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The ocean-coast effect re-examined

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Summary. The Wiener-Hopf technique is used to obtain an analytic solution in closed form for an ocean-coast model that was studied in a recent paper with the aid of a numerical method of solution. The model consists of a uniformly conducting half-space representing the solid Earth overlain by a perfectly conducting half-sheet representing the ocean, with the inducing magnetic field uniform and perpendicular to the edge of the sheet (*E*-polarization). The primary purpose of the investigation is to resolve a large discrepancy between the published values of the horizontal magnetic field over the land surface and those obtained by another more general numerical method applied to a model in which the ocean is represented by a thin sheet of very large finite conductance. The analytic solution reveals errors in the previously published values of the horizontal magnetic field on the land surface but confirms the general accuracy of the results for the other field components. The errors are shown to be responsible for the abnormal behaviour of the Parkinson vectors in the previously published work. For reference the values of the surface field components obtained from the analytic solution are tabulated to three figure accuracy.

1 Introduction

Two recently published papers (Fischer, Schnegg & Usadel 1978; Green & Weaver 1978) have dealt with the mathematical solution of a two-dimensional problem in electromagnetic induction in which a local region of the Earth is represented by a uniformly conducting half-space overlain by a thin sheet whose conductance is variable in one horizontal direction, and in which the harmonic inducing field with time dependence $\exp(i\omega t)$ is horizontal and uniform. Fischer *et al.* were primarily interested in examining the 'coast effect' and so restricted their analysis to a model in which the surface sheet consisted of a single half-plane of perfect conductance, and considered only the (*E*-polarization) solution with the electric

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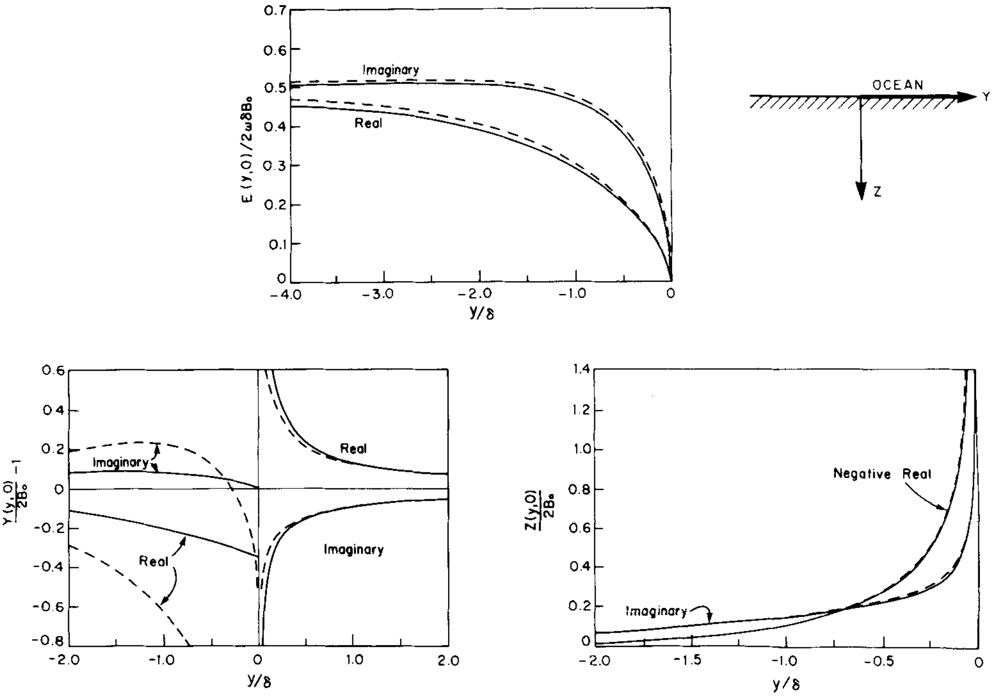

Figure 1. Variations of the electric field E, the horizontal magnetic field Y and the vertical magnetic field Z over the surface of the ocean-coast model according to Green & Weaver (1978) (solid line) and Fischer *et al.* (1978) (broken line). Real and imaginary parts of the dimensionless quantities $E/2B_0\omega\delta$, $Y/2B_0 - 1$ and $Z/2B_0$ are plotted against distance from the coastline in units of skin depth. The fields E and Z vanish over the ocean ($y > 0$).

field parallel to the edge of the half-plane. It is of interest to compare their results with those obtained by the more general method of Green & Weaver where the (finite) conductance $\tau(y)$ of the sheet may vary arbitrarily in the horizontal direction y. In this latter method the model of Fischer *et al.* can be approximated by giving τ some very large constant value for $y > 0$ and putting $\tau = 0$ for $y < 0$.

