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DATA PROCESSING AND INTERPRETATION IN GEOMAGNETIC DEEP SOUNDING

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This paper is a review of techniques of processing magnetic variation data from the point of view of their effectiveness in determining the parameters that describe a particular electromagnetic induction problem. Among the factors that influence the choice of data-processing technique are: (1) the relative importance of induction by vertical and horizontal magnetic field variations, and (2) the relative importance of local induction in the conductivity anomaly as compared with its influence on the flow of worldwide current systems.

The response of an anomaly can be calculated by transfer-function methods, and presented in the form of frequency-dependent induction vectors or ellipses. The usefulness of internal/external field separation procedures is limited by the problems involved in estimating the spatial behaviour of the normal variation fields.

1. Introduction

The objective of a sequence of data-processing operations applied to a given situation should be to present the information in such a way as to narrow down to a minimum the range of models which need to be considered in the interpretation. The interpretation of a geomagnetic deep-sounding experiment inevitably involves the use of modelling techniques, whether the model is physical or numerical, and the smaller the number of parameters, and the narrower the limits on those that it is essential we specify, the better.

The parameters that have to be specified in electromagnetic induction problems can be divided into two groups – those relating to the conductivity anomaly itself, and those that relate to the external inducing field.

1.1. The conductivity anomaly

1.1.1. The lateral extent of the conductor can be estimated from the spatial behaviour of the anomalous internal magnetic variation fields. If sufficient data are available, the fields can be separated into internal and external parts, and the normal internal field removed. Alternatively, one of the so-called "transfer function" methods can be used, which rely on persistent correlations between magnetic field components to detect the presence of anomalous internal currents. From the resulting pattern of anomalous fields, it should be possible to form an idea of how realistic an interpretation of the anomaly is likely to be if a two-dimensional model is used.

1.1.2. The depth and/or thickness of the conductor is very often one of the parameters that we wish to determine. It is usually assumed that the frequency response of the anomaly provides the best test of the correctness of the selected model, although the horizontal scale of the anomaly will also influence the choice. The frequency response is calculated in the form:

$$Q(f) = B_{\mathrm{ia}}(f) / B_{\mathrm{n}}(f) \qquad (1)$$

$B_{\mathrm{ia}}(f)$ is an anomalous internal field component at frequency f, and $B_{\mathrm{n}}(f) = B_{\mathrm{in}}(f) + B_{\mathrm{e}}(f)$ is the total normal field – the sum of the normal internal and external parts. $B_{\mathrm{ia}}(f)$ and $B_{\mathrm{n}}(f)$ need not be the same component of the magnetic field; often the most diagnostic response is the ratio $Z_{\mathrm{ia}}(f)/H_{\mathrm{n}}(f)$ where Z and H are the vertical and horizontal components respectively.

1.1.3. The conductivity of the anomalous body is similarly estimated by comparing the observed response $Q(f)$ with that computed for theoretical models.

1.1.4. The conductivity of the "normal" region surrounding the anomaly is often taken to be zero in cases where local induction is assumed. In theory, it should be known from the long-period data and the solutions obtained for the global conductivity distribution, but in practice the conductivity at shallow depths (up to 400 km) is not resolved by the longer period variations.

1.2. The inducing field

1.2.1. The distribution of the currents producing the external field variations can be specified, or alternatively the spatial behaviour of the normal magnetic field variations over the earth's surface: $B_n(f)$. Another approach is to determine the variation of the response $Q(f)$ with the direction of the inducing field, either by calculating the induction ellipse from the transfer functions, or by determining the instantaneous response to a particular external field configuration by performing an internal/external separation at a given instant of time. The chief problem in establishing the nature of $B_n(f)$ is that measurements can only be made over a limited area, and only those fields whose spatial wavelength is less than the horizontal extent of the array of instruments can be separated.

1.2.2. Do we need to consider induction only by the vertical component of the field, or by the horizontal component, or both? This decision may affect the choice of approach to the processing and presentation of the data. For instance, users of transfer-function methods often assume that Z_n is small, and that induction is by some horizontal component of the field.

1.2.3. We must also decide whether the anomalous fields are the result of local induction in an isolated conducting body, or whether they are caused by the channelling through a local conductor of a worldwide current system induced elsewhere, e.g., in the oceans.

In this paper, I shall first discuss those points that influence the choice of data-processing technique, and then go on to consider methods of determining the frequency response of an anomaly. The determination of $Q(f)$ involves the separation of $B_{ia}(f)$ from the observed field, and two approaches to this problem have been used. The transfer-function method is generally applied to the results of surveys using only a small number of instruments. The second approach, involving the separation of the fields into parts of internal and external origin by the use of surface integral formulae, requires simultaneous measurements by a large number of magnetometers. The two techniques should not, however, be regarded as mutually exclusive.

