978) and compared with numerical 3D modelling (Hibbs *et al.*, 1978; Ramaswamy *et al.*, 1980).

New analogue laboratories to analyse a more general class of conductivity structures (subsurface and deep inhomogeneities in a multi-layered normal model) appeared in the USSR (Moroz *et al.*, 1975, 1978) and in Hungary (Ádám *et al.*, 1981).

The first laboratory is generally specialized in the examination of the resolving power of 3D MT and magnetovariational methods and the study of different excitation mechanisms of EM responses to deep crustal and asthenospheric conductivity anomalies (Berdichevsky *et al.*, 1980, 1982). The Hungarian laboratory was designed to solve interpretation problems of EM controlled-source frequency sounding and profiling applied in 3D environments (Ádám *et al.*, 1981). These and the latest results (Morrison *et al.*, 1982; Olm and Frischknecht, 1982; Villegas-Garcia *et al.*, 1982) prove the effectiveness of the scale modelling approach to the solution of methodical and interpretational problems of deep and prospecting EM methods.

The comparison of 3D numerical and analogue modelling results was performed by several authors (Hibbs *et al.*, 1978; Ramaswamy *et al.*, 1980; Lee *et al.*, 1981; Das and Verma, 1982) and it can be seen that modern analogue models are often more detailed and at least as accurate as numerical ones (Ramaswamy *et al.*, 1980).

2. Outlines of Solution of 3D Inverse Problems

Our minimal interpretative capabilities are directly dependent on the development of 3D modelling. The simplest idea of how to interpret 3D anomalies is the selection of models available. In several recent investigations this approach provided quite satisfactory model fitting of EM array observations (Nienaber *et al.*, 1981; Ramaswamy *et al.*, 1980; Weaver, 1982; etc.). An automatic search for the best fitting 3D model in an exisiting bank of modelling results seems to be realistic in simple geoelectric situations, as in the 2D case (Pelton *et al.*, 1978).

We can also use accurate 3D modelling techniques to study the conditions under which 3D anomalies are suitably approximated by 2D EM fields (high frequencies) or 3D stationary fields (low frequencies). Comparative analysis of 2D and 3D modelling results (Ting and Hohmann, 1981) proved the assumption that 2D H-polarization interpretation methods could be useful to resolve the cross-section of an elongated 3D inhomogeneity, while its strike extent could be traced from the 3D stationary field approximation.

However, the main problem in the development of 3D interpretation system is the advancement of formalized approaches to the solution of general 3D inverse problems. Significant progress can be seen in the same field of 2D induction studies and this experience stimulates corresponding 3D activity.

2.1. General remarks on the scheme of 3D interpretation

The process of 3D EM data interpretation can be decomposed into the following main stages:

(i) selection of the EM data to be interpreted;

(ii) construction of a geoelectric model, i.e. definition of the normal (base) conductivity distribution, primary excitation structure and formal description of the unknown anomalous (excess) conductivity distribution;

(iii) choice of a modelling operator to relate the anomalous conductivity structure to the observed responses;

(iv) mathematical formulation of the inverse problem and its regularization (reduction of an initially ill-posed problem to a well-posed one);

(v) development of a constructive algorithm to solve the latter problem and its numerical implementation.

The first two stages provide the formalization of initial information available and are based on the joint analysis of observed EM data and geoelectric models produced by the previous steps of 1D, 2D, and 3D interpretation and existing a priori geophysical and geological ideas. Though informal considerations can not be avoided at these stages, several formalized techniques have been devised to separate conductivity structures and EM responses into normal and anomalous parts, to distinguish between the effects of subsurface and deep EM anomalies, and to localize geoelectric inhomogeneities using the EM field analytical continuation technique (Beamish, 1977; Banks, 1979; Zhdanov, 1980; Berdichevsky and Zhdanov, 1981, 1983). Moreover, the resolving power of different 3D EM responses was studied in some classes of models (Jones and Vozoff, 1978; Ting and Hohmann, 1981; Berdichevsky *et al.*, 1982; Lam *et al.*, 1982; etc.).

The choice of modelling operator is dependent on the structure of the data selected and the geoelectric model constructed, the computational resources available and the requirements of inverse problem formulation.

Now we consider the last two stages of the above scheme, namely the two main approaches to formulate and solve inverse problems (direct inversion and formalized model fitting) and to show the specific aspects of their 3D applications.

2.2. Methods of 3D direct inversion

This approach is based on the direct inversion of the modelling operator involved in inverse problem. The main results were obtained in quasi-3D models using both electric and magnetic simultaneous EM field observations. Berdichevsky and Zhdanov

(1981, 1983) formulated a scalar Fredholm integral equation expressing the anomalous surface conductivity ΔS in terms of the tangential electric and vertical magnetic field components in a model with an inhomogeneous Price-Sheinmann thin sheet and known 1D normal structure below. This equation requires, however, a numerical solution even in the simplest cases. Vasseur and Weidelt (1977) found a closed form inversion of the vector integral operator describing the anomalous electric field in (1.14).

