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GLOBAL ELECTRICAL CONDUCTIVITY OF THE EARTH

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The Banks (1969, 1972) and Parker (1970) models of the electrical conductivity distribution are critically reviewed along with classical models by Chapman (1919), Lahiri and Price (1939), Rikitake (1950a, b, c) and others. The modern models do not seem to account for the geomagnetic variations having a continuum spectrum and S_q at the same time. A large difference in response between the 1-day and 0.5-day period components of S_q is suspected to be caused by a resonance-like induction in the superficial layer of the earth. Dufficulties in determining the conductivity of the earth's top layer are also emphasized.

An overall distribution of conductivity within the earth which seems to be the most reliable at present, is drawn mostly on the basis of Banks' model.

1. Introduction

Time-varying magnetic fields arising outside the earth, which is more or less conducting, induce electric currents in it and so magnetic fields are produced originating from inside the earth. The theory of electromagnetic induction incorporated in the relationship between the external and internal origin parts of transient geomagnetic variations as obtained by analyses of world-wide data provides a means of inferring the electrical conductivity within the earth.

Since the penetration depth of induced currents depends upon the rapidity of the geomagnetic variation, it is possible to investigate the radial distribution of the electrical conductivity, provided that analyses of geomagnetic variations covering a wide frequency range are available. Bailey (1970) in fact proved that, if the electromagnetic response of the earth is known for all frequencies, the conductivity distribution within the earth can be uniquely determined under certain conditions.

The power of the geomagnetic variation spectrum indicates a steep rise in the spectrum at frequencies less than 10^{-3} c/day. This is caused by the geomagnetic secular variation which is originated entirely inside the earth, so that the above method based on the external–internal relationship of the geomagnetic variation cannot be extended beyond this frequency limit. No exact estimate of the electrical conductivity is therefore possible for the deep interior of the mantle of the earth.

As attempted by McDonald (1957) and Yukutake (1959), however, the attenuation through the mantle of the geomagnetic secular variation originating in the core of the earth may be used for determining the conductivity in the lower mantle with certain assumptions about the origin of the variation. For instance, McDonald assumed a random distribution of secular variation sources at the core–mantle interface. It is clear that such a procedure is something like determining the nature of a filter only from its output without knowing its input. No accurate estimate of the conductivity in the lower mantle can therefore be reached from studies of this sort.

Near-surface lateral inhomogeneities of the conductivity as represented by the distribution of the oceans and the local undulation of a high-temperature mantle layer (Rikitake, 1966, chap. 19; Bullard and Parker, 1970) impose other limitations on the method based on the external–internal relationship of geomagnetic variations at a frequency range higher than 0.2 c/day or thereabouts. Typical geomagnetic variations such as the main phase of a magnetic storm (D_{st}), the daily variation on quiet days (S_q), the geomagnetic bay, the solar flare effect (s.f.e.), the sudden storm commencement (s.s.c.) and the like are all involved in the frequency range. Care must be taken in avoiding the effect of a conductivity anomaly on the determination of the radial conductivity distribution either by choosing geomagnetic observations at non-anomalous places or by smoothing out such an effect on the basis of an extremely large set of data. In any case it seems no easy matter to determine the mean conductivity in the upper mantle down to a depth of a few hundred kilometres.

There is very little to add to what has been mentioned in the excellent papers recently published by Banks (1969, 1972) and Parker (1970) which brought out very clearly modern aspects of the problem of the overall conductivity distribution of the earth. It is aimed, in this paper, to summarize the conductivity distribution within the earth along with raising some comments on the discrepancies between the observed and calculated electromagnetic responses for a few geomagnetic variations.

2. Uniform core model

In the early stage of electromagnetic induction study within the earth, it was customary to assume that the earth consisted of an insulating shell underlain by a sphere having a uniform conductivity. Chapman (1919), who called such an earth model the uniform core model, determined the radius and the conductivity of the conducting sphere on the basis of his analyses of S_q.

