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Decomposition of Magnetotelluric Impedance Tensors in the Presence of Local Three-Dimensional Galvanic Distortion

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There are many occasions on which the magnetotelluric impedance tensor is affected by local galvanic distortion (channelling) of electric currents arising from induction in a conductive structure which is approximately two-dimensional (2-D) on a regional scale. Even though the inductive behavior is 2-D, the resulting impedance tensor can be shown to have three-dimensional (3-D) behavior. Conventional procedures for rotating the impedance tensor such as minimizing the mean square modulus of the diagonal elements do not in general recover the principal axes of induction and thus do not recover the correct principal impedances but rather linear combinations of them. This paper presents a decomposition of the impedance tensor which separates the effects of 3-D channeling from those of 2-D induction. Where the impedance tensor is actually the result of regional 1-D or 2-D induction coupled with local frequency independent telluric distortion, the method correctly recovers the principal axes of induction and, except for a static shift (multiplication by a frequency independent real constant), the two principal impedances. Also obtained are two parameters (twist and shear), which partially describe the effects of telluric distortion. It is shown that the tensor operator which describes the telluric distortions can always be factored into the product of three tensor sub-operators (twist, shear, local anisotropy) and a scalar. This product factorization allows assimilation of local anisotropy, if present, into the regional anisotropy. The method of decomposition is given in the paper along with a discussion of the improvements obtained over the conventional method and an example with real data.

INTRODUCTION

Interpretation of experimental magnetotelluric results is easiest in those cases where the surveyed structure is one-dimensional (1-D) or 2-D. However, experimentally determined magnetotelluric impedance tensors rarely conform to the ideal 2-D impedance tensor. That is, there is no rotation of the coordinate axes such that the diagonal elements of the tensor are both exactly zero. This may occur either (1) because of data errors in the case of 1-D or 2-D induction, (2) because of 3-D induction, or (3) because of 1-D or 2-D induction coupled with the effects of galvanic (frequency-independent) telluric distortion. For historical reasons connected with the ease of calculating inductive responses for 2-D structures and the difficulty of doing the same for 3-D structures, it has been customary to assume the first of the above possibilities in presenting data and to rotate the coordinate axes so as to make the measured tensor as close as possible to an ideal 2-D tensor (one with zero diagonal elements) in some sense (usually a least squares sense [e.g. *Swift*, 1967]).

Improvements in data quality in recent years have made it obvious that the third possibility (1-D or 2-D induction coupled with 3-D telluric distortion) is important in practice. The measured impedance tensor, if such distortion is present, need not be close to a true 2-D impedance tensor, and rotation or decomposition methods based on this assumption make no sense in this situation. A number of alternative decomposition methods have been proposed [e.g. *Eggers*, 1982; *Spitz*, 1985; *LaTorraca et al*, 1986; *Yee and Paulson*, 1987] which do not make any simplifying assumptions about the physical model and use as many parameters to represent the tensor as there are data (eight real parameters in contrast to the five kept by rotation to an idealized 2-D tensor). In the case of induction in one or two-dimensions coupled with 3-D galvanic scattering, then these general decompositions may not be optimal since they fail to take advantage of the simplicities of the underlying model.

Recent work by *Bahr* [1988] has indicated galvanic distortion or current channelling does not destroy most of the information present about an underlying 2-D inductive process. Bahr demonstrates possible ways in which this information can be recovered and shows an application to a field situation. Therefore the physical approach we take to the decomposition problem is to make the specific assumption that a measured impedance tensor is produced by local galvanic distortion, by arbitrary 3-D structures, of the electric currents induced on the large scale in a regionally 1-D or 2-D structure. Even when this model is not true for all frequencies of the data set, it may still be true over limited frequency ranges since the definition of a "regional" scale can be different for different frequency ranges. We present a decomposition appropriate to this particular physical model which, although not the most general model, has considerably wider application than the strictly two-dimensional model which implicitly underlies the Swift decomposition.

In summary, the purpose of our decomposition is to separate local and regional parameters as much as possible under the assumption that the regional structure is at most 2-D and the local structure causes only galvanic scattering of the electric fields, and to do so in the form of a product factorization.

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The Distortion Model

We now review, in more detail, the idealized physical model which underlies the decomposition. Assume that the Earth has a 2-D regional conductivity structure and that any superimposed 3-D structures are inductively small. In the principal axes of the 2-D structure, the regionally averaged electric field $\mathbf{e}_r$ and magnetic field $\mathbf{h}_r$ are related by

$$\mathbf{e}_r = \mathbf{Z}_2\, \mathbf{h}_r = \begin{pmatrix} 0 & a \\ -b & 0 \end{pmatrix} \mathbf{h}_r \qquad (1)$$

where a and b are the impedance elements for the regionally averaged 2-D structure.The measured fields $\mathbf{e}$ and $\mathbf{h}$ at any point may be perturbed by local variations from their regional values. The electric field $\mathbf{e}$ can be very strongly perturbed by local charges that accumulate on conductivity gradients or boundaries. The magnetic field $\mathbf{h}$ is not perturbed nearly as strongly, as it is determined by a weighted spatial average of the telluric current density. Thus we make the simplifying assumption that $\mathbf{h} = \mathbf{h}_r$. For a more complete discussion of this assumption and the nature and magnitudes of the errors involved, see *Dmitriev and Berdichevsky* [1979], *Wannamaker et al* [1984b] and *Groom* [1988]. On the other hand, $\mathbf{e}$ must be related to $\mathbf{e}_r$ by a distortion tensor $\mathbf{C}$, which in the absence of galvanic distortions reduces to the identity tensor.

Following *Bahr* [1988],

$$\mathbf{e} = \mathbf{C}\, \mathbf{e}_r = \begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix} \mathbf{e}_r \qquad (2)$$

Because the distortion structures are here assumed to be inductively weak, all the elements of $\mathbf{C}$ can be assumed real. Thus four real parameters are required to represent distortion. This physical model corresponds to fundamental model II of *Berdichevsky and Dmitriev* [1976] for a 3-D surface inhomogeneity, in which $\mathbf{C}$ is defined in their notation as

$$\mathbf{C} = \begin{pmatrix} F_x^{(x)} & F_x^{(y)} \\ F_y^{(x)} & F_y^{(y)} \end{pmatrix} \qquad (3)$$

The conditions of validity of such inductively weak (phase-free) distortions has been discussed by a large number of authors [e.g. *Berdichevsky and Dmitriev, 1976*; *Cox et al.*, 1980; *Larsen*, 1975, 1977; *Dmitriev and Berdichevsky*, 1979; *Hermance*, 1982; *LeMouel and Menvielle*, 1982; *Jones*, 1983; *Park et al.*, 1983; *Ranganayaki*, 1984; *Wannamaker et al.*, 1984a,b; *Park*, 1985; *West and Edwards*, 1985; *Bahr*, 1988], to which the reader is referred for details.

That four independent parameters are required to represent the most general distortion tensor may not be obvious. Fig. 1 shows a contrived example which demonstrates this requirement. A moderately conductive region of overburden (dotted) is shown on an insulating substratum (white). Inside the circular central region of overburden, an elliptical and highly conducting surface region (e.g., swamp) exists (black). Measurements are made at the centre of the swamp. The regional telluric currents are first twisted through an angle θ_t. The elongation of the swamp then leads to anisotropy with principal axes parallel and perpendicular to the direction $\mathbf{a}$. Applying the transformations implied by each of these distortion operations in turn to the regional electric field yields a final relation of the measured electric field to the regional which can be represented as

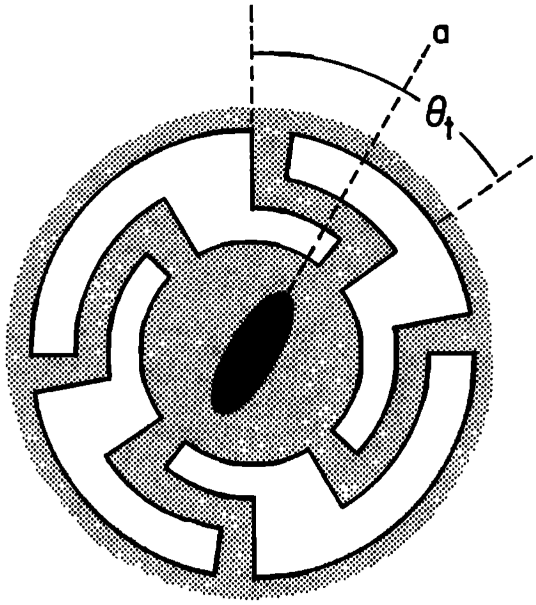

Fig. 1. A contrived example of distortion. See text for details

$$\mathbf{e} = \mathbf{C}\, \mathbf{e}_r = \mathbf{Q}\, \Lambda\, \mathbf{Q}^T\, \mathbf{T}\, \mathbf{e}_r \qquad (4)$$

where, in full,

$$\mathbf{C} = \begin{pmatrix} \cos\theta_a & -\sin\theta_a \\ \sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \times \begin{pmatrix} \cos\theta_a & \sin\theta_a \\ -\sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix} \qquad (5)$$

The matrix $\mathbf{T}$ performs the initial twist, $\mathbf{Q}$ and its transpose $\mathbf{Q}^T$ rotate to the principal axes of the swamp, and Λ imposes the anisotropy caused by the elongation and conductivity contrast of the swamp. As can be seen, four real parameters θ_a, θ_t, λ_1 and λ_2 are required.

