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The Fundamental Model of Magnetotelluric Sounding

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Abstract—It is widely believed that the Tikhonov-Cagniard model can be applied in the magnetotelluric method of exploration geophysics if the horizontal field changes are sufficiently slow. Actually, this is overly restrictive and may give rise to unjustified doubts about the validity of the Tikhonov-Cagniard model. This paper shows that the condition determining the applicability of the model is the linearity of the horizontal field changes, rather than the slowness of these changes. This considerably extends the sphere of practical applications of the Tikhonov-Cagniard model.

I. Introduction

TIKHONOV [1] and Cagniard [2] have suggested a method for determining the electrical conductivity of the Earth's crust and upper mantle from the horizontal components of the magnetotelluric (MT) fields (the natural electric and magnetic fields at the Earth's surface) when these fields are measured simultaneously at a single point of the Earth's surface. This method has become known as *MT sounding.* The fundamental model behind the Tikhonov-Cagniard method of magnetotelluric sounding is extremely simple. A uniform monochromatic electromagnetic plane wave, incident vertically, excites the plane-layered Earth (Fig. 1), the electrical conductivity σ of which is a continuous or piecewise continuous function of the depth z (the z-axis is directed downwards) and whose magnetic permeability is everywhere equal to $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. The time dependence of the field is expressed by the factor $\exp(-i\omega t)$. Displacement currents are neglected. In this one-dimensional model

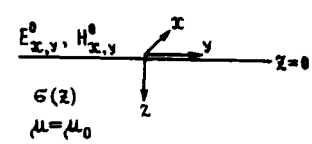

Fig. 1. Tikhonov-Cagniard model.

$$E_x = ZH_y$$
$$E_y = -ZH_x \quad (1)$$

where Z is *the Tikhonov-Cagniard impedance* which satisfies the Riccati equation

$$\frac{dZ}{dz} - \sigma Z^2 = i\omega\mu_0 \quad (2)$$

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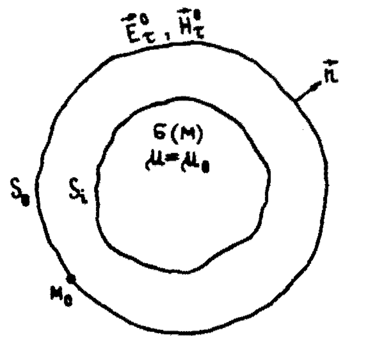

Fig. 2. Generalized MT model.

and is a functional of electrical conductivity. The MT problem, therefore, reduces to reconstruction of the impedance $Z^0 = Z(z = 0)$ at the Earth's surface and to determination of the electrical conductivity $\sigma(z)$ from the parametric dependence of the value Z^0 upon the frequency ω. A central question in the theory of MT sounding is the selection of a class of fields permitting application of the Tikhonov-Cagniard model. Wait [3], [4] and Price [5] arrived at the conclusion that the Tikhonov-Cagniard impedance can be reconstructed for a class of fields with "sufficiently slow horizontal variations." In this paper the main concepts of MT sounding are reconsidered and the classical Wait and Price criteria are supplemented with more extended criterion, thus considerably enlarging the range of practical applicability of the Tikhonov-Cagniard model.

II. The Generalized Magnetotelluric Problem

In general one should assume that the external fields are of arbitrary configuration and that the electrical conductivity of rocks varies not only in vertical but in horizontal directions also. The problem of MT sounding can then be formulated as follows (Fig. 2): find the distribution of the electrical conductivity $\sigma(M)$ within the earth from the tangential components $\vec{E}^0_\tau = [[\vec{n} \times \vec{E}(M_0)] \times \vec{n}]$, $\vec{H}^0_\tau = [[\vec{n} \times \vec{H}(M_0)] \times \vec{n}]$ of the MT fields, measured simultaneously at all M_0 points of the Earth's surface S_0. In order to solve this generalized MT problem, the linear operator $\tilde{Z}^0_{\sigma,\omega}$ transforming $\vec{H}^0_\tau$ to $\vec{E}^0_\tau$ must be introduced:

$$\vec{E}^0_\tau(M_0') = \tilde{Z}^0_{\sigma,\omega}[\vec{H}^0_\tau(M_0)]. \tag{3}$$

The operator $\tilde{Z}^0_{\sigma,\omega}$ is related to the distribution of $\sigma(M)$ and parametrically depends on ω. This generalizes the impedance relations (1) and $\tilde{Z}^0_{\sigma,\omega}$ can therefore be referred to as the *Tikhonov-Cagniard operator*. Transformation (3) is the *Tikhonov-Cagniard transformation*.