Rikitake (1950a, b) determined uniform core models which were compatible with then-available analyses of S_q, D_{st}, the geomagnetic bay, s.f.e. and the variation with a 27-day period. It was found for all these models that the conductivity, that takes on a practically insulating value of about 10^{-15} e.m.u. in the upper mantle, jumps up to a value of about 10^{-12} e.m.u. at a depth of 400 km. On the basis of a rough estimate of the depth, at which the induced currents are most likely to contribute to the surface magnetic field, a radial distribution of the electrical conductivity was obtained (Rikitake, 1950c). A gradual increase in the conductivity below the depth of 400 km was also made clear.

Lahiri and Price (1939) developed a theory of electromagnetic induction within a non-uniform sphere. Applying the theory to interpreting the analyses of S_q and D_{st}, a sharp increase in the conductivity at a depth of several hundred kilometres was concluded.

A common conclusion from these classical studies is a steep increase in the conductivity at a 400–600 km depth. As the theory of electromagnetic induction has been developed only for particular distributions of conductivity including the uniform one because of mathematical difficulty, no further investigation into the conductivity distribution was possible in the 1950's. Since around 1960, the development of high-speed computers has enabled us to tackle the electromagnetic induction problem in a model which has an arbitrary distribution of conductivity as long as spherical symmetry holds good (Takeuchi and Saito, 1963; Eckhardt, 1963). A detailed investigation into the conductivity distribution within the earth thus becomes possible with the aid of extensive analyses of geomagnetic data in recent years.

3. Spectral analysis of geomagnetic data

The spectral-analysis technique has become widely applied to geophysical data in recent years. The development of high-speed computers enables us to deal with a large set of data. According to an analysis of the geomagnetic records at a typical observatory at middle latitude, it is noticeable that the spectrum in the frequency range 10^{-3}–0.5 c/day consists of a number of lines, i.e., 1 and 2 c/year, 1/27, 2/27, and 3/27 c/day, . . . etc., superimposed on an approximately white continuum.

Banks (1969) examined the nature of these lines and continuum spectra. High coherence in the spectrum between widely spaced observatories indicates that the world-wide distribution of these variations is simple. For example the magnetic potential of the 27-day variation and its harmonics as well as the continuum is most likely to be described approximately by a P_1 spherical harmonic only. Although no extensive analysis of spatial distribution has been carried out, these variations seem likely to be caused by fluctuations of an equatorial current ring. On the other hand, P_2 seems to match the annual variation, though the semi-annual one seems likely to be expressed by P_1 (Currie, 1966).

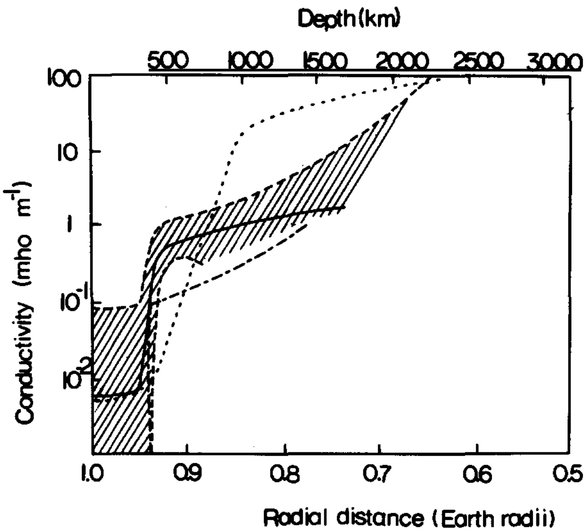

Fig. 1. The Banks model of the conductivity distribution in the mantle. Classical models by Rikitake and McDonald are also shown. (After Banks, 1969.) —— = the 'best-fitting' model, - - - = lower and upper limits on the 'best-fitting' model, — - — = Rikitake (from Rikitake, 1966), ... = Price-McDonald model.