The results of such a comparison are shown in Fig. 1 (see also Green 1978). The value of τ for $y > 0$ was chosen to be 10^{12} in units of $\sigma\delta$ where σ is the conductivity and δ the skin depth of the underlying medium. It can be seen that excellent agreement between the two methods of calculation is obtained for the electric and vertical magnetic fields and also for the horizontal magnetic field over the highly conducting sheet (the ocean) in the half-plane $y > 0$. However, over the half-plane $y < 0$ (land) there are quite large differences between the two sets of calculated values for both the real and imaginary parts of the horizontal magnetic field. It remains to be decided whether this is a result of a computational error in one of the solutions (and if so which one) or whether it represents some non-uniformity in the solution as $\tau \to \infty$ in $y > 0$. After all it is known that the finite jump discontinuity in the horizontal magnetic field at $y = 0$ becomes an algebraic singularity like $y^{-1/2}$ (as $y \to +0$) when τ is made infinite in $y > 0$ (Weidelt 1971).

This question is best resolved by finding the analytic solution of the ocean-coast model in closed form, thereby avoiding the solution of an integral equation for the electric field by approximate numerical procedures as was necessary in the methods of both Fischer *et al.* and of Green & Weaver. Weidelt (1971) has already obtained an analytic solution of the closely related problem of E-polarization induction in two adjacent half-sheets of different finite conductances located in a region of free space, using a method of contour integration to solve the dual integral equations that describe the problem. It would be possible to adapt Weidelt's method of solution to incorporate the underlying half-space of conductivity σ, and also to modify his boundary conditions to account for the ocean of perfect conductance appearing in the model of Fischer *et al.* However, we shall follow the more traditional approach in which the region above the Earth is initially assumed to have a small non-vanishing conductivity so that the standard Wiener-Hopf technique can be applied to solve the problem. The required solution can then be found by letting this small conductivity tend to zero.

2 A closed form solution

The z-axis is chosen to be vertically downwards and the electric field $E(y, z)$ is assumed to be everywhere parallel to the x-axis. The half-space $z > 0$ represents the conducting Earth while the region $z < 0$ containing the source of the inducing magnetic field is assumed to be a poorly conducting medium with correspondingly large skin depth λ (where $\lambda > \delta$). For our final solution we shall let $\lambda \to \infty$. Vacuum permeability is assumed everywhere.

With displacement currents neglected and a time dependence $\exp(i\omega t)$ understood, the electric field satisfies

$$\nabla^2 E = \begin{cases} (2i/\lambda^2)E & (z < 0) \\ (2i/\delta^2)E & (z > 0) \end{cases} \tag{1}$$

subject to the continuity of E across $z = 0$ together with the boundary conditions

$$E'(y, -0) = E'(y, +0), (y < 0); \qquad E(y, 0) = 0, (y > 0). \tag{2}$$

Here E' denotes the derivative of E with respect to z. In addition we require $E \to 0$ as $z \to +\infty$ and $E(y, z) \sim E_\pm(z)$ as $y \to \pm\infty$ where $E_\pm(z)$ are the appropriate one-dimensional solutions given by

$$E_+(z) = -\omega B_0 \lambda (1+i) \begin{cases} \sinh[(1+i)z/\lambda] & (z \leqslant 0) \\ 0 & (z \geqslant 0) \end{cases}$$

and

$$E_-(z) = \frac{\omega B_0 \lambda (1+i)}{\delta + \lambda} \begin{cases} \delta \cosh[(1+i)z/\lambda] - \lambda \sinh[(1+i)z/\lambda], & (z \leqslant 0) \\ 2\delta \exp[-(1+i)z/\delta], & (z \geqslant 0) \end{cases}$$

where the constant B_0 is the amplitude of the source magnetic field (in the y-direction) measured on the surface $z = 0$. Now the function