2. Induction by vertical and horizontal field variations

Three principal arguments have been advanced in favour of induction by either vertical or horizontal field variations:

(a) Above a horizontally stratified conductor, Z_n should be zero or very small when the spatial wavelength of the external field is large (e.g., Everett and Hyndman, 1967).

(b) Regions of anomalous conductivity are likely to have horizontal dimensions substantially greater than their vertical dimensions. Induction in sheet-like bodies by vertical field variations should be much more effective than induction by the horizontal component (Bullard and Parker, 1971).

(c) The observed high correlations of Z and some horizontal component at anomalous mid-latitude stations indicates induction by the horizontal field (Hyndman and Cochrane, 1972).

2.1. The relative importance of normal horizontal and vertical field variations

Price (1950) showed that, for induction in a half-space of uniform conductivity by a spatially uniform external field:

$$Z_n = Z_{in} + Z_e = 0 \qquad (2)$$

i.e., the field of the internal currents cancels the external field variations. The horizontal field, on the other hand, is enhanced by the field of the internal currents.

From this result it appears that the existence of normal vertical field variations requires non-uniform external fields, and the question arises of how rapidly the importance of Z_n increases as the wavelength of the external field decreases. As a measure of the relative importance of Z_n and H_n we can take:

$$|W_l(f)| = |A_{Zl}(f)/A_{Hl}(f)| \qquad (3)$$

where $A_{Zl}(f)$ and $A_{Hl}(f)$ are the coefficients for the term of order l in a spherical harmonic expansion of

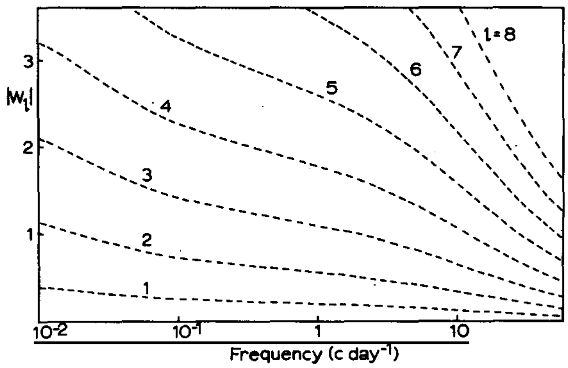

Fig. 1. Relative amplitude of normal vertical and horizontal field variations plotted as a function of frequency for different spherical harmonics of the external field.

the vertical and horizontal magnetic variation fields at frequency f. In Fig. 1, $|W_l(f)|$ is plotted as a function of frequency for different spherical harmonic orders. The conductivity model is one based on the long-period data, and its principal feature is a steep rise in conductivity from 10^{-2} to $2\ \Omega^{-1}.\text{m}^{-1}$ concentrated in the depth range 500–700 km (Banks, 1972). Its usefulness for predicting the response at the higher frequencies is limited by the inability of the long-period variations to resolve the conductivity of the top 400 km. Fig. 1 shows that, for periods less than an hour, $|W_l| \approx 1$ for l greater than 6 or 7, corresponding to wavelengths of 7000 km or less. In the vicinity of

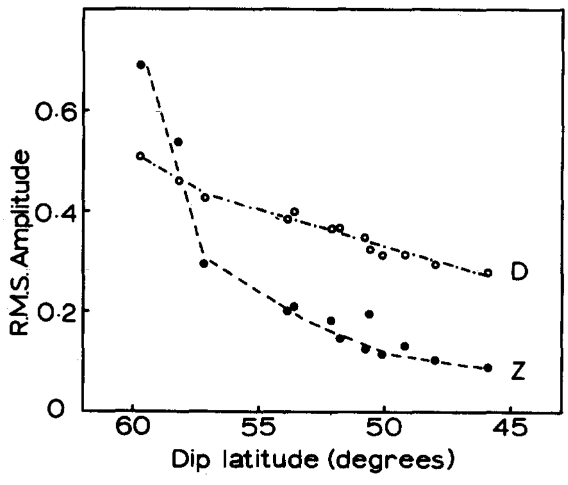

Fig.2. The amplitude of vertical and horizontal field variations at a frequency of 5.5 c day^{-1} plotted against geomagnetic dip latitude.

the auroral or equatorial electrojets, external fields with wavelengths of less than 5000 km are quite possible. In middle latitudes, the source wavelength may be 10,000 km or more. The amplitude of vertical field variations shows a rapid increase at geomagnetic dip latitudes of 50–55° (Fig.2), presumably where the non-uniform fields of the electrojets become important. It appears that Z_n may be larger than H_n at high and very low magnetic latitudes, while H_n is dominant elsewhere.

