*Physics of the Earth and Planetary Interiors*, 7 (1973) 389–398


# AN INTRODUCTION TO ELECTROMAGNETIC INDUCTION IN THE OCEAN

J.C. LARSEN
*Joint Tsunami Research Effort, National Oceanic and Atmospheric Administration, University of Hawaii, Honolulu, (Hawaii)*

Accepted for publication February 2, 1973

The governing equations for the induction of electromagnetic fields in the ocean by ionospheric and oceanic sources are presented. A uniformly conducting layered model and a nonuniformly conducting thin-sheet model are discussed with reference to the interpretation of fields observed in the ocean. A procedure for the separation of the electric field continuum into parts of ionospheric and oceanic origin is presented.

## 1. Introduction

Natural electromagnetic signals in the ocean for periods less than a few years are caused by sources of ionospheric and oceanic origin. Sources in the ionosphere or above induce electric currents in the ocean that are modified through the mutual induction coupling between the ocean and conducting mantle. The dynamo action of the electrically conducting sea water moving in the geomagnetic field also induces oceanic electric currents. Observations of oceanic electromagnetic fields, therefore, can give information about the electrical conductivity structure of the mantle and the fluid motion in the ocean provided the fields can be separated into parts of ionospheric and oceanic origin. Signals from the earth's core are insignificant for periods less than three years (Curie, 1968).

Types of oceanic electromagnetic observations have been the in-situ magnetic and electric fields on the sea floor and sea surface, the horizontal electric field from instruments drifting with the fluid velocity, i.e., the G.E.K. (geomagnetic electrokinetograph), and the voltage differences from long submarine cables. Measurements for conductivity studies have been made on the deep sea floor off southern California (Cox et al., 1970; Greenhouse, 1972), on the sea surface at the dip equator off Peru (Richards, 1970) and on drifting ice stations in the Arctic Ocean by various Russians such as Shneyer (1971). Island sites in the deep ocean have been Christmas Island (Mason, 1963a), Oahu, Hawaii (Mason, 1963b; Rogers, 1966; Klein, 1972), Iceland (Hermance and Grillot, 1970), San Miguel, Azores (Thayer and Hermance, 1972), and Macquarie Island (Swift and Wescott, 1964).

## 2. Governing equations

Maxwell's equations in MKS-units (Panofsky and Phillips, 1955) for a moving conducting medium are:

$$\left.\begin{aligned} &\nabla \cdot \boldsymbol{D} = q,\ \nabla \cdot \boldsymbol{B} = 0,\ \nabla \times \boldsymbol{E} = -\,\partial \boldsymbol{B}/\partial t \\ &\nabla \times \boldsymbol{B} = \mu(\boldsymbol{J} + \partial \boldsymbol{D}/\partial t + q\boldsymbol{V} + \nabla \times \boldsymbol{P} \times \boldsymbol{V})\,. \end{aligned}\right| \quad (1)$$

Continuity of electric current requires:

$$\nabla \cdot \boldsymbol{J} = -\,\partial q/\partial t$$

and continuity for incompressible fluid flow requires:

$$\nabla \cdot \boldsymbol{V} = 0 \quad (2)$$

The material of the earth is assumed isotropic so that the constitutive equations are:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P},\ \boldsymbol{P} = \epsilon_0(\kappa - 1)(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}),\ \boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}).$$

In these equations $\boldsymbol{J}$ is the electric current (Ap/m$^2$), $\boldsymbol{E}$ is the electric field (Vt/m), $\boldsymbol{B}$ is the magnetic field (Wb/m$^2$), $\boldsymbol{D}$ is the electric displacement (C/m$^2$), $\boldsymbol{P}$ is the polarization (C/m$^2$), $\boldsymbol{V}$ is the fluid particle velocity (m/sec), $q$ is the net electric charge density (C/m$^3$), $\mu_0 = 4\pi \cdot 10^{-7}$ is the magnetic permeability (H/m) of free space, $\epsilon_0 = 1/(\mu_0 c^2)$ is the electric permittivity (F/m) of free space where $c = 3 \cdot 10^8$ (m/sec) is the speed of light, $\kappa$ is the dielectric constant and $\sigma$ is the electrical conductivity ($\Omega^{-1}\mathrm{m}^{-1}$).

Sea water has a dielectric constant $\kappa = 78$ (Liebermann, 1962) that can be assumed here to be uniform. The ocean's conductivity depends mainly on temperature and salinity but only slightly on pressure (Horne and Frysinger, 1963). On the surface of the open ocean, values lie between 2.9 and 5.8 $\Omega^{-1}\mathrm{m}^{-1}$ while the vertical mean lies mostly between 2.8 and 3.7 $\Omega^{-1}\mathrm{m}^{-1}$ with a probable value near 3.3 $\Omega^{-1}\mathrm{m}^{-1}$ (Bullard and Parker, 1970). For induction studies of the upper mantle it is valid to assume $\mu = \mu_0$ everywhere (Tozer, 1959).

Displacement and polarization currents, compared with the true electric current, can be ignored, i.e., the quasi-stationary approximation is appropriate (Bullard and Parker, 1970; Sanford, 1971) because useful frequencies for oceanic induction problems are less than one cycle per minute. Higher frequencies are completely absorbed within the ocean. Advection of charge can be neglected because the fluid velocity will be of order 1m/sec or less (Sanford, 1971). Finally, the time-varying magnetic field $\boldsymbol{B}$ will be very small compared with the steady geomagnetic field $\boldsymbol{F}$. For example, tidal induced magnetic fields are not likely to be greater than 10 $\gamma$ (1 $\gamma = 10^{-9}$ Wb/m$^2$) compared with the geomagnetic field of order 30,000 $\gamma$ (Larsen, 1968). This means that magnetohydrodynamic effects are not important for oceanic induced signals. The electromagnetic equations then simplify to:

$$\nabla \cdot \boldsymbol{D} = q, \; \nabla \cdot \boldsymbol{B} = 0, \; \nabla \times \boldsymbol{E} = -\,\partial \boldsymbol{B}/\partial t, \; \nabla \times \boldsymbol{B} = \mu \boldsymbol{J} \quad (3)$$

and the constitutive equations are:

$$\boldsymbol{D} = \epsilon_0 \kappa \boldsymbol{E} + \epsilon_0(\kappa - 1)\boldsymbol{V} \times \boldsymbol{F}, \qquad \boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{F}) \quad (4)$$

for which $\nabla \times \boldsymbol{F} = \nabla \cdot \boldsymbol{F} = 0$.