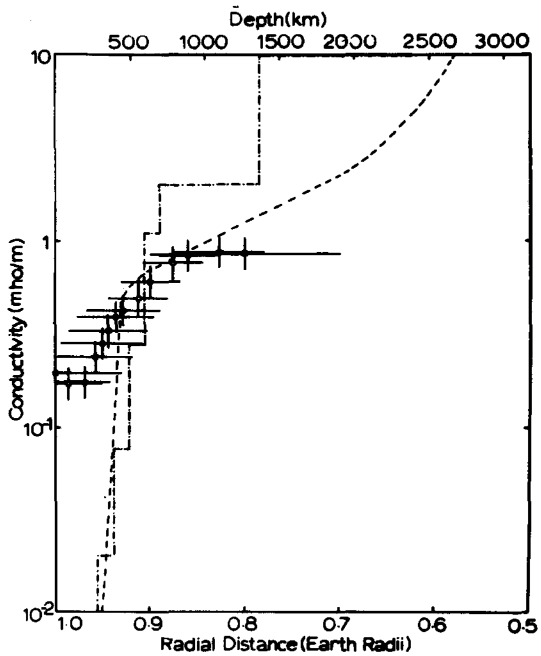

Fig.2. The Banks (1969, 1972) and Parker (1970) models, (after Banks, 1972.) + = Parker (1971), —— = Banks (1969), —·—· = Banks (1972).

A model that is compatible with the continuum response and the response for the annual and semi-annual variations, as well as the 27-day variations and its harmonics, was obtained by Banks (1969) as reproduced in Fig. 1 in which the error range is also shown. The results of previous workers (Rikitake, 1950c; McDonald, 1957) are also illustrated in the figure. The Price-McDonald model has been obtained by combining one of the Lahiri-Price distributions of the conductivity with the McDonald distribution in the lower mantle as surmised from analyses of the geomagnetic secular variation.

Because the Banks model as shown in Fig. 1 is derived from a frequency range 0.01–0.25 c/day, nothing accurate can be said about the conductivity of the top 400 km of the earth. A much more rapid variation should be made use of for looking into the conductivity in the upper mantle although an analysis of such a variation suffers from noise arising from near-surface conductivity irregularities. Any conductivity less than 10^{-12} e.m.u. seems compatible with the observed data. The situation is much the same for the previous models.

Banks (1972) improved his model by taking into account a better fit of the phase relation, getting a little higher conductivity below a depth of 600 km as seen in Fig. 2.

Parker (1970) advanced an inversion technique applicable to the induction problem and determined the conductivity on the basis of the continuum data due to Banks. The essential point of Parker's method is to look for a distribution of conductivity σ under the condition that $\int_0^a (\delta\sigma/\sigma)^2 \, dr$ is a minimum, a and $\delta\sigma$ being the earth's radius and a small deviation of σ. Applying a variational technique, the condition yields a set of simultaneous equations of which the unknowns involve the frequency-dependent radial functions that appear in the theory of electromagnetic induction in a spherical conductor. Starting from an initial choice of conductivity distribution, a small deviation $\delta\sigma$ that tends to satisfy the above condition is obtained as a linear combination of the solutions of such simultaneous equations. The same procedure is repeated until a converged distribution is eventually obtained.

In Parker's method, the first choice must be sufficiently close to the true distribution for a linear approximation to be valid. In Fig.2 is also shown the distribution obtained by Parker along with the two-dimensional uncertainties. Parker's distribution differs from Banks' one in the fact that it gives a conductivity for the top 400 km of the earth about one order of magnitude higher although the cause of such a discrepancy is not quite clear. Logically, an inversion method is superior to the method of best fit based on trials though Parker's technique gives a conductivity estimate averaged over a certain spread of depths.