2.2. *Anomaly dimensions*

If the problem we are dealing with is one of induction in an isolated conducting body, the ratio of the horizontal dimension to the thickness of the body is of critical importance in determining the effectiveness of induction by Z_n and H_n. Bullard and Parker (1971) show that for induction in a disc of radius a, thickness h and conductivity σ, by a uniform vertical field B_0 of frequency f, the relative importance of the induced and inducing fields is given by:

$$\gamma_1 = B_I/B_0 \approx \pi\mu_0\sigma fah/2 \qquad (4)$$

($\mu_0 = 4\pi \cdot 10^{-7}$ henry .m^{-1})
When the inducing field is horizontal, the relevant parameter for induction in a slab of width $2a$ and thickness h is:

$$\gamma_2 = B_I/B_0 \approx \pi\mu_0\sigma fh^2/2 \qquad (5)$$

If the vertical and horizontal variation fields have the same amplitude, the effectiveness of induction by Z as compared with H is given by the ratio:

$$\gamma_1/\gamma_2 = a/h \qquad (6)$$

Provided that Z_n/H_n is greater than h/a, induction in an isolated conducting body should be dominantly by the vertical field, even allowing for its smaller amplitude.

The maximum plausible value for the thickness of an isolated conductivity anomaly is 500 – 600 km, i.e., the depth of the conductivity "step". In any anomaly whose lateral dimensions are more than a few thousand km, induction by the vertical component of the field must be the more important. In practice, the value of h is likely to be a good deal less than 500 km; in the oceans it is only 5 km, and the worldwide current system in the oceans must be induced by the vertical component.

A major difficulty in explaining a magnetic variation anomaly in terms of local induction by H_n is the problem of the location of the return currents. The magnetic effects of the currents induced in an isolated sheet-like conductor will almost cancel at the earth's surface. Hyndman and Cochrane (1972) suggest that the return currents may actually flow at depths of several hundred km. It has generally been assumed in the past that highly conducting regions at or near the surface are effectively insulated from the deep conducting mantle. Hyndman and Cochrane suggest that regions where connections exist may play an important part in controlling the flow of the internal currents. However, by allowing the return currents to flow elsewhere, we are moving towards a global approach to the induction problem, and leaving behind the concept of an isolated conducting body.

2.3. Observed correlations of Z and H

Hyndman and Cochrane (1972) argue that the large correlations observed at middle latitudes between Z and some horizontal component of the field must indicate induction of Z_{ia} by H_n. Their argument rests on the assumption that H_{ia}/H is much less than one, while it is quite probable that Z_{ia}/Z should be of order one.

We can see this by writing:

$$\left. \begin{aligned} Z(f) &= Z_c(f) + N_Z(f) \\ H(f) &= H_c(f) + N_H(f) \end{aligned} \right| \qquad (7)$$

where $Z_c(f)$ correlates perfectly with $H_c(f)$, while $N_Z(f)$ and $N_H(f)$ are the uncorrelated parts of the records. The Z/H coherence is:

$$R^2_{ZH} = \frac{S_{ZcZc}}{S_{ZZ}} \cdot \frac{S_{HcHc}}{S_{HH}} \qquad (8)$$

where S_{ZcZc} is the power spectrum of Z_c, etc. S_{ZcZc}/S_{ZZ} and S_{HcHc}/S_{HH} cannot be determined individually, but it must be true that:

$$\frac{S_{ZcZc}}{S_{ZZ}} > R^2_{ZH} < \frac{S_{HcHc}}{S_{HH}} \qquad (9)$$

It is quite possible to observe Z/H correlations as high as 0.8 at anomalous stations (H here is the horizontal component in such a direction as to maximise R^2_{ZH}), which implies that S_{ZcZc}/S_{ZZ} and S_{HcHc}/S_{HH} are both greater than 0.8. If this result is accounted for in terms of induction by Z, then $Z_c = Z_n$ is the inducing field, and $H_c = H_{ia}$, assuming there to be no normal Z_n/H_n correlation. The implication is that $H_{ia}/H \approx 0.9$, which seems unlikely in view of the apparent uniformity of H at mid-latitudes. Induction by H, on the other hand, would require $Z_c = Z_{ia}$ to be induced by $H_c = H_n$. The consequence, that $Z_{ia}/Z \approx 0.9$, seems to be acceptable to most workers.

2.4. Conclusions

The outcome of the theoretical arguments in sections 2.1 and 2.2 appears to be that for isolated conductivity anomalies of almost any plausible dimensions, the vertical component will be much the most effective in inducing currents, in spite of the relatively small amplitude of normal vertical field variations. When the anomaly with which we are dealing is a feature on a global scale, then it is probably better dealt with in the context of the global induction problem.

The experimental observations discussed in section 2.3 might appear at first sight to contradict the theoretical findings. However, they probably indicate that the majority of magnetic variation anomalies are not the result of local induction, but are instead caused by the channelling of a worldwide current system through local conducting bodies. If this is the case, the arguments about the effectiveness of induction by Z and H are irrelevant. For reasons discussed in section 3.5, such perturbations of the induced current flow can be expected to give rise to the observed Z/H correlations.