The boundary conditions at the surface between two media (Panofsky and Phillips, 1955) are:

$$\boldsymbol{k} \cdot \Delta \boldsymbol{D} = Q, \; \boldsymbol{k} \cdot \Delta \boldsymbol{B} = 0, \; \boldsymbol{k} \times \Delta \boldsymbol{E} = 0, \; \boldsymbol{k} \times \Delta \boldsymbol{B} = \mu \boldsymbol{I} \quad (5)$$

where $\boldsymbol{k}$ is a unit vector normal to the surface, $Q$ is the surface charge, $\Delta$ stands for the change in the field across the surface, and $\boldsymbol{I}$ is a surface current which vanishes if neither media is infinitely conducting.

Eq.3 and 4 imply that:

$$\nabla \cdot \boldsymbol{J} = 0 \quad (6)$$

and that the net charge distribution is:

$$q = \epsilon_0 \kappa \, \nabla \rho \cdot \boldsymbol{J} - \epsilon_0 \boldsymbol{F} \cdot \nabla \times \boldsymbol{V} \quad (7)$$

where $\rho = 1/\sigma$ is the resistivity. Thus, there are charges associated with conductivity gradients and with the vorticity of the fluid. Consequently these charges set up fields that affect the flow of electric current. From eq.6 and 7, the boundary conditions at a surface between two media are:

$$\boldsymbol{k} \cdot \Delta \boldsymbol{J} = 0, \qquad Q = \epsilon_0 \kappa \Delta \rho \boldsymbol{k} \cdot \boldsymbol{J} + \epsilon_0 \boldsymbol{k} \cdot \boldsymbol{F} \times \Delta \boldsymbol{V} \quad (8)$$

The electric field measured from a site drifting with a velocity $\boldsymbol{V}^{\mathrm{m}}$ will be:

$$\boldsymbol{E}^{\mathrm{m}} = \boldsymbol{E} + \boldsymbol{V}^{\mathrm{m}} \times \boldsymbol{F} = \rho \boldsymbol{J} + (\boldsymbol{V}^{\mathrm{m}} - \boldsymbol{V}) \times \boldsymbol{F} \quad (9)$$

The G.E.K. field assumes that $\boldsymbol{V}^{\mathrm{m}} = \boldsymbol{V}$, i.e., the electric field apparatus drifts with the fluid. Then $\boldsymbol{E}^{\mathrm{m}}$ is a measure of the electric current.

## 3. Induction in horizontal layers of uniform conductivity

The electromagnetic equations governing this case reduce to the differential equation in the magnetic field:

$$\nabla^2 \boldsymbol{B} - \mu\sigma \, \partial \boldsymbol{B}/\partial t = -\,\mu\sigma(\boldsymbol{F} \cdot \nabla)\,\boldsymbol{V} \quad (10)$$

when the conductivity $\sigma$ is uniform within each layer and the geomagnetic field $\boldsymbol{F}$ is assumed uniform. The second term on the left contains the effect of self and mutual induction, and the source term due to fluid motion is on the right.

### 3.1. Solutions for flat earth

Choose $x, y, z$-coordinates with $z$ vertically up from the earth's surface. Let solutions have the form:

$$[\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{J}, \boldsymbol{V}] = [\widetilde{\boldsymbol{B}}(z), \widetilde{\boldsymbol{E}}(z), \widetilde{\boldsymbol{J}}(z), \widetilde{\boldsymbol{V}}(z)] \exp[i(kx - \omega t)] \quad (11)$$

where $\widetilde{\boldsymbol{B}}, \widetilde{\boldsymbol{E}}, \widetilde{\boldsymbol{J}}$ and $\widetilde{\boldsymbol{V}}$ are the Fourier transforms in $k$ and $\omega$ of, respectively, $\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{J}$ and $\boldsymbol{V}$. Substitution of eq. 11 into eq. 2, 3, 4, and 6 yields the following relations valid for each layer:

$$\left.\begin{aligned}
&\tilde{B}_x = (i/k)\,\mathrm{d}\tilde{B}_z/\mathrm{d}z,\ \tilde{V}_x = (i/k)\,\mathrm{d}\tilde{V}_z/\mathrm{d}z\\
&\tilde{E}_x + F_z\tilde{V}_y - F_y\tilde{V}_z = -(\mu\sigma)^{-1}\,\mathrm{d}\tilde{B}_y/\mathrm{d}z = \tilde{J}_x/\sigma\\
&\tilde{E}_y = (\omega/k)\tilde{B}_z = \tilde{J}_y/\sigma + F_z\tilde{V}_x - F_x\tilde{V}_z\\
&\tilde{E}_z + F_y\tilde{V}_x - F_x\tilde{V}_y = ik(\mu\sigma)^{-1}\,\tilde{B}_y = \tilde{J}_z/\sigma
\end{aligned}\right| \quad (12)$$

When $V$ is presumed given, the electromagnetic problem reduces to solving for $\tilde{B}_y$ and $\tilde{B}_z$ which are solutions of the differential equations:

$$\mathrm{d}^2\tilde{B}_y/\mathrm{d}z^2 = (k^2 - i\mu\sigma\omega)\tilde{B}_y - ik\mu\sigma F_x\tilde{V}_y - \mu\sigma F_z\,\mathrm{d}\tilde{V}_y/\mathrm{d}z \quad (13)$$

$$\mathrm{d}^2\tilde{B}_z/\mathrm{d}z^2 = (k^2 - i\mu\sigma\omega)\tilde{B}_z + ik\mu\sigma(F_z\tilde{V}_x - F_x\tilde{V}_z) \quad (14)$$

Assuming the conductivity is nowhere infinite, the boundary conditions require $\boldsymbol{B}$ and horizontal $\boldsymbol{E}$ to be continuous across the horizontal boundaries of the layers. This means that $\tilde{B}_y$, $\tilde{B}_z$, $\mathrm{d}\tilde{B}_z/\mathrm{d}z$ and $(\mu\sigma)^{-1}\mathrm{d}\tilde{B}_y/\mathrm{d}z + F_z\tilde{V}_y - F_y\tilde{V}_z$ are continuous at the boundaries.