Although this factorization (5) of $\mathbf{C}$ is instructive, it is not useful in the representation of real data because none of its four parameters can be recovered uniquely from a measured impedance tensor. As *Bahr* [1988] has shown in the case of general galvanic telluric distortion and *Zhang et al.* [1987] in the case of 2-D galvanic distortion, it is not necessary to explicitly solve for the elements of $\mathbf{C}$ in order to recover information about the underlying 2-D impedances. However, since we are here seeking an explicit decomposition (parametrization) of the impedance tensor in which the determinate and indeterminate parts are clearly distinguished, it is necessary first to describe quantitatively the exact way in which $\mathbf{C}$ is indeterminate.

It is given that

$$\mathbf{e} = \mathbf{Z}_m\, \mathbf{h} \qquad (6)$$

where $\mathbf{Z}_m$ is the measured impedance tensor. In the regional or principal axes system, we can express the measured impedance tensor using (1) and (2) as

$$\mathbf{Z}_m = \mathbf{C}\, \mathbf{Z}_2 \qquad (7)$$

or in the measurement axis system, as

$$\mathbf{Z}_m = \mathbf{R}\,\mathbf{C}\,\mathbf{Z}_2\,\mathbf{R}^T \tag{8}$$

where $\mathbf{C}$ is the distortion tensor, again expressed in the regional inductive principal axis system, and $\mathbf{R}$ is a rotation matrix which rotates vectors through an angle θ to the measurement axis system from the regional measurement axis system.

Although the factorization (8) of the measured impedance tensor expresses the underlying physical model, it is clear that any extraction of parameters from this factorization cannot be performed uniquely for measured data. This is because there are nine real parameters required: a rotation angle θ in $\mathbf{R}$, four distortion tensor elements, and the two complex principal impedances. The nature of the nonuniqueness of any decomposition is best illustrated by the following argument. Consider the transformations

$$\mathbf{Z}_2' = \mathbf{W}\,\mathbf{Z}_2 = \begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix} \mathbf{Z}_2 \tag{9}$$

and

$$\mathbf{C}' = \mathbf{C}\,\mathbf{W}^{-1} \tag{10}$$

where w_1 and w_2 are arbitrary non-zero real numbers. The new factorization

$$\mathbf{Z}_m = \mathbf{R}\,\mathbf{C}'\,\mathbf{Z}_2'\,\mathbf{R}^T \tag{11}$$

is also valid, in that it fits $\mathbf{Z}_m$ just as well as (8), since $\mathbf{C}'$ is still real and $\mathbf{Z}_2'$ still has the ideal 2-D form. In fact, it can be shown that a diagonal matrix like $\mathbf{W}$ produces the most general form of nonuniqueness that can be introduced. Thus although (8) is a physically based decomposition of the impedance tensor, (11) shows that it is not yet a useful one.

A USEFUL FACTORIZATION OF THE DISTORTION TENSOR

The previous section discussed the physical model and a rather general decomposition of the corresponding impedance tensor. In this section we will develop a factorization of the distortion tensor C which is the basis for our decomposition of the impedance tensor.

For representation of matrices it is convenient at this point to follow the example of *Spitz* [1985] and introduce a modified form of the Pauli spin matrices as

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{12a}$$

$$\mathbf{\Sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{12b}$$

$$\mathbf{\Sigma}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{12c}$$

$$\mathbf{\Sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{12d}$$

Although any rank 2 tensor $\mathbf{M}$ can be represented as a sum of these matrices (12) as

$$\mathbf{M} = \alpha_0\mathbf{I} + \alpha_1\mathbf{\Sigma}_1 + \alpha_2\mathbf{\Sigma}_2 + \alpha_3\mathbf{\Sigma}_3 \tag{13}$$

we suggest that a useful factorization of $\mathbf{C}$ is as the product

$$\mathbf{C} = g\,\mathbf{T}\,\mathbf{S}\,\mathbf{A} \tag{14}$$

g is a scalar and the tensor factors $\mathbf{T}$, $\mathbf{S}$, and $\mathbf{A}$ are defined by

$$\mathbf{T} = N_2\,(\mathbf{I} + t\mathbf{\Sigma}_2) \tag{15a}$$

$$\mathbf{S} = N_1\,(\mathbf{I} + e\mathbf{\Sigma}_1) \tag{15b}$$

$$\mathbf{A} = N_3\,(\mathbf{I} + s\mathbf{\Sigma}_3) \tag{15c}$$

It should be noted that this factorization is not a singular-value decomposition (SVD). Physical interpretations of each of $\mathbf{T}$, $\mathbf{S}$, and $\mathbf{A}$ will be discussed below. The normalizing factors N_i are defined for convenience in such a way that $\mathbf{T}$, $\mathbf{S}$ and $\mathbf{A}$ individually preserve power (but not isotropy) when applied to an isotropically polarized random electric field, i.e.

$$N_1 = 1/\sqrt{1+e^2} \tag{16a}$$

$$N_2 = 1/\sqrt{1+t^2} \tag{16b}$$

$$N_3 = 1/\sqrt{1+s^2} \tag{16c}$$

The matrix $\mathbf{T}$ is in fact made into an ordinary rotation matrix by this normalization. The practical purpose of this normalization is to ensure that the elements of $\mathbf{T}$, $\mathbf{S}$, and $\mathbf{A}$ remain bounded during any computations.

Some physical insight into this factorization can be obtained by examining the effects of each factor in turn on the regional electric field (i.e., the regional electric field in the natural coordinate system of the regional 2D structure). The "anisotropy" or "splitting" tensor

$$\mathbf{A} = N_3\,(\mathbf{I} + s\mathbf{\Sigma}_3) = N_3 \begin{pmatrix} 1+s & 0 \\ 0 & 1-s \end{pmatrix} \tag{17}$$

simply stretches the two field components by different factors, generating an anisotropy due to the distortions which is simply added to the anisotropy already existing in the regional induction impedance tensor $\mathbf{Z}_2$, as it is developed along the same axes. Note that any rank two diagonal matrix and thus any anisotropy operator can be expressed in this form. This distortion anisotropy is indistinguishable experimentally from the inductive anisotropy except in circumstances when the anisotropy of $\mathbf{Z}_2$ is known independently. (*Berdichevsky and Dmitriev* [1976] in their example have implicitly used this form for the distortion operator by the selection of their measurement location and the symmetries of their inhomogeneity.) Fig. 2 shows the effects of $\mathbf{A}$ on a family of unit vectors for positive s. Note that electric fields lying along either of the principal axes are not changed in direction.