3. Transfer functions and their presentation

The "transfer function" method for the detection and separation of anomalous internal fields from magnetic variation data arose largely from the work of Parkinson (1959) and Wiese (1962). A more formal approach was developed by Schmucker (1964), Everett and Hyndman (1967), and Filloux (1967), and described in detail by Schmucker (1970). Cochrane and Hyndman (1970) and Edwards et al. (1971) have introduced other ways of presenting the transfer functions.

3.1. A theoretical approach

We assume that there exists a linear relationship between the Fourier transforms of the anomalous internal fields and the normal field components, measured at some point:

$$Z_{ia}(f) = W_{ZX}(f)X_n(f) + W_{ZY}(f)Y_n(f) + W_{ZZ}(f)Z_n(f)$$
$$Y_{ia}(f) = W_{YX}(f)X_n(f) + W_{YY}(f)Y_n(f) + W_{YZ}(f)Z_n(f)$$
$$X_{ia}(f) = W_{XX}(f)X_n(f) + W_{XY}(f)Y_n(f) + W_{XZ}(f)Z_n(f) \quad (10)$$

Although this formulation allows for variations in the direction of the inducing field, it does not allow for the effect of changes in the wavelength (λ) of the external field. Strictly speaking, the transfer functions (the matrix of W's) should be functions of λ as well as of f. This difficulty apart, the W's should characterize the anomalous conductivity structure in the vicinity of the station. Z_{ia}, Y_{ia}, X_{ia}, Z_n, Y_n, X_n, have to be determined from the observational data. If this can be done, the W's can be estimated by calculating suitable cross-spectra.

Provided the normal fields are known, the anomalous fields can be calculated from:

$$Z_{ia}(f) = Z(f) - Z_n(f) \text{ etc.} \quad (11)$$

which leads to the set of equations:

$$Z(f) = W_{ZX}(f)\,X_n(f) + W_{ZY}(f)\,Y_n(f) + [1 + W_{ZZ}(f)]\,Z_n(f) \quad (12)$$

etc.

The problem is reduced to that of estimating the normal field components $X_n(f)$, $Y_n(f)$, and $Z_n(f)$ at the point of observation. If simultaneous records are available from a station not affected by the anomaly, one that can be classed as "normal", the normal fields at the anomalous station can be approximated by the total observed fields at the normal station. Such an assumption can only be used if the normal field is effectively uniform over the distance separating the anomalous and normal stations.

3.2. Criteria for choosing normal stations

A practical definition of a normal station is that it should be one situated above a horizontally stratified conductivity structure, and sufficiently distant from any lateral discontinuities as to be unaffected by the associated anomalous internal currents. A more satisfying definition would involve the specification of the horizontally stratified conductivity structure as being itself normal, but the practical use of such a definition would require measurements on a global scale.

The response at a normal station, as defined above, will be independent of the direction of the normal horizontal variation field, and the vertical field will not show any persistent correlation with either X or Y (X and Y can be, as usual, geographic north and east components, or geomagnetic north and east components, i.e., H and D). In other words, when the spectral estimates are averaged over sufficient quantities of data, the coherence between any pair of the components X, Y, and Z should be zero.

However, if the coherence is not zero, it is not necessarily an indication of the presence of persistent internal currents; it can equally well be the result of persistent external currents. The observed magnetic field variations at a particular point can be considered to be the sum of contributions from a series of linearly independent current systems. When the relative amplitude of the contributions from these sources changes from one record to another, the effect is to cause the phase difference $\phi_{ZX}(f)$ between Z and X (for instance) to vary in a random manner between records. Consequently, when averages are taken over a sufficient number of different disturbances, we expect to find that the cross-spectrum of $Z(t)$ and $X(t)$ averages to zero, and $R^2_{ZX}(f) = 0$.

If, on the other hand, the relative positions and configurations of the ionospheric currents remain unchanged from one record to another, the measured coherence will not be zero. Magnetic variations caused by persistent electrojets, by the S_q current system, by the ring current, and by persistent internal currents, will show a significant coherence between the field components.

In passing, it is perhaps worth noting that the coherence of a particular component of the field, determined between different stations, can be used to map the position of internal currents. The interstation coherence of the vertical component should fall off most gradually in a direction parallel to the strike of any anomalous internal current concentration, and most rapidly in a direction at right angles to it.

3.3. Calculation of transfer functions in practice

The set of equations (12) is not often used in practice. This is due in part to the very real problems involved in finding a station that satisfies the criteria outlined in section 3.2, and which is operating simultaneously with the variometers deployed around the anomaly.

Instead, some further simplifying assumptions are introduced:

(1) That Z_n is small compared with Z, and the terms W_{ZZ}, W_{YZ}, W_{XZ} can be ignored.