4. Discrepancy between conductivity distributions from S_q and the continuum

Banks (1969, 1972) and Parker (1970) did not put much stress on analyses of S_q because it seems likely that S_q is contaminated by the effect of near-surface irregularities of the conductivity distribution. In Table I, the observed and calculated responses of S_q are shown as given by Banks (1972). It appears to the writer, however, that the discrepancy between the observed and calculated amplitude ratios is not quite negligible.

The Parker model gives rise to a set of responses which are too high. In order to have a reasonable fit to S_q-data, the conductivity values in the top several hundred kilometres of the earth should be considerably lowered. The Banks models also lead to slightly larger amplitude ratios especially for the P_2^1-response.

TABLE I

Observed and calculated responses of S_q (Banks, 1972)

n	m	Observed*	Price-McDonald (1957)	Banks (1969)	Parker (1970)	Banks (1972)
Amplitude ratio						
2	1	0.376 ± 0.013	0.376	0.454	0.545	0.418
3	2	0.442 ± 0.008	0.369	0.457	0.617	0.425
4	3	0.433 ± 0.017	0.350	0.435	0.654	0.416
Phase difference in degree						
2	1	12.4 ± 2.5	8.4	5.5	9.6	7.9
3	2	14.6 ± 1.2	13.1	6.6	10.2	12.1
4	3	15.3 ± 2.5	17.9	8.4	10.9	16.4

* The data for annual means during the I.G.Y. are 0.357, 0.435 and 0.455 for amplitude ratio and 13°, 13° and 12° for phase difference, respectively (Matsushita and Maeda, 1965).

As S_q has been one of the best analyzed geomagnetic variations since Chapman (1919), the writer feels that the results of S_q-analyses should be properly taken into consideration for the conductivity determination within the earth, although care must be taken of noise arising from near-surface irregularities. In order to incorporate the S_q-responses in those for the P_1-responses as obtained by Banks, the P_2^1- and P_3^2-responses (Q_2^1 and Q_3^2) are converted to the P_1-one (Q_1) in the following way.

Assuming a uniform core model, the radius of the core in units of the earth's radius (q) and the conductivity (σ) are determined from the amplitude ratios and phase differences as given in Table I. Combinations ($q = 0.93$, $\sigma = 8.2 \cdot 10^{-13}$ e.m.u.) and ($q = 0.96$, $\sigma = 6.5 \cdot 10^{-13}$ e.m.u.) are respectively obtained for the observed responses of the P_2^1- and P_3^2-constituents. P_1-responses for such models are then calculated for the 1-day and 0.5-day period variations. The combinations of amplitude ratio and phase difference thus obtained are (0.353, 7.6°) and (0.400, 6.0°), respectively. A simple extrapolation of the P_1-responses of Banks' data (Banks, 1972, fig. 3) in a frequency range $5 \cdot 10^{-2}$–$3 \cdot 10^{-1}$ c/day to 1–2 c/day indicates Q_1-values amounting to about 0.43 and 0.45, respectively. It thus turns out that the Q_1-values deduced from the S_q-data are substantially smaller than those expected for an earth which gives rise to a continuum response as studied by Banks.

In the light of the above discussion, it appears to the writer that the Banks and Parker models must be modified in such a way that the response at higher frequencies becomes a little smaller. Although no exact estimate has been made, the conductivity in the upper mantle must be smaller than those obtained by Banks and Parker. It is not known whether a conductivity distribution which harmonizes with both the continuum and S_q-data actually exists.

5. Effect of near-surface irregularities

Attention should be drawn to the fact that the observed P_2^1-amplitude ratio is considerably smaller than those for P_3^2 and P_4^3 as can be seen in Table I, such a tendency having been noticed even in the case of Chapman's classical analysis (Chapman, 1919). The Q_1-values converted from the P_2^1- and P_3^2-data also indicate that the response for the 1-day period variation is substantially smaller than that for the 0.5-day period one.

The writer suspects that induced currents in the oceans might give rise to such an effect. The theory of electromagnetic induction in a non-uniform sheet indicates a possibility of a resonance-like induction when the wavelength of the inducing field is about the same as that of this conductivity distribution (Rikitake, 1968). It seems likely that the P_3^2-response could be affected by such an effect more seriously than the P_2^1-one judging from the global land–sea distribution.