The "shear" tensor (here so named by analogy to the theory of deformation)

$$\mathbf{S} = N_1\,(\mathbf{I} + e\mathbf{\Sigma}_1) = N_1 \begin{pmatrix} 1 & e \\ e & 1 \end{pmatrix} \tag{18}$$

develops anisotropy on axes which bisect the regional inductive principal axes. The effect of $\mathbf{S}$ on a family of unit vectors is shown in Fig. 3 for positive shear e. Note that the

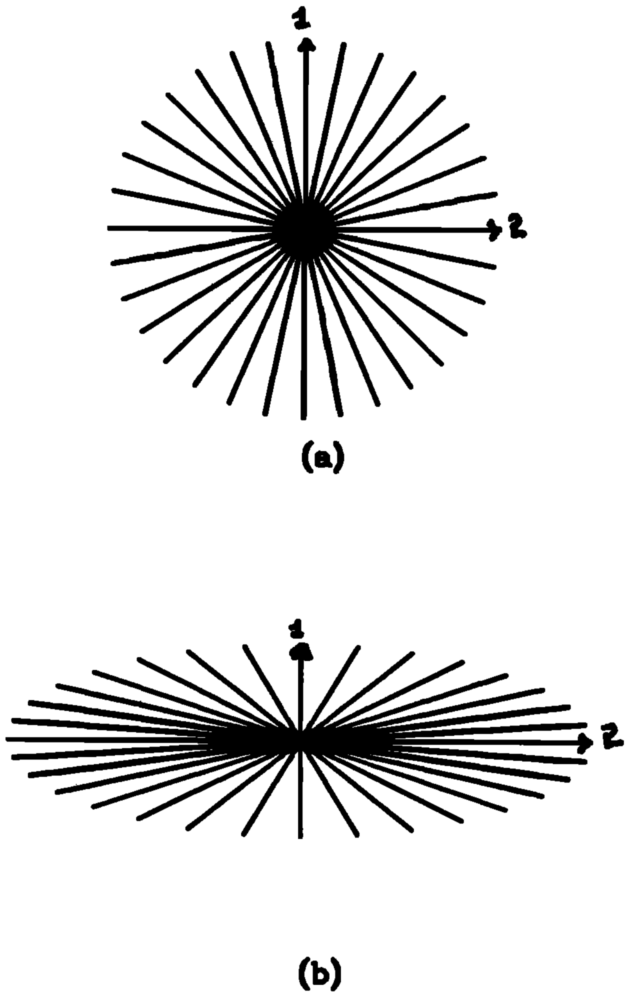

Fig. 2. A family of unit vectors (*a*) before and (*b*) after the application of the splitting tensor **A**. The x axis is up, the y axis to the right.

maximum angular changes occur for vectors aligned with the principal axes. A vector on the x axis in the figure is deflected clockwise by an angle $\tan^{-1} e$, and a vector along the y axis counter-clockwise by the same angle. It is therefore useful to characterize the shear e by a shear angle $\phi_e = \tan^{-1} e$.

The effect of the "twist" tensor

$$\mathbf{T} = N_2\,(\mathbf{I} + t\Sigma_2) = N_2 \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix} \tag{19}$$

is simply to rotate the electric field vectors through a clockwise angle $\tan^{-1} t$. The twist t is usefully characterized by the twist angle $\phi_s = \tan^{-1} t$.

Finally, g performs an overall scaling of the electric fields. This is necessary because **A**, **S**, and **T** have been normalized in such a way that their product will differ from the true distortion tensor **C** by some scalar factor g. We will refer to this scalar as the "site gain".

The advantage of this factorization of the distortion tensor **C** will now be made apparent. Neither g nor **A** can be determined separately from $\mathbf{Z}_2$, since $\mathbf{Z}_2' = g\ \mathbf{A}\ \mathbf{Z}_2$ looks like an equally valid ideal two-dimensional impedance tensor (i.e., has zero diagonal elements). We therefore adopt the convention of trying only to determine $\mathbf{Z}_2'$ rather than $\mathbf{Z}_2$, knowing that the two principal impedances we determine in $\mathbf{Z}_2'$ will have been separately scaled by unknown but frequency-independent factors. A clear advantage of the above factorization of the distortion tensor **C** is that it allows the unknown parts of **C** to be absorbed into the determined impedance tensor without destroying the ideal 2-D form of that tensor. Neither *Larsen* [1975] nor *Berdichevsky and Dmitriev* [1976] required this absorption of local anisotropy into the regional anisotropy since they assumed the regional structure to be one-dimensional (1-D), in which case all anisotropy is due to the local distortions and can be so attributed. The decomposition presented here can be similarly adapted in the 1-D case.

If the telluric distortion is truly frequency-independent, the absorption of g **A** into $\mathbf{Z}_2$ will not change the principal apparent resistivity curve shapes or the phases. Thus we can determine them correctly except for "static shifts." This is not the case with the conventional method, as will be illustrated later. In addition to implicitly absorbing g **A**

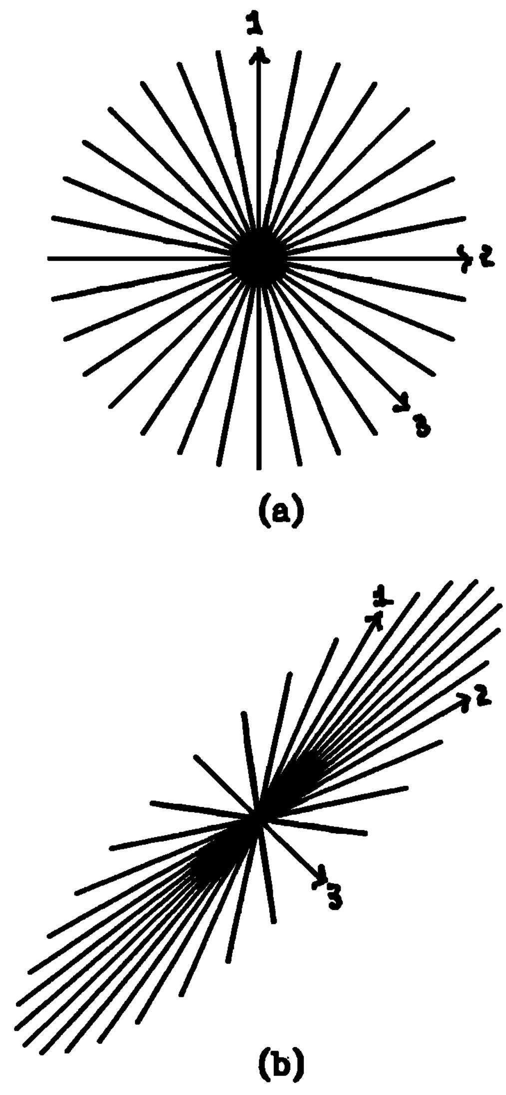

Fig. 3. A family of unit vectors (*a*) before and (*b*) after the application of the shear tensor **S**. The x axis is up, the y axis to the right.

into $\mathbf{Z}_2$, the conventional method also implicitly absorbs $\mathbf{T}$ and $\mathbf{S}$ into $\mathbf{Z}_2$ as well, thus radically distorting it from an ideal 2-D tensor.

That the factorization of $\mathbf{C}$ using real values of g, t, e and s exists and is unique is not obvious for arbitrary $\mathbf{C}$. For example, the classic eigenvalue-eigenvector decomposition of a square matrix will not yield real eigenvalues and eigenvectors unless the matrix has certain properties. Similarly, there is no a priori guarantee that the product factorization proposed in (14) even exists if s, t, e, and g are required to be real.

The remainder of this section is devoted to proving that for all reasonable distortion tensors a unique product factorization (14) exists. If the factorization is multiplied out explicitly, then

$$\mathbf{C} = \frac{g}{\sqrt{(1+e^2)(1+t^2)(1+s^2)}} \times \begin{pmatrix} (1+s)(1-te) & (1-s)(e-t) \\ (1+s)(e+t) & (1-s)(1+te) \end{pmatrix} \tag{20}$$

For the case of "weak" distortions (t, e, and s much less than unity), the factorization can easily be done approximately. If all second and third order terms in e, s and t are neglected, this becomes

$$\mathbf{C} = \begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix} \approx g \begin{pmatrix} 1+s & e-t \\ e+t & 1-s \end{pmatrix}. \tag{21}$$

From this, the factorization parameters can easily be derived as

$$g \approx \frac{C_1 + C_4}{2} \tag{22a}$$

$$e \approx \frac{C_2 + C_3}{C_1 + C_4} \tag{22b}$$

$$s \approx \frac{C_1 - C_4}{C_1 + C_4} \tag{22c}$$

$$t \approx \frac{C_3 - C_2}{C_1 + C_4} \tag{22d}$$

This is the form of the operator utilized by *Larsen* [1975]. In this situation of weak distortion, if the required regional model were 1-D one could (as *Larsen* [1975, 1977] has done with somewhat differently defined parameters) very simply calculate twist, shear, anisotropy and a possibly shifted 1-D impedance, but not the site gain.

For more general distortion, the exact equations (20) must be satisfied. It will be shown that there exist, in general, two solutions to these equations and only one is physically meaningful.

Recall from (20)

$$\mathbf{C} = g' \begin{pmatrix} (1+s)(1-te) & (1-s)(e-t) \\ (1+s)(e+t) & (1-s)(1+te) \end{pmatrix} \tag{20$'$}$$

where g' now includes the normalizing factors. We assume a priori that $\mathbf{C}$ is not of the form

$$\mathbf{C} = \begin{pmatrix} C_1 & 0 \\ C_3 & 0 \end{pmatrix}$$

or

$$\mathbf{C} = \begin{pmatrix} 0 & C_2 \\ 0 & C_4 \end{pmatrix}$$

For these special and physically implausible cases no suitable decomposition exists. If these special cases are excluded, (20) implies $s \neq \pm 1$.