### *3.2. Ionospheric induction*

The case $V = 0$ has been solved many times in both rectangular and spherical coordinates. Recent examples are Wait (1970) and Cox et al. (1970). The solutions reduce to two different modes, the $B$-polarization mode for which $B_z = 0$ and therefore by eq. 12, $B_x = E_y = J_y = 0$ everywhere, and the $E$-polarization mode for which $E_z = 0$ and therefore by eq. 12 $B_y = E_x = J_z = 0$. The $B$-polarization mode, given by eq. 13 is an important mode for the interpretation of signals in the frequency range 5–100 c/sec because of lightning strokes (Liebermann, 1962). At lower frequencies, however, the appropriate mode for the interpretation of ionospheric signals is the $E$-polarization mode (Cox et al., 1970) given by eq. 14.

A desired result of the solution to eq. 14 is the relationship between the electric and magnetic fields, i.e., the tranfer functions:

$$\tilde{Z}(\omega, k) = \tilde{E}_y/\tilde{B}_x, \qquad \tilde{Z}'(\omega, k) = \tilde{B}_z/\tilde{B}_x$$

evaluated at the surface $z = 0$. These contain, in a compact form, the information about the conductivity structure beneath the observation site and are related (Schmucker, 1970) to a common factor:

$$\tilde{Z}(\omega, k) = -i\omega\tilde{C}(\omega, k),\ \tilde{Z}'(\omega, k) = -ik\tilde{C}(\omega, k)$$

For the simple case of a uniformly conducting half space, $\sigma_m$, at a depth $z = -h_m$, and with no other conductors between it and the surface:

$$\tilde{C}(\omega, 0) = h_m + \tfrac{1}{2}p_m + i\tfrac{1}{2}p_m \quad (15)$$

where $p_m = (2/\mu\sigma_m\omega)^{1/2}$ is the skin depth for the conducting half space. Schmucker (1970) has shown, by the use of convolution integrals, that $\tilde{C}$ is a length scale that determines the depth and width of the space that effectively contributes to the inductive response.

An estimate of the conductivity structure can be obtained by comparing the observed $\tilde{C}_{obs}$ with eq. 15 and plotting $\sigma_a = [2\mu\omega(\mathrm{Im}\,\tilde{C}_{obs})^2]^{-1}$ versus $h_a = \mathrm{Re}\,\tilde{C}_{obs}$. This is called the modified apparent conductivity profile (Schmucker, 1970) and differs from the usual apparent resistivity profile in which the apparent conductivity is $\sigma_a^* = [\mu\omega(\mathrm{Mod}\,\tilde{C}_{obs})^2]^{-1}$.

For oceanic mantle conditions it is clear from Cox et al. (1970) that:

$$\tilde{C}(\omega, k) \approx \tilde{C}(\omega, 0) \quad (16)$$

when $k < 6/R_E$ ($R_E$ = 6370 km, earth's radius) for frequencies greater than 1.5 cycles per day. For mid-latitude regions away from the auroral zones (68°N and S) and the electrojet at the dip equator, approximation (16) is valid at even lower frequencies since the continuum will generally have small wavenumbers $k < 2/R_E$ (Banks, 1969). The transfer function is then expressible as:

$$\tilde{Z}(\omega, k) \approx -i\omega\tilde{C}(\omega, 0)$$

and is insensitive to wavenumber. Thus, regardless of the complexity of the layered conductivity structure, the transfer function $\tilde{Z}$ will be a smoothly varying function of frequency that can be estimated from the general background continuum. Surface conductivity structure will not change this result when the induced electric currents are parallel to the conductivity structure.

The transfer function:

$$\tilde{Z}'(\omega, k) \approx -ik\tilde{C}(\omega, 0) \qquad (17)$$

however, is sensitive to the wavenumber and therefore can only be estimated by a study of special geomagnetic events such as bay disturbances for which the wavenumber remains uniform and can be estimated. The general continuum is not suitable because it will contain different types of geomagnetic events so that at any particular frequency there will be a distribution of wavenumbers rather than a single unique wavenumber. The restriction to special events can be relaxed, however, in the case of abrupt surface conductivity structure, e.g., the coastline effect, because large vertical magnetic fields are observed due to the concentration of electric currents along the conductivity contrast. Then the vertical magnetic field tends to be like the horizontal electric field so that $\tilde{Z}'$ will be like $\tilde{Z}$ that is a smoothly varying function of frequency that can be estimated from any part of the background continuum.

Finally, the transfer functions can be inverted to find the layered conductivity structure (Parker, 1970; Weidelt, 1972). If wavenumber is independent of frequency, the conductivity structure for a spherical earth can be found directly from the conductivity structure for a flat earth by an algebraic formula (Weidelt, 1972).

### *3.3. Baroclinic wave motion*

This case involves the electromagnetic response to fluid motion $V$. Barotropic motion is discussed in the next section. The theoretical treatments of baroclinic wave motion have dealt with: (1) surface waves with recent papers by Weaver (1965) and Larsen (1971); (2) internal waves (Beal and Weaver, 1970; Cox et al., 1970); and (3) homogeneous isotropic turbulence (Cox et al., 1970). The magnetic field due to surface waves has been observed (Maclure et al., 1964; Fraser, 1965, 1966; Kozlov et al., 1971) with values amounting to a few gammas near the surface. Long swell is more effective than shorter waves. These solutions suggest the following remarks.

It is essential that theoretical treatments use a velocity field consistent with at least the linearized hydrodynamic equations. This means that frequency and wavenumber are related through the propagation equation.