$$f(y, z) = E(y, z) - E_+(z) \tag{3}$$

clearly satisfies equation (1) subject to the boundary conditions $f \to 0$ as $y \to +\infty$ and as $z \to \pm\infty$, and

$$f \sim \frac{\omega B_0 \lambda \delta (1+i)}{\lambda + \delta} \begin{cases} \exp[(1+i)z/\lambda] & (z \leqslant 0) \\ \exp[-(1+i)z/\delta] & (z \geqslant 0) \end{cases}$$

as $y \to -\infty$. Clearly f is continuous across $z = 0$ and the surface boundary conditions (2) become

$$f'(y, -0) = 2i\omega B_0 + f'(y, +0), (y < 0); \qquad f(y, 0) = 0 \quad (y > 0). \tag{4}$$

The solution of (1) subject to mixed boundary conditions on $z = 0$ can be obtained by the Wiener-Hopf technique. We shall only sketch the method of solution here (see Dawson & Weaver 1979 for a complete treatment of the method applied to a similar induction problem). Defining $f = f_+ + f_-$ where $f_+ = 0$ for $y < 0$, $f_- = 0$ for $y > 0$, and introducing the Fourier transform

$$F(\zeta, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y, z) \exp(iy\zeta)\, dy$$

where $\zeta = \xi + i\eta$, we find that the solution of (1) within the strip $-1/\lambda < \eta < 0$ in transform space is

$$F_+(\zeta, z) + F_-(\zeta, z) = F_-(\zeta, 0) \begin{cases} \exp[z\gamma(\zeta, \lambda)] & (z < 0) \\ \exp[-z\gamma(\zeta, \delta)] & (z > 0) \end{cases} \tag{5}$$

where $\gamma(\zeta, \alpha) = (\zeta^2 + 2i/\alpha^2)^{1/2}$. The functions F_+ and F_-, the Fourier transforms of f_+ and f_-, are analytic in the half-planes $\eta > -1/\lambda$ and $\eta < 0$ respectively. The solution above automatically satisfies all the boundary conditions except the first condition in (4) which in transform space becomes

$$F'_-(\zeta, -0) = F'_-(\zeta, +0) + A/\zeta \tag{6}$$

where $A = \omega B_0 (2/\pi)^{1/2}$. Differentiating (5) at $z = \pm 0$, substituting from (6) and rearranging we obtain

$$\frac{F'_+(\zeta, -0) - F'_+(\zeta, +0)}{K_+(\zeta)} - \frac{A}{\zeta} \frac{K_+(\zeta) - K_+(0)}{K_+(\zeta) K_+(0)} = K_-(\zeta) F_-(\zeta, 0) - \frac{A}{\zeta K_+(0)} \tag{7}$$

where $K_+(\zeta)$ and $K_-(\zeta)$ are non-vanishing and analytic in the respective half-planes $\eta > -1/\lambda$ and $\eta < 0$, with

$$K_+(\zeta) K_-(\zeta) = \gamma(\zeta, \delta) + \gamma(\zeta, \lambda) = K(\zeta). \tag{8}$$

The usual arguments based on analyticity and Liouville's theorem show that each side of equation (7) vanishes, whence by equation (5) and Fourier inversion

$$f(y, z) = \frac{\omega B_0 \lambda \delta (1 - i)}{2\pi (\lambda + \delta)} \int_{-\infty - ia}^{\infty - ia} \frac{R_+(\zeta)}{\zeta R(\zeta)} \exp[-i\zeta y + z\gamma(\zeta, \lambda)]\, d\zeta \quad (z < 0) \tag{9}$$

where $0 < a < 1/\lambda$ and we have defined

$$R(\zeta) = K(\zeta)/K(0), \qquad R_+(\zeta) = K_+(\zeta)/K_+(0). \tag{10}$$

For its evaluation on the surface $z = 0$ it is convenient to transform the integral in (9) by closing the contour at infinity with excursions along the hyperbolic branch cuts $\xi\eta = -1/\lambda^2$ and $\xi\eta = -1/\delta^2$ from $\mp i\infty$ to the branch points at $\pm(1 - i)/\lambda$ and $\pm(1 - i)/\delta$ respectively (see Dawson & Weaver 1979). When $y > 0$ the closed contour C_1 is in the lower half-plane; when $y < 0$ the closed contour C_2 is in the upper half-plane. Around C_1 the integral vanishes by Cauchy's theorem whereas around C_2, which encloses the pole at $\zeta = 0$, the value of the integral is unity by the residue theorem. The substitutions $u = (\eta^2 - 1/\lambda^4\eta^2)^{1/2}$ and $u = (\eta^2 - 1/\delta^4\eta^2)^{1/2}$ for the portions of the contours along the respective branch cuts, permit the solution for the electric field in $z < 0$ to be written, with the aid of (3), in the separate forms for positive and negative y as follows:

for $y > 0$,

$$\frac{E(y,z)}{\omega B_0} = -\lambda(1+i)\left[\sinh\frac{(1+i)z}{\lambda} + \frac{\delta(1-i)}{\pi\beta(\lambda+\delta)}\int_0^\infty \frac{g[u,y,-\gamma(u,\lambda)]}{u+\gamma(u,i\beta)}\sin zu\,du\right] \tag{11}$$

and for $y < 0$

$$\frac{E(y,z)}{\omega B_0} = \frac{\lambda(1+i)}{\lambda+\delta}\left[\delta\cosh\frac{(1+i)z}{\lambda} - \lambda\sinh\frac{(1+i)z}{\lambda}\right]$$
$$+\frac{\beta^2}{\pi}\int_0^\infty \{ug[u,y,\gamma(u,\delta)]\exp[-iz\gamma(u,\beta)]$$
$$-g[u,y,\gamma(u,\lambda)]\,[u\cos zu - i\gamma(u,i\beta)\sin zu]\}\,du \tag{12}$$

where $\beta = \lambda\delta/(\lambda^2-\delta^2)^{1/2}$ and $g(u,y,\alpha) = u\exp(y\alpha)R_+(i\alpha)/\alpha^2$. The associated magnetic field components, Y horizontal and Z vertical, are given at once by the Maxwell equations

$$Y(y,z) = (i/\omega)\,\partial E(y,z)/\partial z, \qquad Z(y,z) = -(i/\omega)\,\partial E(y,z)/\partial y. \tag{13}$$

A standard procedure (see, e.g. Noble 1958, p. 16) for decomposing the function $R(\zeta)$ gives

$$R_+(\zeta) = \exp\left(\int_0^\zeta \phi(w)\,dw\right) \tag{14}$$

where

$$\phi(w) = \frac{1}{2\pi i}\int_{-\infty-ib}^{\infty-ib}\frac{\zeta\,d\zeta}{(\zeta-w)\gamma(\zeta,\lambda)\gamma(\zeta,\delta)} \tag{15}$$

and $0 < b < 1/\lambda$. The integrand in (15) is $R'(\zeta)/\{(\zeta-w)R(\zeta)\}$ obtained from (8) and (10). The integral can be transformed with the aid of Cauchy's theorem by closing the contour in the lower half-plane cut along the line $\zeta = r\exp(-\tfrac{1}{4}\pi i)$ from $r=\infty$ to the branch point at $r=\sqrt{2}/\lambda$ and passing through the other branch point at $r=\sqrt{2}/\delta$. Note that between infinity and its branch point the function γ has a negative sign on the side of the cut facing the imaginary axis, and a positive sign on the other side. Thus the product $\gamma(\zeta,\lambda)\gamma(\zeta,\delta)$ changes sign from one side of the cut to the other only between the branch points $r=\sqrt{2}/\lambda$ and $r=\sqrt{2}/\delta$. The substitution $\sin\theta = \beta(\tfrac{1}{2}r^2 - 1/\lambda^2)^{1/2}$ simplifies the integral further so that (15) becomes

$$\phi(w) = -\frac{(1+i)\beta}{2\pi}\int_0^{\pi/2}\frac{d\theta}{(\sin^2\theta + \beta^2/\lambda^2)^{1/2} - w\beta\sqrt{\tfrac{1}{2}i}}. \tag{16}$$