If $C_4 \neq 0$, let

$$\gamma = \frac{C_2}{C_4} = \frac{e-t}{1+te} \tag{23a}$$

and if $C_1 \neq 0$, let

$$\beta = \frac{C_3}{C_1} = \frac{e+t}{1-te} \tag{23b}$$

The special cases where $C_1 = 0$ or $C_4 = 0$ which have two solutions are straightforward and are not discussed here. If $\gamma = \beta$, then there exists only one solution: $t = 0$ and

$$e = \gamma = \beta \quad g = \frac{C_1 + C_4}{2} \quad s = \frac{C_1 - C_4}{2g} \tag{24a}$$

Again, if $\gamma = -\beta$, the only solution is $e = 0$ and

$$t = -\gamma = \beta \quad g = \frac{C_1 + C_4}{2} \quad s = \frac{C_1 - C_4}{2g} \tag{24b}$$

However, if $\gamma \neq \beta$ and $\gamma \neq -\beta$, then from (23) it can be shown that e and t satisfy quadratic equations

$$(\gamma + \beta)e^2 + 2e(1 - \gamma\beta) - (\beta + \gamma) = 0 \tag{25a}$$

$$(\gamma - \beta)t^2 - 2t(1 + \gamma\beta) - (\gamma - \beta) = 0 \tag{25b}$$

which have real solutions as required:

$$t = \frac{(\gamma\beta + 1) \pm \sqrt{(1+\gamma^2)(1+\beta^2)}}{\gamma - \beta} \tag{26a}$$

$$e = \frac{(\gamma\beta - 1) \pm \sqrt{(1+\gamma^2)(1+\beta^2)}}{\gamma + \beta} \tag{26b}$$

We denote the solution for t with the positive square root as t^+, while t^- denotes the alternate solution, and similarly for e. Note

$$t^+ t^- = -1 \qquad e^+ e^- = -1$$

and the two solution sets are $(e_1, t_1) = (e^+, t^-)$ and $(e_2, t_2) = (e^-, t^+)$.

It is easy to show that when $\gamma\beta = -1$, $t = \pm 1$ and when $\gamma\beta = 1$, $e = \pm 1$. Except for these cases, there exists a pair of solutions. In one solution, $|e| > 1$, and in the other, $|e| < 1$. Similarly, one solution has $|t|$ greater than one and in the other, t has magnitude less than one. To be more specific, it can be shown that if (g, t, e, s) is a solution, then $(-gets, -t^{-1}, -e^{-1}, s^{-1})$ is also a solution.

The two solutions cannot always be divided into "small" ($|t, e| < 1$) and "large" ($|t, e| > 1$) distortion solutions. However, if

$$0 \leq |\gamma\beta| \leq 1$$

then

$$|t_2| < |t_1|$$

$$|e_2| < |e_1|$$

and there is a small distortion solution distinct from a large distortion solution. However, if $|\gamma\beta| > 1$ then the solutions are of "mixed distortion" type. That is, there exists one solution which has small shear and large twist and another with small twist and large shear. However, one can reasonably require that $|e| \leq 1$, since intuitive consideration of the effect of the shear operator (18) would imply that a shear angle of magnitude greater than 45^o is not meaningful. This restriction to the solution when shear has magnitude less than one thus defines a unique solution for the product factorization.

Completion of the uniqueness proof now only requires that shear and site gain be uniquely determined from the distortion tensor. To obtain the anisotropy factors for a known distortion tensor, (20′) yields

$$\frac{1+s}{1-s} = \left(\frac{1+te}{1-te}\right)\frac{C_1}{C_4} \tag{27}$$

if $te \neq 1$ and $C_4 \neq 0$ (The special cases where $te = 1$ and $C_4 = 0$ are easily obtainable). Equation (27) leads to the solutions

$$s_1 = \frac{(C_1 - C_4) + e_1 t_1 (C_1 + C_4)}{(C_1 + C_4) + e_1 t_1 (C_1 - C_4)} \tag{28a}$$

and

$$s_2 = \frac{1}{s_1} \tag{28b}$$

Finally, to determine the scaling parameter g, premultiply $\mathbf{C}$ by $\mathbf{S}^{-1}\ \mathbf{T}^{-1}$. The inverse of $\mathbf{T}$ always exists since its determinant is $1+t^2$ and t is real. The inverse of $\mathbf{S}$ exists if $e \neq \pm 1$. (This case is considered separately.) The resulting matrix $g\ \mathbf{A}$ is diagonal and the sum of the diagonal elements yields

$$2g_i' = \frac{1}{(1-e_i^2)(1+t_i^2)}[C_1(1+e_i t_i) - C_2(e_i + t_i) - C_3(e_i - t_i) + C_4(1 - t_i e_i)] \tag{29}$$

where $i = 1, 2$.

In summary, there are two solutions (sometimes degenerate) for the decomposition of any physically plausible distortion tensor, only one of which need be considered physically meaningful. Although investigations of channelling effects with this factorization may prove useful, an explicit method of factorization of the $\mathbf{C}$ tensor per se was not our primary goal here. The above discussion was necessary to establish that the parameters used (g, e, t, s) are in fact well defined in the proposed factorization.

DECOMPOSITION OF THE IMPEDANCE TENSOR

If the above decomposition (14) of the distortion tensor is substituted into (7), the result is

$$\mathbf{Z}_m = g\ \mathbf{R}\ \mathbf{T}\ \mathbf{S}\ \mathbf{A}\ \mathbf{Z}_2\ \mathbf{R}^T \tag{30}$$

If $g\ \mathbf{A}$ is absorbed into $\mathbf{Z}_2$ to give $\mathbf{Z}_2'$, this can be written as

$$\mathbf{Z}_m = \mathbf{R}\ \mathbf{T}\ \mathbf{S}\ \mathbf{Z}_2\ \mathbf{R}^T \tag{31}$$

where the prime on $\mathbf{Z}_2$ has been dropped since $\mathbf{Z}_2$ and $\mathbf{Z}_2'$ are experimentally indistinguishable, as noted previously. Equation (31) constitutes the desired decomposition. It has seven real parameters, which are (1 and 2) the real and imaginary parts of the major principal impedance a (or equivalently the major apparent resistivity and phase), (3 and 4) the real and imaginary parts of the minor principal impedance b (or equivalently the minor apparent resistivity and phase), (5) the azimuth θ of the major apparent resistivity, (6) the shear angle $\phi_e = \tan^{-1} e$, and (7) the twist angle $\phi_t = \tan^{-1} t$. (An alternative parameter, the local distortion strike, which contains the same information, will be discussed later.)

To calculate these parameters from a measured impedance tensor, they must be explicitly related to the data by multiplying out the decomposition. The datum $\mathbf{Z}_m$ is conveniently represented by its summary decomposition coefficients α_i, where

$$\mathbf{Z}_m = \frac{1}{2}(\alpha_0 \mathbf{I} + \alpha_1 \mathbf{\Sigma}_1 + \alpha_2 \mathbf{\Sigma}_2 + \alpha_3 \mathbf{\Sigma}_3) \tag{32}$$

and

$$\alpha_0 = Z_{xx} + Z_{yy} \tag{33a}$$

$$\alpha_1 = Z_{xy} + Z_{yx} \tag{33b}$$

$$\alpha_2 = Z_{yx} - Z_{xy} \tag{33c}$$

$$\alpha_3 = Z_{xx} - Z_{yy} \tag{33d}$$

If the product decomposition in (31) is multiplied out, it yields after some algebra a nonlinear system of equations:

$$\alpha_0 = t\sigma + e\delta \tag{34a}$$

$$\alpha_1 = (\delta - et\sigma)\cos 2\theta - (t\delta + e\sigma)\sin 2\theta \tag{34b}$$

$$\alpha_2 = -\sigma + et\delta \tag{34c}$$

$$\alpha_3 = -(t\delta + e\sigma)\cos 2\theta - (\delta - et\sigma)\sin 2\theta \tag{34d}$$

where for convenience the definitions

$$\sigma = a + b \quad \text{and} \quad \delta = a - b \tag{35}$$

for the sum of the principal impedances and their difference have been used. The 90^o ambiguity in θ can be resolved by adopting the convention either that $|a| > |b|$, so that a is the major principal apparent resistivity and θ is the azimuth of the electric fields associated with it, or that the azimuth lies between 0^o and 90^o. The latter has been adopted here.

