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Letter to the Editors

Computer Modelling of Electrical Conductivity Structures

K. Williamson, C. Hewlett and H. Y. Tammemagi

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The study of the Earth by means of its natural electromagnetic field has received a stimulus with the development of numerical solutions of Maxwell's equations for two-dimensional conductivity structures (Madden & Swift 1969; Wright 1970). In particular, Jones & Pascoe (1971) and Pascoe & Jones (1972) have published general computer programs which have received widespread application. It is the purpose of this note to reveal and discuss an error in their mathematical derivation.

The Taylor's Series expansions illustrated on p. 7 of Jones & Pascoe (1971) can be solved for $(\partial^2 f/\partial y^2)_0$ and $(\partial^2 f/\partial z^2)_0$ by Cramer's Rule or by algebraic manipulation. Substitution into their equation (10) yields

$$f_0\left(\frac{2}{d_1 d_3}+\frac{2}{d_2 d_4}\right)-\eta^2 g_0$$

$$= f_1\left(\frac{2}{d_1{}^2+d_1 d_3}\right)+f_2\left(\frac{2}{d_2{}^2+d_2 d_4}\right)+f_3\left(\frac{2}{d_3{}^2+d_3 d_1}\right)+f_4\left(\frac{2}{d_4{}^2+d_4 d_2}\right). \quad (1)$$

It is clear that equation (1) is only equivalent to equation (12) of Jones & Pascoe under special circumstances, that is, when $d_1 = d_3$ and $d_2 = d_4$ (or when $d_1 = d_2 = d_3 = d_4$). Similarly, their equation (13) is incorrect and the error is carried forward in the subsequent derivations. The electromagnetic responses calculated by the computer programs of Jones & Pascoe (1971) are only correct when the conducting configuration is defined by a grid of identical rectangles or squares.

In order to illustrate that this error can be significant a simple conductivity configuration, as shown in Fig. 1, was solved using computer programs based on those listed by Jones & Pascoe (1971) with the substitution of Subroutine BYCOND from Pascoe & Jones (1972). The programs were executed twice, using the corrected formulae for the second run. Both the *E*-polarization and *H*-polarization cases were considered, and a forty by forty mesh was chosen, with irregular grid spacings (Table 1).

The apparent resistivities calculated for *H*-polarization by the two methods are shown in Table 2; the results deviate by as much as 55 per cent and generally by about 20 per cent. The *E*-polarization case shows similar discrepancies. (Admittedly an extreme case of irregular grid spacing was chosen.)

To achieve continuity of the electromagnetic fields with the boundary conditions, certain criteria must be observed. The grid size in regions where the field changes rapidly must be sufficiently small to justify the neglect of third and higher order derivatives in the Taylor expansion used to form the finite difference equations. In

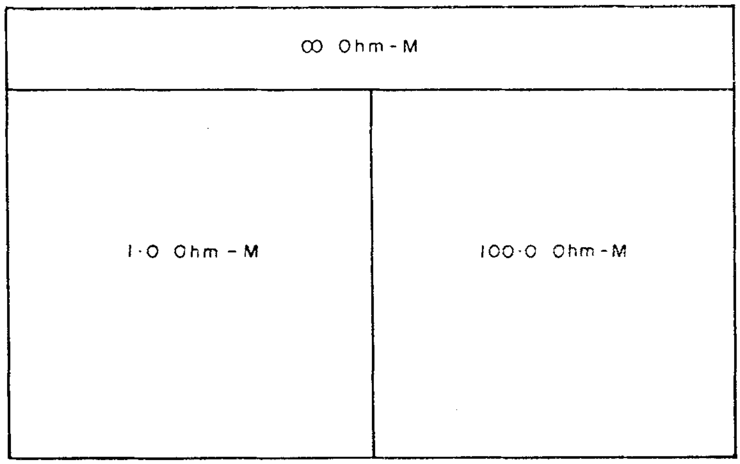

FIG. 1. The resistivity model (not to scale) frequency = 0.01 Hz.

Table 1

Grid spacings in kilometres (K-vertical, H-horizontal). The nine top rows are assigned zero conductivity. The conductivity contrast lies between horizontal grids 20 *and* 21.

Grid number	*H* (km)	*K* (km)	Grid number	*H* (km)	*K* (km)
1	50·0	500·0	21	5·0	2·0
2	20·0	200·0	22	10·0	5·0
3	10·0	100·0	23	5·0	2·0
4	5·0	50·0	24	10·0	5·0
5	2·0	20·0	25	5·0	2·0
6	2·0	10·0	26	10·0	5·0
7	1·0	5·0	27	5·0	2·0
8	2·0	2·0	28	10·0	5·0
9	1·0	1·0	29	5·0	10·0
10	0·5	0·5	30	10·0	20·0
11	1·0	0·2	31	5·0	10·0
12	0·5	0·5	32	10·0	20·0
13	1·0	0·5	33	20·0	10·0
14	0·5	1·0	34	10·0	20·0
15	1·0	0·5	35	20·0	10·0
16	0·5	1·0	36	20·0	20·0
17	1·0	0·5	37	50·0	50·0
18	0·5	1·0	38	100·0	100·0
19	1·0	0·5	39	200·0	200·0
20	0·5	1·0	40	500·0	500·0

Table 2

Apparent resistivities in ohm metres. The percentage difference arising due to the mistake in the original program of Jones & Pascoe is shown in Column 4.

Grid number	Apparent resistivity (computed according to Jones & Pascoe (1971))	Apparent resistivity (using corrected program)	Difference per cent
2	0·74	0·90	18
3	0·74	0·90	18
4	0·75	0·90	17
5	0·77	0·92	16
6	0·78	0·93	16
7	0·79	0·95	17
8	0·79	0·96	18
9	0·78	0·96	19
10	0·77	0·95	19
11	0·75	0·95	21
12	0·72	0·92	22
13	0·70	0·90	22
14	0·64	0·84	24
15	0·61	0·80	24
16	0·51	0·69	26
17	0·46	0·62	26
18	0·32	0·45	29
19	0·24	0·35	31
20	0·08	0·13	38
21	0·05	0·11	55
22	88·33	130·9	33
23	77·66	109·3	29
24	75·39	104·9	28
25	73·30	100·7	27
26	72·72	99·51	27
27	72·12	98·22	27
28	71·96	97·85	26
29	71·84	97·50	26
30	71·84	97·44	26
31	71·92	97·46	26
32	71·98	97·53	26
33	72·13	97·71	26
34	72·46	98·12	26
35	72·61	98·31	26
36	72·86	98·62	26
37	73·02	98·85	26
38	73·22	99·11	26
39	73·31	99·27	26
40	73·46	99·55	26

addition, the distances to the side and bottom boundaries from conductivity contrasts must be large compared with the corresponding skin depths. These essential constraints on grid size can usually only be satisfied by constructing meshes with variable grid spacings. In the vicinity of conductivity contrasts, the grid spacings must be small (relative to the skin depth), increasing in size as they approach the boundaries. Thus, if realistic conductivity models are to be constructed, variable grid sizes are an essential part of the mesh design. This point has not always received the attention it deserves.

Equations (12) and (13) of Jones & Pascoe and the subsequent derivations based on them are incorrect when meshes containing irregular grid spacings are considered. The design of physically realistic conductivity models requires the use of variable grid sizes; equation (1) should therefore be used as a basis for modifying the relevant sections in Jones & Pascoe (1971).

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Geophysics Department
Imperial College
London, SW7

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