If we now let $\lambda\to\infty$, so that $\beta\to\delta$ and $\gamma(u,\lambda)\to|u|$, then (11) and (12) reduce to the solutions for the electric field in a non-conducting region $z < 0$. If we differentiate this solution according to (13) and then set $z = 0$ we finally obtain the desired expressions for the surface electromagnetic field. The non-vanishing components of this field are

$$\frac{E(y,0)}{\omega\delta B_0} = 1 + i + \frac{\delta}{\pi}\int_0^\infty\left\{\frac{u^2R_+[i\gamma(u,\delta)]}{u^2+2i/\delta^2}\exp[y\gamma(u,\delta)] - R_+(iu)\exp(yu)\right\}du,\ (y<0) \tag{17}$$

$$\frac{Y(y,0)}{B_0}-2=\begin{cases}\dfrac{\delta^2}{\pi}\displaystyle\int_0^\infty\left\{\frac{u^2R_+[i\gamma(u,\delta)]}{\gamma(u,\delta)}\exp[y\gamma(u,\delta)]-\gamma(u,i\delta)R_+(iu)\exp(yu)\right\}du,\ (y<0)\\[2ex] \dfrac{(1-i)\delta}{\pi}\displaystyle\int_0^\infty\frac{\exp(-yu)}{R_+(iu)}\,du\quad (y>0)\end{cases}\tag{18}$$

$$\frac{Z(y,0)}{B_0}=-\frac{i\delta^2}{\pi}\int_0^\infty\left\{\frac{uR_+[i\gamma(u,\delta)]}{\gamma(u,\delta)}\exp[y\gamma(u,\delta)]-R_+(iu)\exp(yu)\right\}u\,du\quad (y<0).\tag{19}$$

In the second equation (18) we have noted that $R_+(-iu)=R(iu)/R_+(iu)$ since R is an even function (Noble 1958, p. 17). The limiting form of (16) as $\lambda\to\infty$ can be integrated exactly to give

$$2\pi\phi(w)=\frac{(1+i)\delta}{(1-\tfrac{1}{2}i\delta^2w^2)^{1/2}}\log\frac{(1-\delta w\sqrt{\tfrac{1}{2}i})^{1/2}-(1+\delta w\sqrt{\tfrac{1}{2}i})^{1/2}}{(1-\delta w\sqrt{\tfrac{1}{2}i})^{1/2}+(1+\delta w\sqrt{\tfrac{1}{2}i})^{1/2}},\tag{20}$$

which, substituted in (14) and simplified by the change of variable $\delta w\sqrt{\tfrac{1}{2}i}=-2s/(1+s^2)$ gives $R_+(\zeta)$ in the form

$$R_+(\zeta)=\exp\left(-\frac{2}{\pi}\int_0^{h(\zeta)}\frac{\log s}{1+s^2}\,ds\right),\tag{21}$$

where

$$h(\zeta)=\frac{-\tfrac{1}{2}(1+i)\delta\zeta}{1+(1-\tfrac{1}{2}i\delta^2\zeta^2)^{1/2}}.$$

For the functions $R_+(iu)$ and $R_+[i\gamma(u,\delta)]$ appearing in the integrands of (17), (18) and (19) we note that

$$h(iu)=\frac{\tfrac{1}{2}(1-i)\delta u}{1+(1+\tfrac{1}{2}i\delta^2u^2)^{1/2}},\qquad h[i\gamma(u,\delta)]=\frac{\tfrac{1}{2}(1-i)\delta\gamma(u,\delta)}{1+\delta u\sqrt{\tfrac{1}{2}i}}\tag{22}$$

so that as u varies from 0 to ∞, $h(iu)$ and $h[i\gamma(u,\delta)]$ range from 0 to $-i$ and from 1 to $-i$ respectively with their moduli always less than unity. Thus the contour of integration in (21) can be chosen such that $|s|<1$, allowing us to expand the integrand as an infinite series and then to integrate term by term. The result is

$$\int_0^{h(\zeta)}\frac{\log s}{1+s^2}\,ds=\log h(\zeta)\arctan h(\zeta)-\sum_{n=0}^{\infty}(-1)^n\frac{\{h(\zeta)\}^{2n+1}}{(2n+1)^2}.\tag{23}$$

From equations (21), (22) and (23) the required numerical values of R_+ are readily computed for substitution in (17), (18) and (19), from which the solutions for the surface electromagnetic field can be evaluated by numerical integration.