(2) That there is no correlation of Z_n with X_n or Y_n.

(3) That X_n and Y_n can be replaced by X and Y, respectively, i.e., that X_{ia}/X and Y_{ia}/Y are small.

The validity of these assumptions has already been discussed in sections 2.1 – 2.3 and 3.2; they are probably quite safe in middle latitudes. If they are acceptable, we can replace X_n and Y_n by X and Y, and reduce the status of the term involving $W_{ZZ}(f)$ to that of a small uncorrelated "error". This approach has the attraction that the transfer functions $W_{ZX}(f)$ and $W_{ZY}(f)$ can be estimated from the three components of the field recorded at a single station. The very much simplified equation becomes:

$$Z(f) = A(f)\,X(f) + B(f)\,Y(f) + \epsilon(f) \qquad (13)$$

where $\epsilon(f)$ is a residual part of $Z(f)$ that does not correlate with X or Y – presumably Z_n and instrumental noise. The remaining transfer functions involved in the other two equations can not be estimated by a single-station method of this kind.

3.4. Estimation of the transfer functions using cross-spectra

$A(f)$ and $B(f)$ are estimated by minimizing the residual power $S_{\epsilon\epsilon}(f) = \langle \epsilon(f)\,\epsilon^*(f)\rangle$ with respect to A and B (Schmucker, 1970; Everett and Hyndman, 1967; Cochrane and Hyndman, 1970). A simple calculation supplies the following formulae for A and B:

$$A = \frac{S_{ZX}S_{YY} - S_{ZY}S_{YX}}{\overline{S_{XX}S_{YY}} - \overline{S_{YX}S^*_{YX}}}$$

$$B = \frac{S_{ZY}S_{XX} - S_{ZX}S^*_{YX}}{\overline{S_{XX}S_{YY}} - \overline{S_{YX}S^*_{YX}}} \qquad (14)$$

where S_{ZX} is the cross spectrum of $Z(t)$ and $X(t)$ etc., and the asterisk indicates the complex conjugate. An inherent assumption in this calculation is that $\langle \epsilon(f) X^*(f)\rangle = 0$; in other words it is assumed that $\langle Z_n(f) X^*(f)\rangle = 0$.

The cross-spectra can be estimated either by calculating the Fourier transforms of individual disturbances, and then taking a suitable average over transfer estimates obtained from different stretches of data, or by computing the power and cross-spectra for a complex disturbed record such as that of a magnetic storm. In either case, there must be sufficient power at the frequency of interest for the spectral estimates to be meaningful.

3.5. The significance of the observed relations

The equation $Z_c(f) = A(f)\,X(f) + B(f)\,Y(f)$ defines a "preferred plane" along which magnetic field variations of frequency f tend to occur. Parkinson (1959, 1962) studied preferred planes by a graphical method, and indicated the direction of the plane by plotting the horizontal component of a unit vector orthogonal to the plane. The "Parkinson vector" defined in this way points towards anomalous internal concentrations of current.

Parkinson's method did not allow him to study phase differences between field components. In general $A(f)$ and $B(f)$ are complex, and many authors calculate both real and imaginary induction vectors at each frequency. The length of the real vector is:

$$G_R = [(\mathrm{Re}(A))^2 + (\mathrm{Re}(B))^2]^{\frac{1}{2}} \qquad (15)$$

with an azimuth:

$$\theta_R = \mathrm{arctg}\,(\mathrm{Re}(B)/\mathrm{Re}(A)) \qquad (16)$$

and similar expressions are used to calculate the magnitude and direction of the imaginary vector. As defined in this way, the vector lies in the preferred plane, and points away from internal currents. Some authors (e.g., Edwards et al., 1971) then change the azimuth by 180°, so that its direction is the same as that of the Parkinson vector.

It is generally agreed that the vectors so defined point to anomalous concentrations of the internal current. Many workers would further argue that the existence of the correlations between Z and X and Y must indicate that the anomalous internal current is "induced" by the horizontal component with which Z shows maximum correlation, i.e., the component in the direction θ_R. I think that this argument is prob-

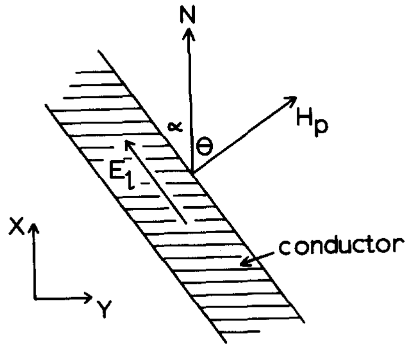

Fig. 3. Geometry of a two-dimensional conductivity anomaly.

ably correct, but that we have to be a little careful in defining what we mean by the word "induced" in this context.