There is, in fact, a unique decomposition (34) for a given impedance tensor if the physical model is correct for the impedance tensor and there is no noise present (after ambiguity for regional azimuth convention is resolved and the low shear restriction is made for the solution of the factorization of $\mathbf{C}$, as explained in the previous section). In practice, experimental data with noise or deviations from the physical model will never exactly fit the proposed decomposition. In this case, a solution of these eight real equations (34) for the seven decomposition parameters must be achieved by a least squares fitting procedure. This necessarily requires good data; the conventional method, in fitting only five parameters, is more stable with respect to data errors.

A COMPARISON OF METHODS

Swift (1967) has defined a 3-D indicator, skew, as

$$|\Gamma| = \left|-\frac{\alpha_0}{\alpha_2}\right| \tag{36}$$

For the distortion model developed here, Γ turns out to have the value

$$\Gamma = \frac{t\sigma + e\delta}{\sigma - et\delta} \tag{37}$$

Even when the induction is 2-D, it is clear from (37) that the skew can be significantly different from zero and a function of frequency if distortions are present. There are two extreme cases which are worth noting. If the impedance tensor $\mathbf{Z}_2$ recovered by this new decomposition is isotropic (i.e., the net effect of the distortion anisotropy and the inductive anisotropy is nil), then δ is zero, and

$$\Gamma = t = \tan\phi_t \tag{38}$$

If the anisotropy of $\mathbf{Z}_2$ is extreme, so that $|a| \gg |b|$ and thus $\delta \approx \sigma$, then

$$\Gamma = \frac{t+e}{1-et} = \tan(\phi_t + \phi_e) \tag{39}$$

using the formula for the tangent function of a sum of angles. This prompts us to define a skew angle γ as $\tan^{-1}\Gamma$ (note that this is to be distinguished from skew angles defined by *LaTorraca et al.*[1986] and *Eggers*[1982]. The skew angle is an approximate estimate of the magnitudes of the twist and shear angles. Thus MT data can be rejected unnecessarily on the basis of a large skew even if induction is 2-D in nature. The new decomposition proposed above will identify such situations and permit the use of such data.

The conventional method of recovering the principal impedances and the inductive strike is to minimize

$$|Z'_{xx}|^2 + |Z'_{yy}|^2$$

as a function of coordinate rotation angle θ' [*Swift*, 1967; *Sims and Bostick*, 1969]. This is equivalent to minimizing $|\alpha_3(\theta')|^2$ [*Spitz*, 1985, *Sims and Bostick*, 1969]. As a function of the chosen coordinate rotation angle θ',

$$\begin{aligned}\alpha_3(\theta') = &-(t\delta + e\sigma)\cos 2(\theta - \theta') \\ &- (\delta - et\sigma)\sin 2(\theta - \theta')\end{aligned} \tag{40}$$

It is clear that minimizing $|\alpha_3|^2$ as given by (40) with respect to θ' will not yield for θ' the true inductive strike θ if distortion is present. What result will be obtained? Rather than solving the general case, we will solve the simpler special cases. Where $\mathbf{Z}_2$ is highly anisotropic ($\delta \approx \sigma$), α_3 can be made zero by choosing

$$\begin{aligned}\theta' &= \theta + \frac{1}{2}\tan^{-1}\left(\frac{t+e}{1-et}\right) = \theta + \frac{1}{2}\gamma \\ &= \theta + \frac{1}{2}(\phi_t + \phi_e)\end{aligned} \tag{41a}$$

Thus the recovered azimuth differs from the principal inductive strike by half the skew angle. The general implication of this special case is that azimuth errors of the conventional method can be of the same order of magnitude as the skew angle. Going to the other extreme, in the isotropic case ($\delta \approx 0$), α_3 can be made zero by choosing

$$\theta' = \theta + \frac{1}{2}\tan^{-1}\left(\frac{1}{t}\right) = \theta \pm \frac{\pi}{4} + \frac{1}{2}\phi_t \tag{41b}$$

if t is not zero.

For the special case of high anisotropy, what principal impedances will be recovered? The answers we would like are of course scalar multiples of the true 2-D impedances, $a(\omega)$ and $b(\omega)$, where $b(\omega) \ll a(\omega)$. Use of an impedance tensor of the form expressed by (31) in the conventional (i.e., Swift) decomposition process shows after a little algebraic manipulation that the impedances a' and b' actually recovered by the conventional method are

$$a'(\omega) \approx a(\omega)\left[\frac{(1-et)(1+\cos\gamma) + (e+t)\sin\gamma}{2}\right] \tag{42a}$$

and

$$b'(\omega) \approx a(\omega)\left[\frac{(1-et)(1-\cos\gamma) - (e+t)\sin\gamma}{2}\right] \tag{42b}$$

Thus the major principal impedance is recovered correctly, except for a scaling factor which is frequency independent if the model is true. However, the minor principal impedance is not recovered correctly at all. The value obtained is the major principal impedance multiplied by some scaling factor. The temptation, on seeing such a result for the two principal impedances, would be to conclude that 1-D induction, modified by distortions in some unspecified way, was occurring and to attempt to fit the impedance curves with a 1-D inductive model. This would of course be incorrect.

Another instructive special case is that of weak distortion (e, t, and s all much less than unity) in a regionally approximately isotropic Earth. In this case, terms of second and third order in e, t and δ/σ can be neglected, giving as approximations for equations (32a) to (32b)

$$\alpha_0 \approx t\sigma \tag{43a}$$

$$\alpha_1 \approx \delta\cos 2\theta - e\sigma\sin 2\theta \tag{43b}$$

$$\alpha_2 \approx -\sigma \tag{43c}$$

$$\alpha_3 \approx -e\sigma\cos 2\theta - \delta\sin 2\theta \tag{43d}$$

Note that if the shear e is zero, $|\alpha_3(\theta')|^2$ is minimized by $\theta' = \theta$, and the conventional method will recover the correct inductive strike no matter what the twist is. The conventional method in fact recovers the correct principal impedances as well in this case (except for the usual static shifts). Note that the skew $|\alpha_0/\alpha_2|$ depends only on the twist and not at all on the shear, whereas it is the shear which is the important parameter in determining the validity of the conventional method. This illustrates a case in which the conventional method gives the right answers even when the skew is nonzero. A corollary of this is that the conventional method can do badly in this case with zero skew ($t = 0$) if the shear is nonzero.

The role of the effective one dimensional impedances [*Berdichevsky and Dmitriev*, 1976; *Ranganayaki*, 1984] such as half the difference of the off-diagonal elements of $\mathbf{Z}_m$ (the Berdichevsky trace impedance) and the square root of the determinant of $\mathbf{Z}_m$ is worth examining. The use of these effective impedances does not necessarily correspond to a belief that the Earth is layered; they are simply convenient condensations of the information in the measured impedance tensor to forms which can be used for modeling and which are thought to be less affected by noise and telluric distortions. In suitable cases they also have the desirable property of reflecting to some extent a horizontally averaged re-