3 The horizontal magnetic field over the land surface

The expression for $Y(y,0)$, $(y<0)$, given by the first of equations (18) is of particular interest because it represents one of the field components that is commonly recorded near a coastline – the horizontal magnetic field over the land surface – and it is in this component that the discrepancy between the calculations of Fischer *et al.* (1978) and Green & Weaver

(1978) occurs. It is possible to transform the integral in this expression into a simpler form that is not only more convenient for numerical calculation but also obviously real at $y = 0$.

The substitution $v = \gamma(u, \delta)$ in the first term of the integral transforms it into

$$\int_\Gamma \gamma(v, i\delta) R_+(iv) \exp(yv)\, dv$$

where the contour of integration Γ runs from $(1 + i)/\delta$ to $+\infty$ along the half-branch of the rectangular hyperbola joining these points in the complex plane. The contour of integration for the second term can now be displaced from the real axis to run along the radial line from 0 to $(1 + i)/\delta$ in the complex plane and thence to $+\infty$ along the hyperbolic contour Γ. When the two terms are combined in equation (18) the integrals along Γ exactly cancel leaving only the integral from 0 to $(1 + i)/\delta$ of the second term. Thus, rearranged so that $\frac{1}{2}\delta u(1 - i)$ is the variable of integration, the expression for the surface magnetic field in the region $y < 0$ becomes

$$\frac{Y(y, 0)}{2B_0} = 1 - \frac{1}{\pi}\int_0^1 (1 - u^2)^{1/2} S(u) \exp[(1 + i)yu/\delta]\, du \tag{24}$$

where $S(u) = R_+[(1 - i)u/\delta]$. By (21) we see that $S(u)$ is purely real because

$$h[(i - 1)u/\delta] = \{1 - (1 - u^2)^{1/2}\}/u.$$

It follows that

$$\frac{Y(-0, 0)}{2B_0} = 1 - \frac{1}{\pi}\int_0^1 (1 - u^2)^{1/2} S(u)\, du \tag{25}$$

is also real, so that $\operatorname{Im} Y(-0,0) = 0$. A Gauss–Legendre evaluation of (25) gives $\operatorname{Re} Y(-0,0)/2B_0 = 0.650$.

Both of these results agree with the curves in Fig. 1 based on the Green & Weaver (1978) calculations. Fischer *et al.* (1978) claim that a detailed investigation of the behaviour of the field as $y \to -0$ shows that the real and imaginary parts of $Y(-0, 0)/2B_0$ approach 1.14 and 0 respectively, but it is not clear how these limiting values could be reached by the broken line curves in Fig. 1, both of which appear to be leading towards large negative values as $y \to -0$, without the horizontal gradient of the field undergoing unreasonably large and sudden changes close to the origin. In any case their value of 1.14 for the limiting value of the real part of the field does not agree with the results obtained from (25).

4 Numerical results

A Gauss–Legendre numerical integration was used to evaluate the surface field components from equations (17), (19), (24) and the second of equations (18), for various values of y. The electric and vertical magnetic field values so obtained were indistinguishable, when plotted graphically, from the Green & Weaver values depicted by the solid line graphs in Fig. 1. The plotted values of the horizontal magnetic field were also coincident with the solid line graphs for $y < 0$, but for $y > 0$ they tended to follow more closely the broken line graphs of Fischer *et al.* as the field approached its singularity at $y = +0$. A detailed examination of the asymptotic behaviour as $y \to -0$ of the integrals in equations (18) and (19) shows that this singularity, and also that in Z, is algebraic and of order $y^{-1/2}$.

The values of the real and imaginary parts of the dimensionless field components $E/2B_0\omega\delta$, $Y/2B_0$, and $Z/2B_0$ are given in Table 1 to three figure accuracy for selected values

Table 1. Surface field values obtained from the analytic solution.