Not resorting to intuition, we shall prove that the Tikhonov-Cagniard operator does exist. The proof arises directly from an analysis of the following boundary-value problem. Let the Earth's surface S_0 bound the inhomogeneous Earth which has an arbitrary distribution $\sigma(M)$. The magnetic field $\vec{H}$ within the Earth satisfies the equation

$$\Delta\vec{H} + k^2\vec{H} - \frac{1}{\sigma}[\operatorname{curl}\vec{H} \times \operatorname{grad}\sigma] \tag{4}$$

where

$$k^2 = i\omega\mu_0\sigma, \qquad \operatorname{Re}(k) > 0.$$

On the interfaces S_i of discontinuities of the electrical conductivity $\sigma(M)$ certain conjugation conditions must be satisfied:

$$[\vec{H}_\tau]_{S_i} = 0, \qquad \left[\frac{1}{\sigma}\operatorname{curl}_\tau \vec{H}\right]_{S_i} = 0$$

where the brackets represent function discontinuity, i.e., the difference between limiting values on the inner and outer sides of the interface S_i. On the surface of the Earth the values of $\vec{H}^0_\tau$, and hence also those of $\partial H^0_n/\partial n = -\operatorname{div}^S \vec{H}^0_\tau$, are given ($\operatorname{div}^S$ is a symbol for two-dimensional divergence). It is known from the theory of partial differential equations that this problem has only one solution. It can be expressed as

$$\vec{H}(M) = \iint_{S_0} \hat{G}_{\sigma,\omega}(M, M_0)\,\vec{H}^0_\tau(M_0)\,dS_{M_0}$$

where $\hat{G}_{\sigma,\omega}$ is the matrix Green's function governed by the distribution of $\sigma(M)$ and is parametrically dependent on ω. The solution obtained provides the electric field

$$\vec{E}(M) = \frac{1}{\sigma(M)}\operatorname{curl}\vec{H}(M) = \frac{1}{\sigma(M)}\operatorname{curl}\iint_{S_0} \hat{G}_{\sigma,\omega}(M, M_0)\,\vec{H}^0_\tau(M_0)\,dS_{M_0}.$$

Now all that is necessary is to move the point M to the Earth's surface and, confining oneself to determination of $\vec{E}^0_\tau$, to write down

$$\vec{E}^0_\tau(M_0') = \iint \hat{K}_{\sigma,\omega}(M_0', M_0)\,\vec{H}^0_\tau(M_0)\,dS_{M_0} \tag{5}$$

where the matrix function $\hat{K}_{\sigma,\omega}$ is the result of the Green's function differentiation and reduction. It is evident that $\vec{E}^0_\tau$ is determined by linear transformation of $\vec{H}^0_\tau$, which was to be proved. The matrix function $\hat{K}_{\sigma,\omega}$ is the kernel of the integral operator $\tilde{Z}^0_{\sigma,\omega}$ determined completely by the distribution of $\sigma(M)$.

To each distribution of $\sigma(M)$, i.e., to each geoelectrical model, corresponds its own matrix $\hat{K}_{\sigma,\omega}$, i.e., its own Tikhonov-Cagniard operator. A concrete form of the Tikhonov-Cagniard operator (i.e., of the matrix $\hat{K}_{\sigma,\omega}$) can be found by solving a direct problem. The inverse problem is reduced to reconstruction of the Tikhonov-Cagniard operator from the known values $\vec{E}^0_\tau$ and $\vec{H}^0_\tau$, and to the determination of the distribution of $\sigma(M)$ in a selected class of geoelectrical models.