The electromagnetic response, for two-dimensional flow, separates into $E$- and $B$-polarization modes. Irrotational flow, e.g., surface waves, result in only $E$-polarization mode whereas rotational flow, e.g., baroclinic waves or tidal motion on a rotating earth, result in both $E$- and $B$-polarization modes.

Self and mutual induction play a critical role and are related to the time variation and the size of the induced electric currents and the relationship between them as given by the propagation equation. The long surface wave solution (Larsen, 1971) shows that self induction acts to inhibit the electric current flow and that mutual induction tends to cancel the effects of self induction. Self induction is more important for large ocean depths and, subject to the propagation equation, high frequency and small wavenumber. For small frequency and wavenumber the conducting mantle plays a significant role through mutual induction. The conductivity of the ionosphere, however, can be ignored (Larsen, 1968) because its conductivity, $\sigma \lesssim 10^{-3}\ \Omega^{-1}\mathrm{m}^{-1}$, is relatively small compared with sea water.

The $B$-polarization mode is controlled by the leakage of electric currents from the ocean into the conducting crust and mantle that are connected only by ohmic paths to the ocean. Selfinduction plays a role by modifying the distribution of these electric currents.

### *3.4. Steady oceanic currents*

This has been the important case studied by oceanographers (Longuet-Higgins et al., 1954). Two-dimensional flow is pure $B$-polarization mode. Measurements have consisted of voltage differences from submarine cables beneath ocean currents such as the Gulf Stream (Wertheim, 1954) and G.E.K. measurements. For a recent theoretical discussion of G.E.K. measurements, see Sanford and Schmitz (1971).

The electric field is expressible by a potential function as:

$$E = -\nabla\phi$$

and the governing equation is then:

$$\nabla^2\phi = F \cdot \nabla \times V$$

Theoretical treatments have ignored lateral conductivity changes within the ocean (Longuet-Higgins et al., 1954), but oceanic currents are a different water mass from the surroundings. For example, eastward at 35°N across the western boundary of the Gulf Stream, the temperature can change from 16° to 20°C and the salinity from 34 to 35‰ (Stommel, 1958). This corresponds to a conductivity change from 4.4 to 4.9 $\Omega^{-1}m^{-1}$, an increase of 10%. A simple model is then to assume a velocity and conductivity everywhere uniform except at some interface where there is a step $\Delta V$ and $\Delta\sigma$. At the interface the boundary conditions from eq. 5 and 8 are:

$$\phi^+ = \phi^-, \quad \partial\phi^+/\partial x = (1 - \Delta\sigma/\sigma^+)\partial\phi^-/\partial x$$
$$- \boldsymbol{k} \cdot \boldsymbol{F} \times [V^+ - (1 - \Delta\sigma/\sigma^+) V^-]$$

where $x$ is the coordinate normal to the interface. The question is whether a small change in conductivity associated with the ocean current boundary can significantly alter the flow of electric current. This problem is being investigated by L. Spielvogel (private communication, 1972).

Harvey (1972) has measured the time variations in the vertical electric field from a site in the deep ocean near Hawaii. Vertical electric currents tend to be vanishingly small so that self induction will be small. Steady-state theory is therefore appropriate and his signals are a measure of the fluid velocity in the magnetic east–west direction.

## 4. Induction in a thin sheet of variable conductivity and thickness

The conductivity of air is less than $10^{-12}\ \Omega^{-1}m^{-1}$ and the conductivity of basalt beneath the ocean is probably less than $10^{-2}\ \Omega^{-1}m^{-1}$ (Cox, 1971). Therefore the ocean and its sediments are assumed here to be bounded above and below by perfect insulators. This approximation prevents electric current from leaking into the conducting mantle, i.e., $B$-polarization is suppressed.

In the open Pacific, the ocean depths lie between 3 and 6 km with a mean near 4–4.5 km and the sediment thickness is usually less than 600 m and near 300 m for areas not covered by abyssal plains (Shor et al., 1970). The conductivity of sea-floor sediments depends mainly on porosity and is probably no more than one half as conducting as sea water (Bullard and Parker, 1970). Fig.1 shows the vertical section of the conductivity model that includes a conducting mantle and allows for variations in surface ($z = \zeta$) and bottom ($z = -h_1$) topography. In the following let the subscripts $s$ and $z$ represent, respectively, horizontal and vertical components.

The electromagnetic equations for the ocean with sediments can be simplified to thin-sheet approximations following Price (1949) for signals less than one cycle per hour because the ocean plus sediment depth will be small compared with the horizontal wavelength of the signal and the depth of penetration of the signal into the mantle. The long-wavelength results of Sanford (1971) for induction by fluid motion in a layer having small perturbations in bottom and surface topography agree with the thin-sheet approximations. These are found by vertically integrating the electromagnetic equations (3 and 4) over the ocean plus sediment depth, applying the boundary conditions at the top and bottom and letting the depth become small. Then:

$$\boldsymbol{k} \cdot \Delta\boldsymbol{D} = Q,\ \boldsymbol{k} \cdot \Delta\boldsymbol{B} = 0,\ \boldsymbol{k} \times \Delta\boldsymbol{E} = 0,\ \boldsymbol{k} \times \Delta\boldsymbol{B} = \mu \boldsymbol{I}_s \quad (18)$$

where $\boldsymbol{k}$ is a vertical unit vector, $\Delta\boldsymbol{B} = \boldsymbol{B}(z = \zeta) - \boldsymbol{B}(z = -h_2)$, $Q$ is the surface charge, and the vertically averaged electric current is:

$$I_s = \int_{-h_2}^{\zeta} J_s \mathrm{d}z$$

Note the similarity of eq. 18 with the boundary conditions (eq. 5).

Ohm's law for a moving conductor becomes:

$$I_s = S(E_s + \bar{V}_s^* \times k F_z) \quad (19)$$

where $S$ is the layer conductivity:

$$S(x, y) = \int_{-h_2}^{\zeta} \sigma\, \mathrm{d}z$$

and the conductivity weighted averaged velocity (Sanford, 1971) is:

$$\bar{V}_s^* = \frac{1}{S} \int_{-h_2}^{\zeta} \sigma_1 V_s\, \mathrm{d}z$$

The main contribution to $\bar{V}_s^*$ is the barotropic motion as the baroclinic motion tends to be averaged out.