This operator can be considered as a convolution integral:

$$\mathbf{E}_S^a(\mathbf{r}_s) = \iint\limits_{-\infty}^{\infty} \hat{G}_S^E(\mathbf{r}_s - \mathbf{r}_s')\mathbf{J}_s(\mathbf{r}')\,\mathrm{d}\mathbf{r}_s', \qquad \mathbf{r}_s = (x, y),\ \mathbf{r}_s' = (x', y') \tag{2.1}$$

and an application of the deconvolution theorem produces:

$$\mathbf{J}_s = F_{xy}^{-1} \cdot [(\hat{g}_S^E)^{-1} \cdot F_{xy}[\mathbf{E}_s^a]], \quad \hat{g}_S^E = F_{xy}[\hat{G}_S^E]. \tag{2.2}$$

The anomalous conductivity ΔS is easily obtained from the equation $\mathbf{J}_s = \Delta S \cdot \mathbf{E}_s^a$. When a thin sheet is placed inside a conducting medium, algorithm (2.1), (2.2) can be generalized to determine the 3D sheet current vector $\mathbf{J}_v$ from the anomalous magnetic field:

$$\mathbf{J}_v = F_{xy}^{-1} \cdot [(\hat{g}^H)^{-1} \cdot F_{xy}[\mathbf{H}^a]], \quad \hat{g}^H = F_{xy}[\hat{G}^H]. \tag{2.3}$$

The first, and still the only practical results in the application of direct inversion techniques were presented by Banks (1979). The simplest model of a thin sheet in a non-conducting space was considered to interpret geomagnetic array studies in the region of the Kenya Rift. These calculations, being of practical interest, proved once more the ill-posed character of the direct inversion problem and the need to regularize the methods applied to obtain a stable solution. Really, the spatial frequency domain filtering operators in (2.2), (2.3) are not bounded at infinity, as is the case in the algorithm for the analytical continuation of the EM field downward from the earth's surface. The regularized low spatial frequency filtering devised in the latter case (Zhdanov *et al*, 1978; Berdichevsky and Zhdanov, 1981, 1983), seems to be useful in direct inversion applications. Another way to stabilize the solution is based on the numerical approximation of the modelling problem by a discrete system like (1.3) and the use of a generalized matrix inversion technique to solve this system for the extended multi-component and multi-frequency set of surface EM observations (Vasseur and Weidelt, 1977).

In the general 3D case the reconstruction of the excess current distribution, $\mathbf{J}_v$, in modelling operators (1.2) looks also quite realistic, though the convolution formalism like (2.1)–(2.3) is not valid here and the inversion of (1.2) has to be performed numerically. The anomalous conductivity, $\Delta\sigma$, can be found from the equation $\mathbf{J}_v = \Delta\sigma \cdot \mathbf{E}^a$, and the anomalous electric field inside the region V is determined by substitution of the known distribution J_v in Equation (1.1) and further integration.

2.2.1. *Formalized Techniques of 3D Model Fitting*

This approach is not as specific to the 3D case as is direct inversion. In fitting techniques the inverse problem is reduced to the minimization of a functional expressing the misfit between interpreted EM data and corresponding modelling responses. This problem is slightly dependent on the structure of the modelling operator (and on its derivatives with respect to unknown model parameters in gradient minimization schemes). Therefore, the functional minimization technique may be considered as a very broad and flexible interpretative tool which makes it possible to fit arrays of single-point estimates of conductivity functions (impedance, apparent resistivity, magnetic transfer functions, etc) as well as simultaneous EM field observations in the most general classes of geoelectric models.

However, the computational effort here involves tens of modelling problem solutions and in general looks enormously high if accurate 3D modelling operators are applied. To construct effective 3D formalized fitting procedures, we could deal now only with the fastest approximate modelling techniques such as rough discretization IE solutions with predetermined Green's tensor arrays. At the stages of model parametrization, misfit functional formulation and minimization problem solution, the experience accumulated in 2D cases (Weidelt, 1975b, 1978; Jupp and Vozoff, 1976; Oristaglio and Worthington, 1980; Zhdanov *et al.*, 1980; Berdichevsky and Zhdanov, 1981, 1983; Cerv and Pek, 1981; etc.) can be applied with evident modifications. For example, a 3D version of the tightening surfaces method (Zhdanov and Varentsov, 1980, 1983; Varentsov, 1981) is easily formulated if based on the express-modelling technique (1.5). The main advantages of this low-frequency approach consist in the formulation of the misfit functional in the spatial frequency domain, compact one-function description of the anomalous conductivity structure and stable functional minimization performed in Tikhonov's regularization scheme. Another even more simple, but still useful, possibility is to interpret EM field observations in the spatial frequency domain by fitting the spectral EM responses of dipole distributions composed of single 'equivalent' dipoles, lines, sheets and boxes of dipoles with optimized locations, moments and orientations. These reponses can be presented analytically by elementary functions and so the fitting procedure could be very fast compared with the 3D direct inversion scheme involving the evaluation of currents, $\mathbf{J}_v$, mentioned in the previous section.

3. Summary

A conclusion of this review is that several significant theoretical and numerical improvements could be introduced into existing 3D modelling approaches. At the same time, increased attention is to be paid to the numerical investigation of particular techniques, their comparative analysis in different classes of models and the evolution of technical aspects of 3D modelling in practical applications.

3D inverse problems should become the main subject of theoretical studies and the family of inverse problem formulations ought to be extended. By using the experience of 2D induction studies and advances in the theory of ill-posed problems, stable methods for the solution of 3D inverse problems will be constructed. It is quite realistic to expect the first, simplest techniques of this kind very soon.

Finally, the great necessity to integrate 3D interpretation approaches (on theoretical and technological levels) with methods and results of 1D and 2D interpretation seems to be apparent now.

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