In addition to the surface irregularities of the conductivity as represented by the land–sea contrast, the effects of possible undulation of a highly conducting mantle layer on the geomagnetic variations are sometimes serious. Actually, Reitzel et al. (1970) reported that even S_q is controlled by an underground conductivity anomaly in the western U.S.A.

According to a study of electromagnetic induction in a sphere of which the shape slightly deviates from a true sphere (Rikitake, 1964), the response of such a deformed sphere is appreciably different from that of a sphere having a radius equal to the mean radius of the former. The magnetic field of internal origin at the earth's surface is largely controlled by electric currents induced in the uplifted portions of the conducting layer. It is therefore possible that the mean depth at which a steep rise of the conductivity vs. depth curve occurs is underestimated. In view of the fact that anomalously high conductivity in the upper mantle has been found in many parts of the world, no meaningful determination of the conductivity, rigorously speaking, can be made in the top 400 km of the earth as long as spherical symmetry is assumed.

6. Overall distribution of the conductivity within the earth

Summarizing what the writer stated in the foregoing sections, the most reliable model representing the electrical conductivity distribution in the mantle would be the one due to Banks (1972), although it is suspected that the model will shortly be subjected to a modification by improved analyses of geomagnetic data. Especially, it is the writer's desire to find a model which achieves a better fit to the S_q-data.

It is disappointing that no accurate determination of the conductivity in the top layer of the mantle is possible even if geomagnetic variations of shorter period are analyzed. This is caused not only by noise arising from lateral inhomogeneity of the conductivity but also by the fact that the physical meaning of the 'mean' conductivity is not clear.

Fig.3 is a smoothed version of Bank's distribution of conductivity in the mantle supplemented by his previous distribution (Banks, 1969) for the top layer. The conductivity in the lower mantle seems one order of magnitude smaller than that of the McDonald model.

As for the conductivity of the earth's core, no convincing way of estimation has been developed although there are a few studies (Rikitake, 1966, chap. 16; Stacey, 1969, p. 150). We herewith take $3 \cdot 10^{-7}$ e.m.u. as a typical value of the conductivity in the core.

Although further improvement based on geomagnetic data of high quality and a proper theory of inversion is required, Fig.3 seems to be the most reliable distribution of electrical conductivity throughout the earth at the present stage of investigation.

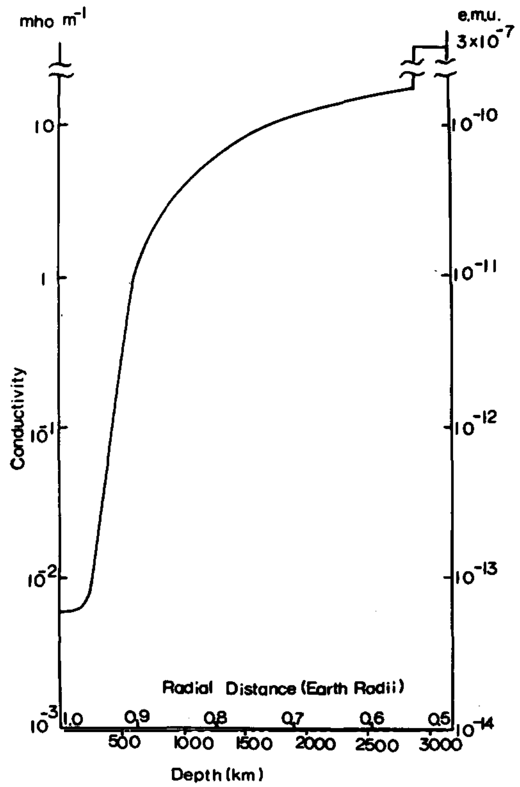

Fig.3. Overall conductivity distribution mostly based on Banks (1972).

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