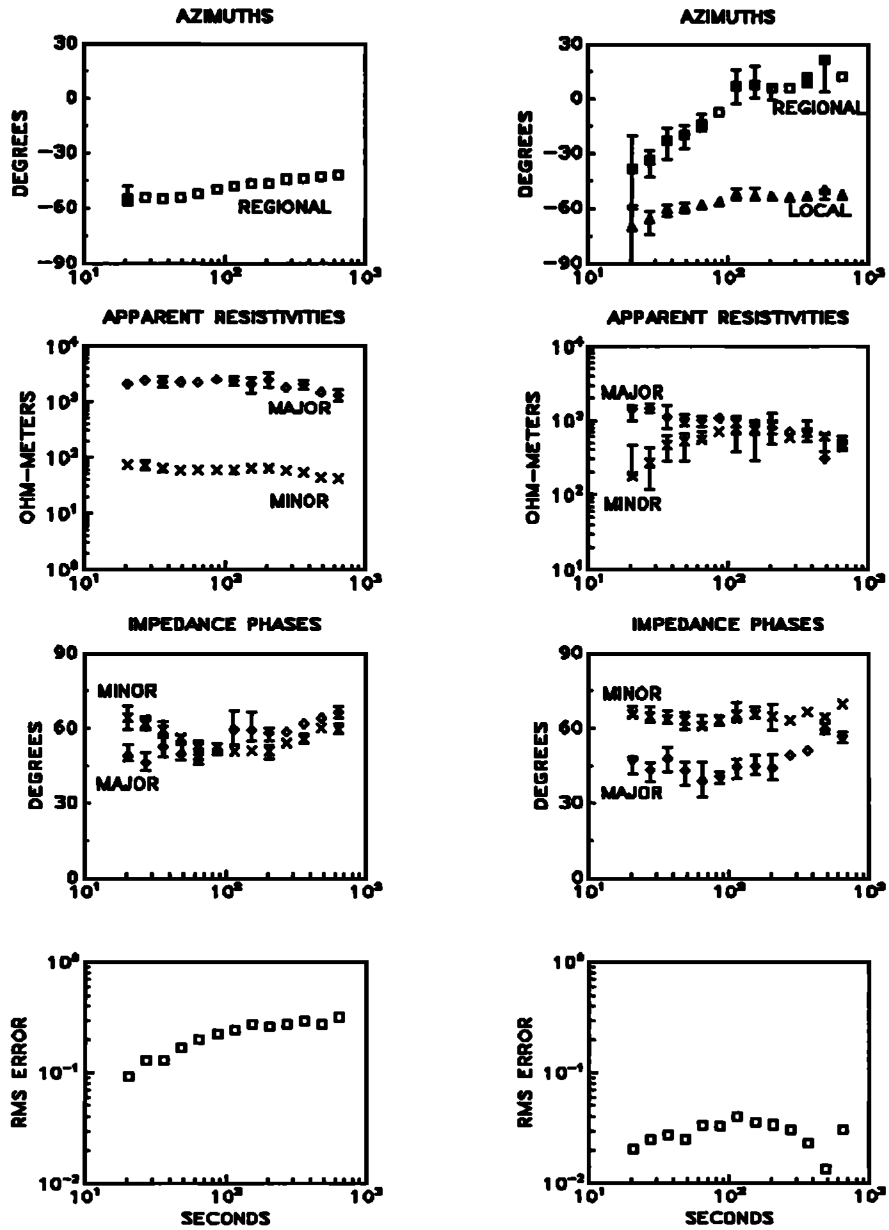

Fig. 4. Comparison of parameters by different methods. The left column contains graphs of the parameters, inductive strike, apparent resistivities, impedance phases, and error of fit by the conventional technique. The right column contains the same information by the new technique when the distortion parameters are not constrained to be independent of frequency.

gional inductive structure. In the absence of distortions, the Berdichevsky impedance is preferable, since its errors of estimation are lower. Where distortions are present, however, the Berdichevsky impedance is no longer simply the arithmetic mean of the two principal impedances, but an unknown linear combination of them, and, as such, cannot be modeled. The determinantal impedance, on the other hand, is unaffected by distortion as described here except for multiplication by a frequency-independent scalar (the determinant of $\mathbf{C}$). This result would appear to contradict a result of *Berdichevsky and Dmitriev* [1976]. They showed that the trace impedance was less distorted than the determinantal impedance, measured outside a vertical elliptically cylindrical inhomogeneity. The discrepancy, however, is explained by the fact that their measurement location lay on an axis of symmetry where the shear and twist would be zero.

So far in this section, the new decomposition has been compared to other approaches which, explicitly or implicitly, make too many simplifying assumptions to be useful for the physical model used here (i.e., 3-D galvanic distortion over 2-D induction). It is also instructive to compare our decomposition with approaches which make too few simplifying assumptions for this particular physical model (i.e., fail to take advantage of the model in deciding what quantities to extract).

We emphasize again prior to this comparison that the purpose of our decomposition presented here is to separate local and regional parameters as much as possible under the assumption that the regional structure is at most 2-D and the local structure causes only galvanic scattering of the electric fields. Recently, there have been a number of eight parameter decompositions developed [e.g. *Eggers*, 1982; *LaTorraca et al.*, 1986; *Yee and Paulson*, 1987]. These decompositions are based on mathematical characteristics of the impedance tensor and do not attempt to separate local and regional effects. The eigenstate formulation of *Eggers* [1982] is a reasonably characteristic example of these methods. Four of Eggers' parameters are the two complex eigenvalues $\lambda^{\pm}$ which are given by

$$\lambda^{\pm} = -\frac{\alpha_2}{2} \pm \frac{1}{2}\sqrt{\alpha_2^2 - 4 \ \det|\,\mathbf{Z}\,|} \tag{44}$$

where α_2 is defined by (33c). These are intended to be analogous to the two principal impedances of the Swift method and in fact reduce to those when the Swift model of simple 2-D induction is appropriate. When the physical model of intermediate complexity defined here is appropriate, these eigenvalues *do not* relate simply to the principal impedances.

In the physical case of local 3-D galvanic scattering of electric fields due to a regional 2-D structure (7),

$$\lambda^{\pm} = -\frac{\alpha_2}{2} \pm \frac{1}{2}\sqrt{\alpha_2^2 - 4 \ \det|\,\mathbf{Z}_2\,| \ \det|\,\mathbf{C}\,|} \tag{45}$$

where now α_2 is given by (34c). Thus we see that the eigenvalues mix the regional 2-D impedances in a similar manner to the conventional method. This mixing is dependent on the electric distortion matrix $\mathbf{C}$. A more detailed analysis of this comparison can be found in the work by *Groom* [1988].

STRONG OR 2-D DISTORTIONS

Zhang et al. [1987] have applied the same physical ideas as *Bahr* [1988] to the special case where the distortion structure is two-dimensional in nature. For comparison, we examine the strong distortion case (not necessarily 2-D) with the decomposition proposed here. Consider the net effect of the tensor $\mathbf{T}\ \mathbf{S}$ on the regional electric fields produced by $\mathbf{Z}_2\ \mathbf{h}$. If the distortion is strong ($|e|$ approaching unity), $\mathbf{S}$ highly polarizes the electric field along azimuth $\pi/4$ with respect to the principal inductive coordinate system (or $-\pi/4$ if the shear is negative). The twist tensor $\mathbf{T}$ then rotates this axis of polarization by the twist angle. In the measurement coordinates the final azimuth of the strong local electric field polarization direction is thus

$$\theta_l = \theta + \phi_t \pm \frac{\pi}{4} \tag{46}$$

where the sign is chosen the same as that of the shear. We can define θ_l as the distortion strike or local strike; it will be either perpendicular or parallel to the strike of the strong distortion structure. The distortion strike can be used as a decomposition parameter in place of the twist, as it contains all the information about the twist and describes more directly the distortion structure. The above result (46) can be derived rigorously for 2-D distortion by using the result of *Zhang et al.* [1987] for the form of a 2-D distortion tensor (it has only three independent parameters, as it is symmetric) and a strong distortion condition ($|e| \approx 1$).

INDICATORS OF 3-D INDUCTION

The assumption of 3-D distortion acting on 2-D induction will not be true in all cases and it is important to be able to see when this is the case. As noted above, the skew is not a suitable indicator for this purpose. There are two ways in which deviations from the ideal distortion model can be detected. The distortion model leads to a decomposition with only seven real parameters and therefore cannot exactly fit all possible impedance tensors, which need in general eight real parameters to describe them. The root mean square relative error of fit ϵ of the channeling decomposition is given by

$$\epsilon^2 = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|\hat{Z}_{ij} - Z_{ij}\right|^2}{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|Z_{ij}\right|^2} \tag{47}$$

where Z_{ij} and $\hat{Z}_{ij}$ are the measured and modeled impedance tensor elements respectively. This error parameter ϵ should be small compared with unity. It can be calculated at every frequency and thus can be used to define frequency ranges in which the ideal distortion model is significantly in error. Note that non-zero estimated values for ϵ may not be significant if data errors are taken into account. A conventional chi-square test with one degree of freedom may be used to assess this significance if the errors in $\mathbf{Z}$ can be assumed normally distributed. This is not the only way to assess the model validity; *Bahr* [1988] has defined a somewhat different measure of deviation from this model.

A second way to detect deviations from the ideal distortion model is to examine the frequency dependence of the distortion parameters. If the ideal distortion model is a realistic model in a range of frequencies, these parameters will be approximately independent of frequency in that range. In practice, a structure which acts as part of the regional inductive structure at high frequencies may act as a frequency-independent distortion structure at much lower frequencies.