No electric currents leak into the assumed non-

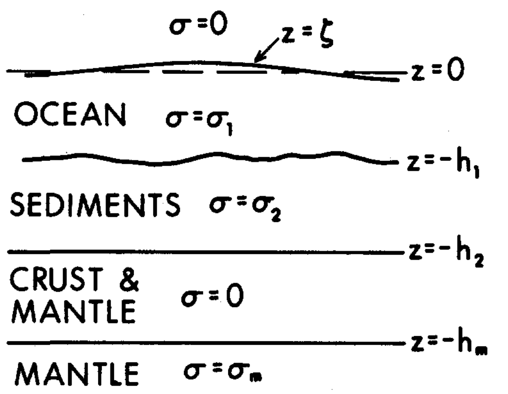


Fig. 1. Model of electrical conductivity of ocean and upper mantle.

conducting air or crust. Thus, even with surface topography, $\nabla_s \cdot \boldsymbol{I}_s = 0$ and $\boldsymbol{I}_s$ is describable by a stream function:

$$\boldsymbol{I}_s = -\boldsymbol{k} \times \nabla_s \psi \tag{20}$$

where $\nabla_s$ is the horizontal gradient operator. Continuity of fluid flow requires:

$$\nabla_s \cdot (h_1 \overline{V}_s) = -\partial\zeta/\partial t \tag{21}$$

where $\overline{V}_s$ is the vertically averaged velocity. Define an electrical equivalent depth as:

$$d(x, y) = S(x, y)/\sigma_0 \tag{22}$$

where $\sigma_0$ is the mean conductivity of the ocean and $d_0$ is the mean depth determined from the mean of $S$ divided by $\sigma_0$.

The electric field is:

$$\boldsymbol{E}_s = (\nabla_s\psi/S - \overline{V}_s^* F_z) \times \boldsymbol{k} \tag{23}$$

and the G.E.K. field is:

$$\boldsymbol{E}_s^m = [\nabla_s\psi/S + (\boldsymbol{V}_s - \overline{V}_s^*)F_z] \times \boldsymbol{k}$$

The G.E.K. observations lead to the estimates $\boldsymbol{V}_s - \overline{V}_s^*$, the local effects, but clearly the observations need to be corrected for $\nabla_s\psi$, i.e., the effects of large-scale electric current flow that, in principle, are dependent on the overall conductivity structure.

The magnetic field can be expressed in terms of the stream function by the Biot-Savart law:

$$\boldsymbol{B} = \frac{\mu}{4\pi} \iiint \frac{\boldsymbol{J} \times \boldsymbol{r}}{r^3}\, dx'\, dy' dz' \tag{24}$$

where $\boldsymbol{r}$ is directed from an electric current element at $(x', y', z')$ to the observation point $(x, y, 0)$ on the thin sheet. Substitution of eq. 20 into eq. 24 and integration over the thickness of the sheet gives:

$$\left.\begin{aligned} \boldsymbol{B}_s &= -\frac{\mu}{4\pi} \iint_A (\boldsymbol{k} \cdot \boldsymbol{G})\, \nabla_s\psi\, dx'dy' \\ B_z &= \frac{\mu}{4\pi} \iint_A \boldsymbol{G} \cdot \nabla_s\psi\, dx'dy' \end{aligned}\right| \tag{25}$$

where the integration is over the surface area $A$ that contains the electric currents. Letting $\boldsymbol{J}_s = \boldsymbol{I}_s/d_0$, the kernel at mid-ocean depth for the oceanic electric currents is:

$$\boldsymbol{G}_0 = r_s^{-2}\,(r_s^2 + \frac{1}{4}d_0^2)^{-1/2}\, \boldsymbol{r}_s$$

Just above and below the sheet the horizontal magnetic field due to the oceanic electric currents can be written, respectively, as:

$$\boldsymbol{B}_s^+ = -\tfrac{1}{2}\mu\nabla_s\psi, \qquad \boldsymbol{B}_s^- = \tfrac{1}{2}\mu\nabla_s\psi$$

The simplest case for the mantle is to assume it is infinitely conducting at an apparent depth:

$$h_a = h_m + (2\mu\sigma_m\omega)^{-1/2}$$

Note that $h_a$ is the real part of $\widetilde{C}$ in eq. 15. The effect of the mantle in this case can be given by electric currents at a depth $2h_a$ that are the negative image of the oceanic electric currents. For the spherical case, the image will be at a radius $R = (R_E - h_a)^2/R_E$ (Bullard and Parker, 1970). For the flat earth, when $d_0 \ll 2h_a$, the kernel for the mantle electric currents is:

$$\boldsymbol{G}_m = (r_s^2 + 4h_a^2)^{-3/2}\,(\boldsymbol{r}_s + 2h_a\boldsymbol{k})$$

The combined ocean and mantle will be $\boldsymbol{G} = \boldsymbol{G}_0 - \boldsymbol{G}_m$ where the negative sign is due to the negative image currents within the mantle.

The vertical magnetic field, by the use of the divergence theorem, can also be written (Larsen, 1968) as:

$$B_z = \frac{\mu}{4\pi}\left[\iint_A \psi\nabla_s \cdot \boldsymbol{G}\, dx'dy' + \oint_B \psi_B \boldsymbol{n} \cdot \boldsymbol{G}\, ds\right] + \frac{\mu}{d_0}\psi \tag{26}$$

The second term on the right is a line integral about the boundary of the conducting region $A$ and $n_s$ is the horizontal normal pointed inwards from the boundary. The boundary is a stream line as the region outside A is nonconducting. If there is only one boundary $\psi_B$ can be set to zero and the line integral vanishes. The third term is due to the line integral about the $1/r_s$ singularity in $G_0$. The kernel $\nabla_s \cdot G_0$, however, has no singularity, a useful feature for numerical calculations.