Jones and Price (1970) have investigated the perturbation of the internal current flow and the associated electromagnetic fields produced by a two-dimensional conductor boundary or structure such as that shown in Fig. 3. They show that the equations can be split into two groups dealing with two sets of field components. In the H-polarization case, the current flow is perpendicular to the boundary, and the field components are E_p, H_l, and E_z (E_p is the electric field component perpendicular to the boundary, H_l is the magnetic field component parallel to the boundary, and E_z is the vertical electric field). Jones and Price show that, in this mode, the magnetic field at the earth's surface is unaffected by the conductor boundary. If the external field is essentially uniform, there will be no contribution to the vertical field variations from the H-polarization fields.

In the E-polarization case, the current flow is parallel to the boundary, and the field components involved are E_l, H_p, and Z. The current flow can be split into two parts: the normal horizontal current sheets, and a current which represents the perturbation of the normal currents by the conductor. The normal current sheets produce a magnetic field at the earth's surface that cancels the external vertical field variations. The only contribution to the vertical component comes from the anomalous internal current flow associated with E_l, which is itself a perturbation of the main internal current flow parallel to the conductor boundary. The vertical field variations must therefore correlate with the component of the normal current flow parallel to the boundary, which on a global scale is induced by H_p. It follows that there should be a linear relationship between Z and H_p:

$$Z(f) = G(f)\, H_p(f) \tag{17}$$

By writing $H_p(f) = X(f) \sin \alpha + Y(f) \cos \alpha$, we can show that this equation is equivalent to eq.13 provided that:

$$\text{tg}\, \theta = B(f)/A(f) \tag{18}$$

$$\text{and: } |G(f)|^2 = (|A(f)|^2 + |B(f)|^2) \tag{19}$$

(θ is the azimuth of H_p, rather than the strike of the conductor) which is in agreement with the way the induction vectors are calculated. We conclude that the observed correlation of Z and H_p can be interpreted as indicating that the vertical field variations are largely produced by anomalous internal currents that are perturbations by a conductor boundary of normal currents induced on a worldwide scale by H_p.

3.6. Induction ellipses

Very often, $G(f)$ has a phase close to zero, and in such cases the separate calculation of real and imaginary vectors is a satisfactory way of presenting the transfer functions. However, when $G(f)$ has a substantial phase, it is probably preferable to compute an induction ellipse, as described by Everett and Hyndman (1967). They point out that $A(f)$, $B(f)$ can be

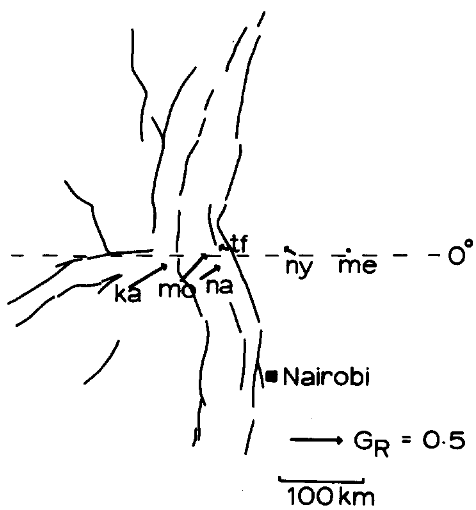

Fig. 4. Real induction vectors at 50-minutes period for stations around the East African Rift Valley.

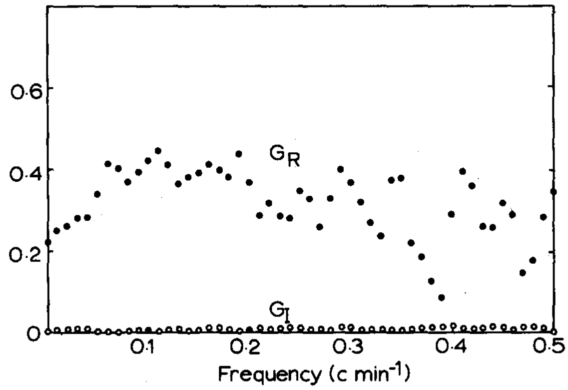

Fig. 5. Magnitudes of real (G_R) and imaginary (G_I) induction vectors at Kabianga, plotted against frequency.