$\frac{\|y\|}{\delta}$	$E(y,0)/2B_0\omega\delta$		$Y(y,0)/2B_0$				$Z(y,0)/2B_0$	
	Re $y<0$	Im	Re $y<0$	Im	Re $y>0$	Im	Re $y<0$	Im
0	0	0	0.650	0	∞	-∞	-∞	∞
0.05	0.058	0.137	0.658	0.0079	2.091	-0.555	-1.324	0.598
0.1	0.084	0.190	0.666	0.0152	1.704	-0.382	-0.894	0.438
0.2	0.121	0.261	0.682	0.0286	1.441	-0.258	-0.574	0.327
0.3	0.151	0.310	0.698	0.0403	1.330	-0.203	-0.424	0.276
0.4	0.177	0.348	0.714	0.0503	1.266	-0.170	-0.330	0.245
0.5	0.200	0.377	0.729	0.0590	1.224	-0.147	-0.265	0.221
0.6	0.221	0.401	0.743	0.0663	1.194	-0.131	-0.216	0.202
0.7	0.240	0.421	0.758	0.0724	1.172	-0.118	-0.178	0.185
0.8	0.258	0.437	0.771	0.0774	1.154	-0.107	-0.147	0.171
0.9	0.274	0.450	0.784	0.0814	1.140	-0.099	-0.122	0.157
1.0	0.290	0.461	0.796	0.0846	1.128	-0.092	-0.101	0.145
1.25	0.323	0.482	0.825	0.0892	1.106	-0.078	-0.063	0.119
1.5	0.350	0.494	0.849	0.0902	1.090	-0.069	-0.038	0.098
1.75	0.372	0.501	0.870	0.0888	1.079	-0.061	-0.022	0.080
2.0	0.390	0.506	0.888	0.0856	1.070	-0.055	-0.012	0.065
2.25	0.404	0.508	0.903	0.0814	1.063	-0.050	-0.005	0.053
2.5	0.416	0.508	0.915	0.0766	1.057	-0.046	-0.001	0.043
2.75	0.426	0.508	0.925	0.0717	1.053	-0.043	0.001	0.035
3.0	0.434	0.508	0.934	0.0668	1.049	-0.040	0.002	0.028
3.25	0.440	0.507	0.941	0.0622	1.045	-0.037	0.003	0.023
3.5	0.445	0.506	0.947	0.0578	1.042	-0.035	0.003	0.019
4.0	0.454	0.505	0.955	0.0501	1.037	-0.031	0.003	0.014
5.0	0.464	0.503	0.966	0.0388	1.030	-0.026	0.001	0.014
∞	0.5	0.5	1.0	0	1.0	0	0	0

of $|y|/\delta$. These exact results attest to the accuracy of the general numerical method of Green & Weaver (1978) for E-polarization problems. They also reveal errors in the values of the horizontal magnetic field for $y < 0$ (land) obtained by Fischer *et al.* (1978), but confirm the general accuracy of their results for the electric field, the vertical magnetic field and for the horizontal magnetic field in the region $y > 0$ (ocean). In this regard it is interesting to note that in a subsequent paper Fischer (1979) found that 'something went wrong' when calculating the real and imaginary Parkinson vectors whose lengths are defined by $v_r = \sin\theta_r$ and $v_i = \sin\theta_i$ respectively, where

$$\tan\theta_r = -\operatorname{Re}[Z(y,0)/Y(y,0)], \qquad \tan\theta_i = -\operatorname{Im}[Z(y,0)/Y(y,0)]. \tag{26}$$

His calculated variations of v_r and v_i over the land surface were clearly in error, especially v_r which even went negative near the coastline. Fischer attributed this abnormal behaviour to the extreme features (i.e. the thin sheet and perfect conductivity) of the ocean model. Now the local field $Y(y,0)$ in definition (26) is really only an approximation to the regional field $2B_0$ because the existence of Parkinson vectors for real data recorded at any given site depends on an assumed linear relation between the induced vertical field and the regional horizontal field. When Fischer replaced $Y(y,0)$ by $2B_0$ in (26) be obtained quite reasonably behaved Parkinson vectors. The explanation of this is now clear; the original errors in v_r and v_i were due not to any unrealistic features of the model but to the fact that incorrect values for $Y(y,0)$ were used in the calculation. When the accurate values of Y and Z given in Table 1 are substituted in equation (26) it is found that both v_r and v_i vary in the expected manner, their magnitudes increasing steeply as the ocean coast is approached with v_r positive and v_i negative. Moreover it is apparent from columns 4 and 5 in Table 1 that $Y(y,0)/2B_0$ does not deviate too far from the regional value of $1 + i0$ over the whole of land surface, so that it is indeed possible to approximate the regional field by the local field when defining the Parkinson vectors, even for this highly idealized model.

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