The thin-sheet approximations imply:

$$\boldsymbol{k} \cdot \nabla_s \times \boldsymbol{E}_s = -\partial B_z/\partial t$$

Substitution of eq.22 and 23 yields:

$$\nabla_s \cdot (\nabla_s \psi/S) - \partial B_z^{\mathrm{i}}/\partial t = \partial B_z^{\mathrm{e}}/\partial t + \nabla_s \cdot (F_z \bar{V}_s^*) \qquad (27)$$

where $B_z$ has been split into an external part, $B_z^{\mathrm{e}}$, that is due to the external ionospheric electric currents and an internal part, $B_z^i$, that is given in terms of $\psi$ by either eq.25 or eq.26.

Eq.27 is the governing equation for the induction of electric currents in a thin sheet or shell. It is an integro-differential equation in the two horizontal space coordinates and it therefore lends itself to numerical solution for the world ocean using the actual shapes and topography of the oceans. Bullard and Parker (1970) have numerically solved eq.27 for the induction of electric currents in the world ocean due to diurnal ionospheric variations. Included in eq.27 is the coastline effect, i.e., the enhancement of $B_z$ near a nonconducting boundary. Coastline effects have been observed at deep oceanic islands (Hermance, 1968; Mason, 1963a; Klein, 1972).

The terms of eq.27 from left to right have the following interpretation:

(1) The resistive term that includes the lateral changes in surface conductivity or depth.

(2) The vertical magnetic field due to electric currents in the ocean and mantle. This term contains the self and mutual induction effects.

(3) The term that induces electric currents in the ocean by ionospheric sources. Note that it is the time rate of change of vertical magnetic flux through the ocean that is the most effective component in inducing horizontal electric currents in a thin layer of such an extensive size as the ocean.

(4) The term inducing oceanic electric currents by barotropic fluid motions. Using eq.21, and $d\bar{V}_s \approx h_1 \bar{V}_s$ this source term expands into:

$$\nabla_s \cdot (F_z \bar{V}_s^*) = -(F_z/d)\partial\zeta/\partial t + \bar{V}_s^* \cdot \nabla_s F_z - (F_z/d)\bar{V}_s^* \cdot \nabla_s d \qquad (28)$$

If $d$, $\zeta$, and $F_z$ are uniform, there will be no electric currents even though $\boldsymbol{V} \times \boldsymbol{F}$ may be nonzero. If $d$ is uniform, the first term on the right is the most important for tidal motion. If the fluid motion has wavelengths of order $R_{\mathrm{E}}$, the second term will also be important. If the flow tends to be horizontally nondivergent, e.g., planetary waves, the second term plays the significant role for the very largest scale motion. The third term depends mainly on bottom topography, and whether it plays a significant role or not, outside regions of large topographic gradients, is not obvious. If one evaluates the third term solely on the value of possible bottom slopes, then the third term is the predominant term almost everywhere in the ocean. However, the other terms may be more effective in generating electric currents as their effects are organized whereas topography will be irregular for most regions. Topography also diffracts fluid motion around it. That is, barotropic flow has a tendency to be parallel to the bottom contours so that the effect of topography is reduced. Finally, stratification and friction will reduce the flow near the bottom. Whether all these effects will make the third term of eq.28 insignificant, however, is not readily apparent.

Solutions of eq.27 have been found for a semidiurnal tidal Kelvin wave propagating northward along the California coast (Larsen, 1968). Observations agree with the solutions in which it was assumed that the bottom contours were parallel with the fluid motion.

Measurements of voltage differences from long submarine cables such as across the North Atlantic (Stommel, 1954) and across the Pacific (M. Richards, private communication, 1972) are probably best interpreted in terms of the global solution of eq.27.

## 5. Separation of fields into ionospheric and oceanic generated parts

Sea floor electric field measurements, at a depth of 4.4 km and a distance of 600 km offshore from California, clearly show a pronounced oceanic tidal signal (Larsen, 1968) so that one must conclude that electric currents due to oceanic and ionospheric sources may be of comparable size. Thus, in order to interpret oceanic electromagnetic fields, one must be able to separate the fields into ionospheric and oceanic generated parts.

One method of separation (Malin, 1970) deals just with the tidal variations and is based on the observation that the magnetic field due to ionospheric tides will be negligibly small near midnight. Then by a comparison of night- and daytime variations, the tidal variations can be separated into oceanic and ionospheric parts provided the tidal period is not a harmonic of the diurnal variation. One could also make a separation of the tidal variations by comparing an oceanic site with a continental site (Larsen, 1968).

A separation procedure, presently being explored for electric field observations on Oahu, Hawaii, is based on a cross-correlation of the electric field with the island horizontal magnetic field. Only a brief outline of the separation procedure and some conclusions will be presented here. The detailed calculations will be given in another paper. Imagine, therefore, for simplicity's sake a single component of the horizontal electric and magnetic field, and let the subscripts O and I refer, respectively, to parts of oceanic and ionospheric origin. The Fourier transforms of the time series into frequency space can be written as:

$$\widetilde{E} = \widetilde{E}_0 + \widetilde{E}_\mathrm{I}, \widetilde{B} = \widetilde{B}_0 + \widetilde{B}_\mathrm{I}, \widetilde{E}_\mathrm{I} = \widetilde{Z}_\mathrm{I}\widetilde{B}_\mathrm{I}, \widetilde{E}_0 = \widetilde{Z}_0\widetilde{B}_0$$

where the $\widetilde{Z}$'s are the transfer functions and $\widetilde{E}(\omega)$ and $\widetilde{B}(\omega)$ are the Fourier transforms of the simultaneously observed fields $E(t)$ and $B(t)$. Solving for $\widetilde{E}_0$, one has:

$$(1 - \widetilde{Z}_\mathrm{I}/\widetilde{Z}_0)\widetilde{E}_0 = \widetilde{E} - \widetilde{Z}_\mathrm{I}\widetilde{B}$$

One observes that for frequencies less than one cycle per hour the oceanic generated part compared with the ionospheric part is more prominent in the horizontal electric field than in the horizontal magnetic field (Larsen, 1968). This is because the horizontal electric field is a measure of the in-situ electric current whereas the horizontal magnetic field, by the Biot-Savart law, is an integral over all electric currents. The interpretation of the sea-floor measurements is consistent with the fact that low-frequency electric currents in the ionosphere are very much stronger than the oceanic electric currents induced by either ionospheric or oceanic sources. The horizontal magnetic field on an island will therefore have a very small contribution from oceanic-induced electric currents. This amounts to letting $\widetilde{B}_0 \ll \widetilde{B}_\mathrm{I}$ so that it is valid to assume:

$$\widetilde{E}_0 = \widetilde{E} - \widetilde{Z}_\mathrm{I}\widetilde{B}$$

and the problem has been reduced to estimating the transfer function $\widetilde{Z}_\mathrm{I}$. The part correlated with $\widetilde{B}$ is $\widetilde{E}_\mathrm{I}$ and the uncorrelated part is $\widetilde{E}_0$.