If application of the decomposition to the data suggests

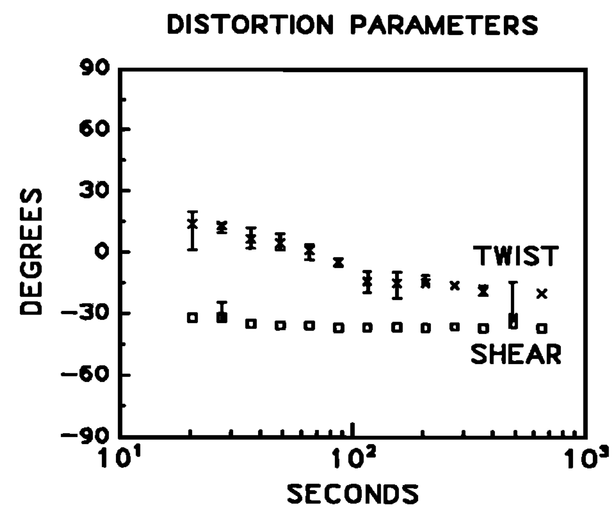

Fig. 5. Distortion parameters (twist and shear) for the interpretation given by Fig. 4

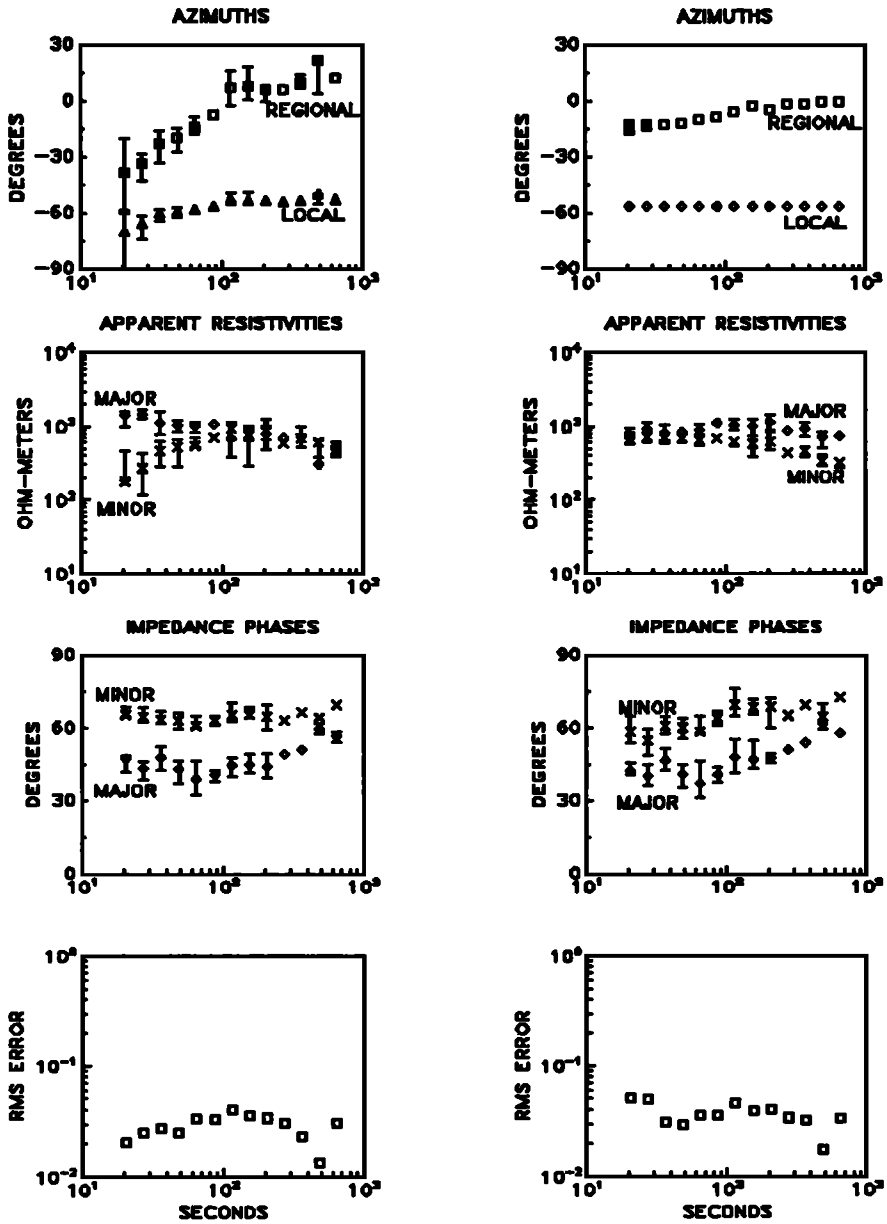

Fig. 6. Comparison of parameters for frequency-independent distortion versus nonconstrained distortion. Right column contains parameters for a decomposition constrained to have frequency-independent shear and local or distortion azimuth.

that the model is valid, at least over a certain frequency range, because e and θ_t are approximately constant for instance, how might one test this? One fits the data at N relevant frequencies with a different set of seven parameters at each frequency, requiring $7N$ parameters in total. One then fits the data using the same shear and local azimuth at all frequencies, requiring $5N+2$ parameters. If the reduction by $2N-2$ in the number of adjustable parameters used to fit the model has not significantly increased the fitting error, the model is probably valid. This approach can be rigorously formulated as a standard partial F-test if necessary.

Finally, as a third test, if nearby sites give quite different impedance tensors, their decompositions can be compared. If the regional inductive parameters are very similar (except for static shift) despite large possible differences in the distortion parameters, it argues that the underlying model here can explain the data.

An Example With Experimental Data

The example presented here is not intended to be a reinterpretation based on the new factorization. It is intended to demonstrate some of the points made in the section on comparisons. The data are long period data obtained recently at a site on the Canadian Shield in northern Ontario about 30 km north of the town of Chapleau [*Cavaliere*, 1987]. The data set is expected to have relatively large errors and was chosen for this reason to test the robustness of the method. The upper crustal rocks are granitic and have resistivities of the order of 10^4 ohm-meters. The topographic relief of a few tens of meters is extensively but erratically covered with glacial debris ranging from clays (resistivity of the order of 10 ohm-meters) to gravels and sands (resistivity of the order of 10^3 ohm-meters). The likelihood of strong telluric distortion with a length scale of several hundred meters is high.

The distortion decomposition was first obtained without any frequency-independent constraints on the distortion parameters. The resulting induction parameters from the conventional method and the new decomposition are compared in Fig. 4 along with the rms error of fit (47). Fig. 5 contains the distortion parameters, twist and shear, obtained by the new decomposition.

There are three specific pieces of evidence in these results which suggest that the channelling decomposition is appropriate. On examination of Fig. 5 it is observed that the distortion parameter shear is essentially frequency-independent without it having been so constrained. Second, the error of fit, ϵ, of the new decomposition is 0.05 or smaller at all periods and typically about 0.02, whereas the conventional errors ranged as high as 0.4. Third, the local strike (shown in the small figure which contains the new decomposition azimuth) is very close to the conventional regional azimuth. That such results occur in the galvanic distortion regime was discussed in a previous section. On the other hand, in opposition to an hypothesis of galvanic distortion there is the presence in these results of a frequency-dependent twist. However, it should be noted that although the regional azimuth may vary slowly with frequency if the model is only approximately valid (e.g., in a nearly 1-D model with a poorly defined strike direction), a strong local strike θ_l (local electric polarization direction) will not do the same. Thus the twist must compensate by varying in an opposite sense from the regional azimuth as per (46). It can be noted from Fig. 4 and 5 that the twist does indeed vary more or less oppositely to the regional azimuth here.

The conventional apparent resistivities are very flat over this period range and appear to be "static shifts" (i.e., multiplication by real but frequency-independent constants) of each other. This result combined with the similarity in the major and minor impedance phases would suggest, conventionally, a 1-D regional conductivity structure with unknown "static shift". However, the error of fit for the 2-D model of the impedance tensor is quite high.

The new inductive strike azimuth differs considerably from the old and varies much more with frequency. The local electric fields seem to have a direction of polarization, namely the apparent local strike or azimuth (which is relatively independent of frequency) as shown on Fig. 4. It seems clear that the inductive strike recovered by the conventional method is wrong as a result of domination by distortion effects; note the near coincidence of the conventional strike and the local strike on Fig. 4.

The model for the decomposition dictates that the local distortion parameters be independent of frequency. As required by the physical model underlying the decomposition, the distortion parameters shear and local strike (e, θ_l) are relatively independent of frequency as seen from Fig. 4 and 5. This decomposition gave a data set error as discussed previously of 0.0009 (the average over frequencies of the square of the rms error). The next stage in the method is to investigate whether the distortion parameters are actually independent of frequency and whether the observed variations in the distortion parameters in Fig. 4 and 5 are due to noise or slight inadequacies in the model. The shear in Fig. 5 is almost frequency-independent, as is the local strike in Fig. 4. If we assume a priori that the shear and distortion strike are independent of period and constrain them to be so in a least squares fit, the average error of the fit of the decomposition increases to 0.0014. This increase is not significant compared to errors in the data and thus supports the model hypothesis. Fig. 6 is a comparison of induction parameters between the unconstrained decomposition (left column) and the constrained decomposition (right column). Fig. 7 gives the shear and twist for the constrained solution with the classical skew angle for comparison.