represented by an ellipse, and the azimuth of the major axis (θ_m) relative to the X-direction is calculated by maximizing the expression:

$$Z_c(\theta)\, Z_c^*(\theta) = (A \cos\theta + B \sin\theta)\,(A^* \cos\theta + B^* \sin\theta) \quad (20)$$

with respect to θ. $Z_c(\theta)$ is the correlated vertical field associated with a horizontal variation of unit amplitude and zero phase at azimuth θ. θ_m is given by the equation:

$$\text{tg}\, 2\theta_m = \frac{AB^* - A^*B}{AA^* - BB^*} \quad (21)$$

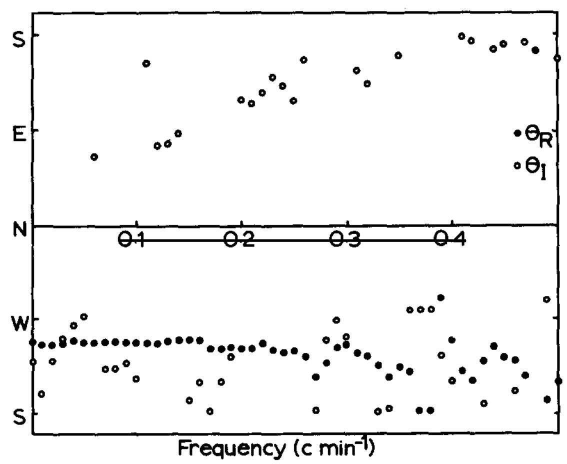

Fig. 6. Azimuths of real (θ_R) and imaginary (θ_I) induction vectors.

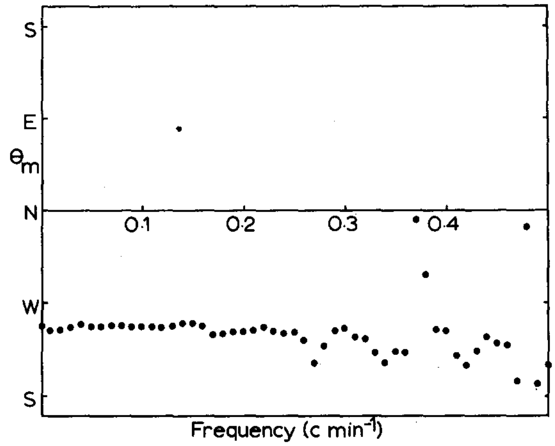

Fig. 7. Azimuth of the major axis of the induction ellipse at Kabianga.

The ellipticity of the ellipse is a measure of how far the two-dimensional assumption is justified. In a true two-dimensional case, the ellipse should degenerate into a straight line perpendicular to the strike of the conductor. Horizontal magnetic variations can be resolved into components along the major and minor axes of the ellipse and we can determine the Z/H_p phase along the major axis. The phase along the minor axis will differ by 90°.

As an example of the different approaches to the presentation of the transfer functions $A(f)$ and $B(f)$, I have calculated induction vectors and the induction ellipse for a station in Kenya, just west of the Rift Valley. In Fig.4, real induction vectors have been plotted for a period of 50 minutes. Their direction has been reversed, so that they point towards the

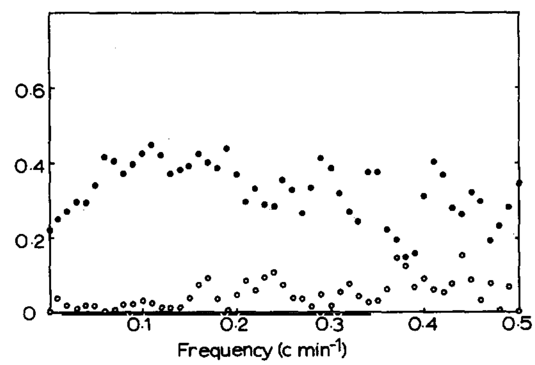

Fig.8. Lengths of the major and minor axes of the induction ellipse (solid circles = major axis, open circles = minor axis).

anomalous currents. They clearly indicate a concentration of current flowing along the Rift. In Fig.5, the magnitudes of the real and imaginary vectors are plotted as functions of frequency. For this station, G_I is very small, indicating that the anomalous vertical field is almost in phase with H_p. The azimuth of the real vector (θ_R) is very consistent; the imaginary vector shows a considerable scatter (Fig.6). The induction ellipse does not throw a lot of extra light on the situation in this case, because the phase of G is small. Fig.7 shows the azimuth of the major axis of the induction ellipse; it coincides almost exactly with θ_R. In Fig.8, the lengths of the major and minor axes of the ellipse are plotted. For periods down to 8 minutes, the ellipticity is large, indicating a close approximation to a two-dimensional situation.

4. Separation of fields into internal and external parts

The formal separation of magnetic variation fields into parts of internal and external origin has not often been attempted, except for some work on two-dimensional problems (e.g., Siebert and Kertz, 1957), and the array studies by Reitzel et al. (1970), Porath et al. (1970), etc. It is only since the design of a relatively inexpensive variometer by Gough and Reitzel (1967) that it has been possible to deploy large two-dimensional arrays of instruments, and to consider the separation of three dimensional fields. What follows is largely by way of comment on the problems that Gough, Reitzel, Porath, and their co-workers have experienced in attempting to separate magnetic variation fields.