The transfer function between horizontal $\widetilde{E}_\mathrm{I}$ and $\widetilde{B}_\mathrm{I}$ can be assumed to be a smoothly varying function of frequency and to be insensitive to wavenumber (see section 3). Then the transfer function is expanded in polynominals of frequency as:

$$\widetilde{Z}_\mathrm{I}(\omega) = \widetilde{Z}_M(\omega) \sum_{n=0}^{N} A_n \omega^n \tag{29}$$

where $A_n$ are the complex terms to be determined and various values of $N$ up to 10 are tried. The factor $\widetilde{Z}_M(\omega)$ is the transfer function based on a first estimate of the possible conductivity structure. This factor is found to be necessary in order that a low-order polynomial can adequately approximate the actual transfer function. This process can be iterated, i.e., having determined $\widetilde{Z}_\mathrm{I}$ and a realistic conductivity model from it, the model is then used to supply a new factor $\widetilde{Z}_M$. One stops when $\widetilde{Z}_\mathrm{I} \approx \widetilde{Z}_M$ for $N$ less than say 10.

The coefficients of the polynomials are estimated using the general background continuum by finding the minimum of:

$$R^2 = \sum_{j=1}^{J} Wt(\omega_j)|\widetilde{E}(\omega_j) - \widetilde{B}(\omega_j)\, \widetilde{Z}_M(\omega_j) \sum_{n=0}^{N} A_n \omega_j^n|^2$$

by least squares where $J$ is the number of frequencies considered for a particular finite record length and $Wt(\omega)$ is a weighting function found to be absolutely necessary in order to arrive at transfer functions that give realistic conductivity structures. The least-square procedure is justified if the residues, here $\widetilde{E}_0$, have a white noise spectrum (Jenkins and Watts, 1969). The weighting function is chosen, therefore, to prewhiten the spectrum of the uncorrelated field. After several trials, it was found that the uncorrelated spectrum had a trend $\omega^{-1}$ and pronounced peaks at the diurnal and semidiurnal tidal frequencies. Therefore, the weighting function was chosen to go as $\omega$ with deep notches around 1 and 2 cycles per day. The correlated and uncorrelated time series were then found by taking the Fourier transform of, respectively, $\widetilde{E}_\mathrm{I}$ and $\widetilde{E}_0$.

Fig. 2 shows the separation of a month of electric

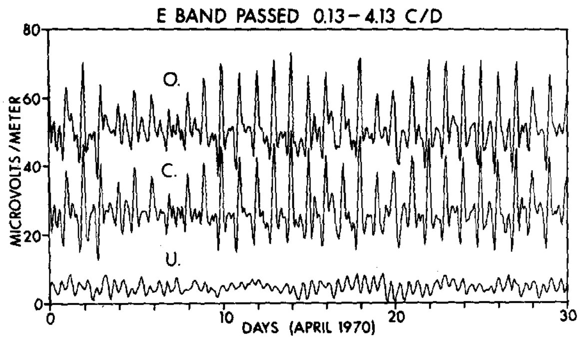


Fig. 2. Monthly sample of separation of electric field on Oahu, Hawaii. Band passed original series labeled *O*, series correlated with Honolulu magnetic field labeled *C*, and uncorrelated series labeled *U*.

field observations on Oahu, Hawaii into parts correlated and uncorrelated with the two horizontal components of the magnetic field at the Honolulu Observatory. The correlated part shows the typical diurnal variations of the ionosphere and, for this record, contains more than 90% of the total energy of the original record. The uncorrelated part has the appearance of a typical sea-level record.

A check on the reliability of the transfer function is provided by a study of the oceanic tidal lines contained in the uncorrelated signal. That is, estimates of different tidal lines from separate, independent segments give nearly identical amplitudes and phases, provided the weighting function is used. This shows that the transfer functions are quite reliable at diurnal and semidiurnal frequencies. The transfer functions are also not biased by the white noise part of $E$, see for example Munk and Cartwright (1966).

It might be argued that there is no real advantage in the polynomial expansion of the transfer function over the usual approach of estimating the transfer function by narrow independent bands of frequencies. Some advantages are the following:

(1) The polynomial expansion leads to a smooth estimate of the transfer function based on the entire frequency range. No further smoothing seems to be required in order to determine realistic conductivity structures.

(2) The smoothed transfer function, since it is a continuous function of frequencies, can be used to separate all frequencies including tides into oceanic and ionospheric generated parts.

(3) Weidelt (1972) has shown, for a layered earth and wavenumber independent of frequency, that the transfer function and its derivatives with respect to frequency have to satisfy certain inequalities. These conditions of inequalities are easily checked for the polynomial expansion case.

The best procedure for the present is to first compute the transfer function by narrow frequency bands in order to determine the degree of smoothing that seems to be allowed and then to apply the proper degree of polynomial smoothing.

It seems likely that the above separation procedure could be applied to midlatitude G.E.K. observations in order to remove ionospheric signals that can be significant (Cox et al., 1964). If the conductivity structure is known at both the base magnetic site and the G.E.K. site, then the transfer functions can be computed and used to estimate the ionospheric electric field at the G.E.K. site using the observed base magnetic variations. The best procedure, since the G.E.K. field is observed for short durations with many interruptions, is to estimate the ionospheric field in the time domain. This will involve a convolution integral between $\widetilde{Z}_I$ and $\widetilde{B}$.