The resulting regional azimuth for the constrained decomposition (Fig. 6) is significantly modified and is now much less variable with frequency. The major apparent resistivity and impedance phase are virtually unaltered from the unconstrained case, and the minor impedance is only slightly altered from the unconstrained case. The twist still varies in an opposite sense to the regional strike. Note that for a constrained local strike, twist is no longer an independent parameter but depends entirely on local strike and regional azimuth. In fact, if there is no preferred regional azimuth (i.e., 1-D regional structure) then the regional azimuth (θ) and the twist (t) can vary arbitrarily so that their total is a constant defined by the constrained local azimuth.

Because of the relative isotropy of the apparent resistivities, it appears possible that the regional strike may be wandering simply because it is intrinsically poorly determined, not because the model is grossly invalid. This prompts the following question. What would the result be if the regional azimuth as well as the shear and twist were constrained to be more or less constant? If the large-scale structure is truly 1-D, this set of constraints should produce an equally accurate parameterization of the measured impedance tensor. Fig. 8 shows the decomposition results when regional strike, shear angle, and twist angle are all constrained in the fit to lie within 2^o of the mean values suggested by Fig. 6 and 7. There is very little increase in the error of fit from the unconstrained decomposition. The number of free parameters used for the entire data set is now less than the number used

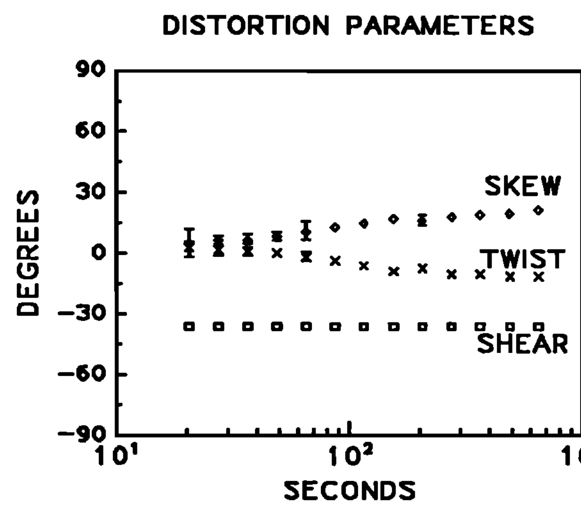

Fig. 7. Distortion parameters (twist and shear) for the decomposition of Fig. 6 (right column). The conventional three-dimensional indicator, skew, is included.

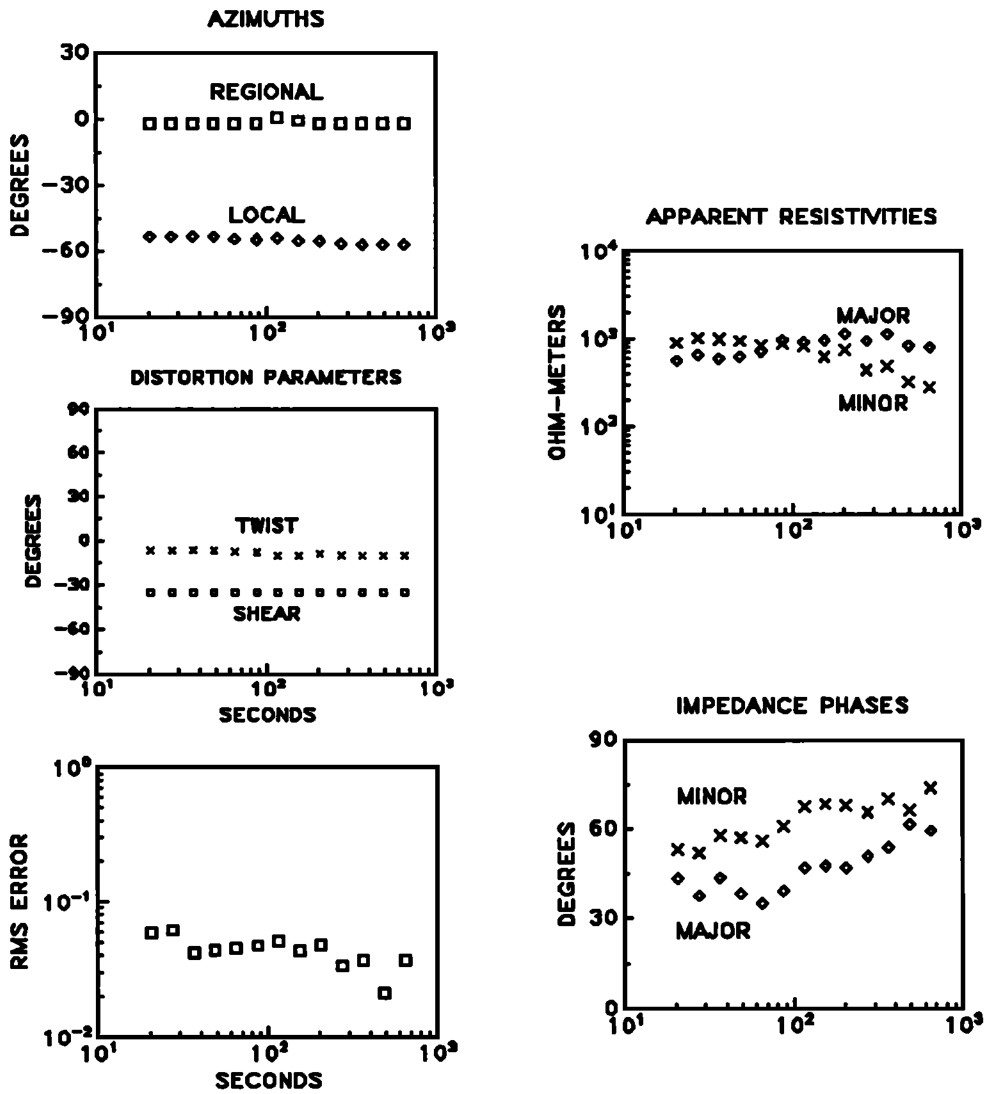

Fig. 8. Induction and distortion parameters of a one-dimensional induction model with constrained frequency-independent twist and shear

for the conventional decomposition. However, the error of fit, as seen by comparing Fig. 8 to 4, is still almost an order of magnitude smaller than the conventional error.

Twist, shear and local azimuth are now essentially frequency-independent, as is the regional azimuth. There are only slight variations in the impedances (Fig. 6) from the previous constrained decomposition. It appears that the large-scale structure can be interpreted as being one-dimensional at these periods, except for one problem. That problem is with the impedance phases in Fig. 8. If the regional structure is truly 1-D, the impedance phases should be identical. However, other physical effects may be present, which have not been considered, causing these phase differences. In particular, an important factor to be considered is the effect on the magnetic field of the galvanically distorted currents. This effect produces a magnetic field out of phase with the primary magnetic field. *Berdichevsky and Dmitriev* [1976], for instance, have considered the effects of this anomalous field for some restricted cases. In fact, a careful analysis of the effect of the anomalous magnetic field due to the distorted currents indicates that the major effect of this anomalous field on the decomposition is to alter the phases of the 2-D impedances. In fact, the majority of the phase differences in Fig. 8 can probably be accounted for by such a magnetic field [*Groom*, 1988]. However, this result does indicate the necessity of recognizing if not actually incorporating galvanic magnetic effects in any decomposition which attempts to account for all the effects of strong 3-D galvanic electric field distortions by conductive overburden over a resistive Earth.

CONCLUSIONS

The discussion of the physics of distortion given in this paper is not new. What is new is an explicit decomposition of the impedance tensor, the parameters of which separately represent the determinate distortion effects, the indeterminate distortion effects, and 2-D inductive effects. It offers, in addition, a relatively robust way of determining these quantities in the presence of noise by virtue of being implemented as a fit to all the available data.

Because the class of magnetotelluric responses described by the distortion model is much larger than that described by ideal 2-D induction, the recovered principal impedances and inductive strikes should be physically meaningful in a much larger proportion of experimental cases. Even in cases where the physical system does not conform exactly to the simple distortion model, this decomposition may still offer a useful standard way to present and compare both real and synthetic magnetotelluric data for 3-D structures.

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