4.1. Surface integral formulae

The separation formulae that Porath et al. (1970) use are:

$$I^{\infty}(X,Y) = 2\pi\,(Z_e - Z_i)|_0 = \int_{-\infty}^{+\infty}\!\!\int \frac{(x-x_0)\,X(x,y) + (y-y_0)\,Y(x,y)}{[(x-x_0)^2 + (y-y_0)^2]^{\frac{3}{2}}}\,dx\,dy$$

$$-I^{\infty}(Z,y) = 2\pi\,(Y_e - Y_i)|_0 = -\int_{-\infty}^{+\infty}\!\!\int \frac{(y-y_0)\,Z(x,y)}{[(x-x_0)^2 + (y-y_0)^2]^{\frac{3}{2}}}\,dx\,dy \qquad (22)$$

$$-I^{\infty}(Z,x) = 2\pi\,(X_e - X_i)|_0 = -\int_{-\infty}^{+\infty}\!\!\int \frac{(x-x_0)\,Z(x,y)}{[(x-x_0)^2 + (y-y_0)^2]^{\frac{3}{2}}}\,dx\,dy$$

and, of course:

$$(Z_e + Z_i)|_0 = Z(x_0, y_0)$$
$$(Y_e + Y_i)|_0 = Y(x_0, y_0) \qquad (23)$$
$$(X_e + X_i)|_0 = X(x_0, y_0)$$

(x_0, y_0) is the point at which separation is being attempted.

The procedure employed by Porath et al. (1970) is:

(a) To draw, by interpolation, contour maps of field components at a single instant of time, or maps of Fourier amplitudes at a particular frequency, calculated from the same stretch of data at each station. (In the latter case, two maps have to be drawn for each component, for the cosine and sine transforms.)

(b) The maps of the horizontal components are adjusted to meet the condition that the vertical component of the curl of the magnetic field is zero:

$$\frac{\partial X}{\partial y} = \frac{\partial Y}{\partial x} \qquad (24)$$

This can be done by drawing the contours of X and Y in a suitable way on the same map, or by relaxing the residuals of the line integrals of the magnetic field evaluated around grid squares (Price and Wilkins, 1963).

(c) A "summation window" is chosen, over which the surface integrals are evaluated around each separation point. For the separation to be possible at points near the edges of the array, the contour maps have to be extrapolated outside the array.

4.2. The problem of the "normal" variation fields

The surface integrals (eq.22) should be evaluated over an infinite domain about the separation point

(x_0, y_0). In practice, they can only be evaluated over the area of the summation window (domain D), with the consequence that only those fields can be separated whose scale-lengths are less than the dimensions of D. The normal variation fields, with scale-lengths greater than D, have to be removed either before or after evaluation of the integrals.

Removal of the normal fields B_n before separation should leave zero fields outside the area of the anomaly. Problems arise in estimating the spatial behaviour of B_n across the array, especially if the array does not cover the whole of the anomaly. The experience of groups working with arrays seems to be that simple graphical methods of interpolating the normal field are as effective as any. More use could perhaps be made of data from standard magnetic observatories, particularly in areas such as Europe with a relatively dense network, although many observatories may themselves be affected by localized anomalous internal fields.

The alternative is to evaluate the surface integrals first, and remove the normal fields afterwards. The consequences of this procedure can be seen if we consider the effects on the separation of, for example, the Y-component. What we would like to be able to determine is:

$$2\pi (Y_e - Y_i) = -I^\infty (Z, y) \qquad (25)$$

whereas what is computed is:

$$2\pi (Y_{es} - Y_{is}) = -I^D (Z, y) \qquad (26)$$

The infinite domain over which the integral in eq.25 is evaluated can be split up into two parts: D, and the remainder. By doing this, it is possible to show that:

$$Y_{is} = Y_{ia} + Y_n/2 + I^D (Z_n, y)/4\pi \qquad (27)$$

$$\text{or: } Y_{is} = Y_{ia} + \beta Y_n \qquad (28)$$

$$\text{Also: } Y_{es} = (1-\beta)\, Y_n \qquad (29)$$

In other words, the internal field that can be separated from the array of observations contains a fraction β of the inseparable normal field. The value of β depends on $I^D (Z_n, y)$, i.e., on the spatial behaviour of Z_n. If β is known, a proper separation is possible, since:

$$Y_n = Y_{es}/(1-\beta) \qquad (30)$$

$$\text{and: } Y_{ia} = Y_{is} - \beta\, Y_n \qquad (31)$$

can then be determined. For instance, if the normal vertical field has zero east–west gradient across the array, $I^D (Z_n, y) = 0$, and $\beta = 0.5$.

It is not clear that any real advantage is gained by postponing the estimation of the behaviour of the normal fields to this stage. Nor is it obvious that an internal/external field separation subject to these uncertainties gives better estimates of the anomaly response than those derived by the approach described in section 3.

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