Attempts to separate the continuum of $\widetilde{B}_z$ at the Honolulu Observatory into oceanic and ionospheric parts have failed. The main reason, it is believed, is that at any particular frequency there can not be a single unique transfer function $\widetilde{Z}'$ because the continuum will contain a distribution of wavenumbers. This will result in low coherence between the vertical and horizontal magnetic fields.

## References

Banks, R.J., 1969. *Geophys. J.*, 17: 457.

Beal, H.T. and Weaver, J.T., 1970. *J. Geophys. Res.*, 75: 6846.

Bullard, E.C. and Parker, R.L., 1970. In: A Maxwell (Editor), *The Sea, IV*. Wiley, New York, N.Y., p.695.

Cox, C.S., 1971. In: J. Heacock (Editor), *Geophysical Monograph 14*. A.G.U., Washington, D.C., p.227.

Cox, C.S., Teramoto, T. and Filloux, J., 1964. In: K. Yoshida (Editor), *Studies on Oceanography. Hidaka Anniv. Univ. of Tokyo Press*, p.449.

Cox, C.S., Filloux, J.H. and Larsen, J.C., 1970. In: A. Maxwell (Editor), *The Sea, IV*. Wiley, New York, N.Y., p. 637.

Currie, R., 1968. *J. Geophys. Res.*, 73: 2779.

Fraser, D.C., 1965. *Nature*, 206: 605.

Fraser, D.C., 1966. *Geophys. J.*, 11: 507.
Greenhouse, J., 1972. *Geomagnetic Time Variations on the Sea Floor off Southern California.* Thesis, Univ. of California, San Diego, Calif.
Harvey, R.P., 1972. Oceanic water motions derived from the measurements of the vertical electric field. *Hawaii Inst. Geophys. Rep.*, 72–7. University of Hawaii, Honolulu.
Hermance, J.F., 1968. *Can. J. Earth Sci.*, 5: 515.
Hermance, J.F. and Grillot, L.R., 1970. *J. Geophys. Res.*, 75: 6582.
Horne, R.A. and Frysinger, G.R., 1963. *J. Geophys. Res.*, 68: 1967.
Jenkins, G.M. and Watts, D.G., 1969. *Spectral Analysis and its Applications.* Holden-Day, San Francisco, Calif.
Klein, D., 1972. Geomagnetic time-variations, the island effect, and electromagnetic depth sounding on oceanic islands. *Hawaii Inst. Geophys. Rep.*, 72–3. University of Hawaii, Honolulu.
Kozlov, A.N., Fonarev, G.A. and Shumov, L.A., 1971. *Geomagn. Aeron.*, 11: 633 (English translation).
Larsen, J.C., 1968. *Geophys. J.*, 16: 47.
Larsen, J.C., 1971. *J. Marine Res.*, 29: 28.
Liebermann, L.N., 1962. In: M. Hill (Editor), *The Sea, I.* Wiley, New York, N.Y., p. 469.
Longuet-Higgins, M.S., Stern, M.E. and Stommel, H., 1954. *Pap. Phys. Oceanogr. Meteorol.*, 13: 1.
Maclure, K.C., Hafer, R.A. and Weaver, J.T., 1964. *Nature*, 204: 1290.
Malin, S.R.C., 1970. *Geophys. J.*, 21: 447.
Mason, R.G., 1963a. Spatial depedence of time-variations of the geomagnetic field in the range 24 hrs. 3 mins, on Christmas Island. *Imp. Coll. Sci. Tech., Lond.*, 3: 1.
Mason, R.G., 1963b. Spatial dependence of time-variations of the geomagnetic field on Oahu, Hawaii. *Trans. A.G.U.*, 44–40.
Munk, W.H. and Cartwright, D.E., 1966. *Philos. Trans, R. Soc. Lon., Ser. A*, 259: 533.
Panofsky, W. and Phillips, M., 1955. *Classical Electricity and Magnetism.* Addison Wesley, Reading, Mass.
Parker, R.L., 1970. *Geophys. J.*, 22: 121.
Price, A.T., 1949. *Q.J. Mech. Appl. Math.*, 2: 283.
Richards, M.L., 1970. *A Study of Electrical Conductivity in the Earth near Peru.* Thesis, Univ. of California, San Diego, Calif.
Rogers, R.A.G., 1966. *The Effect of Islands on Electromagnetic Induction in the Oceans.* Thesis, Imperial College, Univ. of London.
Sanford, T.B., 1971. *J. Geophys. Res.*, 76: 3476.
Sanford, T.B. and Schmitz Jr., W.J., 1971. *J. Marine Res.*, 29: 347.
Schmucker, U., 1970. *J. Geomagn. Geoelectr.*, 22: 9.
Shneyer, V.S., 1971. *Geomagn. Aeron.*, 11: 308 (English translation).
Shor Jr., G.G., Menard, H.W. and Raitt, R.W., 1970. In: A. Maxwell (Editor), *The Sea, IV.* Wiley, New York, N.Y., p. 3.
Stommel, H., 1954. *Arch. Meteorol. Geophys. Bioklimatol. Ser. A.*, 7: 292.
Stommel, H., 1958. *The Gulf Stream.* Univ. California Press, Berkeley, Calif.
Swift, D.W. and Wescott, E.M., 1964. *J. Geophys. Res.*, 69: 4149.
Thayer, R.E. and Hermance, J.F., 1972. Magnetic and telluric field observations on San Miguel, Azores. *Trans. A.G.U.*, 53–65.
Tozer, D.C., 1959. In: L. Ahrens, F. Press, K. Rankama and S. Runcorn (Editors), *Physics and Chemistry of the Earth*, 3. Pergamon, London, p. 414.
Wait, J.R., 1970. *Electromagnetic Waves in Stratified Media.* MacMillan, New York, N.Y., 2nd edition.
Weaver, J.T., 1965. *J. Geophys. Res.*, 70: 1921.
Weidelt, P., 1972. *Z. Geophys.*, 38: 257.
Wertheim, G.K., 1954. *Trans. A.G.U.*, 35: 872.