III. Tikhonov-Cagniard Operator for Plane-Layered Earth

It would be interesting to find the structure of the Tikhonov-Cagniard operator for the class of plane-layered which are the basis of the MT theory. Unlike (5), the operator $\tilde{Z}^0_{\sigma,\omega}$ will be determined in integrodifferential form. This will help to avoid generalized functions and will render the results more visual. Consider a model in which the Earth's surface coincides with the plane $z = 0$ of the Cartesian system x, y, z (Fig. 1) and the electrical conductivity of the Earth is a continuous or piecewise continuous function $\sigma(z)$. The Earth is excited by currents randomly distributed in the ionosphere. The air exhibits a finite electrical conductivity. Hence, two mechanisms involved here are electromagnetic induction and galvanic leakage.

According to (4), the magnetic field within the Earth ($z \geqslant 0$) satisfies the equations

$$\Delta H_x + k^2 H_x - \frac{1}{\sigma}\frac{d\sigma}{dz}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = 0$$

$$\Delta H_y + k^2 H_y - \frac{1}{\sigma}\frac{d\sigma}{dz}\left(\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y}\right) = 0$$

$$\Delta H_z + k^2 H_z = 0. \quad (6)$$

On the surface of the Earth $H_x = H_x^0, H_y = H_y^0$. On the planes $Z = Z_i$, where $\sigma(z)$ is discontinuous, the conjugation conditions

$$[H_x]_{z_i} = 0 \quad [H_y]_{z_i} = 0 \quad \left[\frac{1}{\sigma}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\right]_{z_i} = 0$$

$$\left[\frac{1}{\sigma}\left(\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y}\right)\right]_{z_i} = 0$$

are met. When $z \to \infty$ the magnetic field vanishes. The medium is magnetically uniform and, consequently, the $\vec{H}$ field satisfies

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0. \quad (7)$$

The $\vec{E}$ field is expressed via the $\vec{H}$ field:

$$\vec{E} = \frac{1}{\sigma}\,\text{curl}\,\vec{H}. \quad (8)$$

This three-dimensional problem can be reduced to one-dimensional by Fourier transformations:

$$\vec{h}(z,\alpha,\beta) = \iint_{-\infty}^{+\infty} \vec{H}(x,y,z)\exp - i(\alpha x + \beta y)\,dx\,dy$$

$$\vec{e}(z,\alpha,\beta) = \iint_{-\infty}^{+\infty} \vec{E}(x,y,z)\exp - (\alpha x + \beta y)\,dx\,dy. \quad (9)$$

Here α and β are the spatial frequencies along the x- and y-axes, respectively. The Fourier transforms $\vec{h}$ and $\vec{e}$ are the *spatial spectra* of the electromagnetic field. Transition to a one-dimensional problem greatly simplifies the model analysis.

According to (6) and (9), the $\vec{h}$ spectrum satisfies the equations

$$h_x'' - \eta^2 h_x - \frac{\sigma'}{\sigma}(h_x' - i\alpha h_z) = 0$$

$$h_y'' - \eta^2 h_y - \frac{\sigma'}{\sigma}(h_y' - i\beta h_z) = 0$$

$$h_z'' - \eta^2 h_z = 0 \quad (10)$$

where $\eta^2 = \alpha^2 + \beta^2 - k^2$ and $\text{Re}\,\eta > 0$; the primes represent differentiation with respect to z. On the Earth's surface $h_x = h_x^0$ and $h_y = h_y^0$. The conjugation conditions are written in the form

$$[h_x]_{z_i} = 0 \quad [h_y]_{z_i} = 0 \quad \left[\frac{1}{\sigma}(h_x' - i\alpha h_z)\right]_{z_i} = 0$$

$$\left[\frac{1}{\sigma}(h_y' - i\beta h_z)\right]_{z_i} = 0.$$

When $z \to \infty$ one has $\vec{h} \to 0$. It follows from condition (7) that

$$i\alpha h_x + i\beta h_y + h_z' = 0. \quad (11)$$

According to (8) and (9), the $\vec{e}$ spectrum is determined by the $\vec{h}$ spectrum:

$$e_x = \frac{1}{\sigma}(i\beta h_z - h_y')$$

$$e_y = \frac{1}{\sigma}(h_x' - i\alpha h_z)$$

$$e_z = \frac{i}{\sigma}(\alpha h_y - \beta h_x). \quad (12)$$

Now we express $\vec{h}$ and $\vec{e}$ via scalar potentials,

$$U_h = \alpha h_x + \beta h_y \qquad U_e = \beta h_x - \alpha h_y. \quad (13)$$

These potentials satisfy the equations

$$U_h'' - \eta^2 U_h + \frac{i\omega\mu_0\sigma'}{\eta^2} U_h' = 0$$

$$U_e'' - \eta^2 U_e - \frac{\sigma'}{\sigma} U_e' = 0 \quad (14)$$

as well as conditions of continuity for U_h, U_e, U_h'/η^2 and U_e'/σ at the interfaces of discontinuities of σ, and the conditions that $U_h \to 0$ and $U_e \to 0$ as $z \to \infty$.

According to (10) through (13),

$$h_x = \frac{\alpha U_h + \beta U_e}{\alpha^2 + \beta^2}$$

$$h_y = \frac{\beta U_h - \alpha U_e}{\alpha^2 + \beta_2}$$

$$h_z = \frac{h_z''}{\eta^2} = -\frac{i(\alpha h_x' + \beta h_y')}{\eta^2} = -\frac{i}{\eta^2} U_h' \quad (15)$$

and

$$e_x = \frac{\beta k^2 U_h' + \alpha\eta^2 U_e'}{\sigma\eta^2(\alpha^2 + \beta^2)}$$

$$e_y = \frac{\beta\eta^2 U_e' - \alpha k^2 U_h'}{\sigma\eta^2(\alpha^2 + \beta^2)}$$

$$e_z = -\frac{i}{\sigma} U_e. \quad (16)$$

The potentials U_h and U_e allow separation of the field into two parts, called *H mode* and *E mode*. In the H mode, described by potential U_h, there is no E_z component, whereas in the E mode, described by potential U_e, there is no H_z component.

If the field contains only the H mode, then

$$h_x = \frac{\alpha}{\alpha^2 + \beta^2} U_h \qquad h_y = \frac{\beta}{\alpha^2 + \beta^2} U_h \qquad h_z = -\frac{i}{\eta^2} U_h'$$

$$e_x = \frac{i\omega\mu_0\beta}{\eta^2(\alpha^2 + \beta^2)} U_h' \qquad e_y = -\frac{i\omega\mu_0\alpha}{\eta^2(\alpha^2 + \beta^2)} U_h' \qquad e_z = 0. \quad (17)$$

In this field configuration the Earth is excited by *induction* (the ionospheric currents induce telluric currents). According to (17),

$$e_x = Z_h h_y \text{ and } e_y = -Zh_x \tag{18}$$

where Z_h is the *spectral impedance of the magnetic type:*

$$Z_h = \frac{i\omega\mu_0}{\eta^2}\frac{U_h'}{U_h}. \tag{19}$$

According to (14), Z_h satisfies the Riccati equation

$$\frac{dZ_h}{dz} + \frac{\eta^2}{i\omega\mu_0} Z_h^2 = i\omega\mu_0. \tag{20}$$

If the field contains only the E mode, then

$$h_x = \frac{\beta}{\alpha^2+\beta^2} U_e \quad h_y = -\frac{\alpha}{\alpha^2+\beta^2} U_e \quad h_z = 0$$
$$e_x = \frac{\alpha}{\sigma(\alpha^2+\beta^2)} U_e' \quad e_y = \frac{\beta}{\sigma(\alpha^2+\beta^2)} U_e' \quad e_z = -\frac{i}{\sigma} U_e. \tag{21}$$

In this field configuration the active mechanism is of *galvanic* character (the ionospheric currents overflow into the Earth). According to (21),

$$e_x = Z_e h_y \text{ and } e_y = -Z_e h_x \tag{22}$$

where Z_e is the *spectral impedance of the electric type:*

$$Z_e = -\frac{1}{\sigma}\frac{U_e'}{U_e}. \tag{23}$$

According to (14), Z_e satisfies the Riccati equation

$$\frac{dZ_e}{dz} - \sigma Z_e^2 = -\frac{\eta^2}{\sigma}. \tag{24}$$

If the field simultaneously contains the two modes, it is seen that after eliminating U_h' and U_e' from (15), (16), (19), and (20),

$$\begin{aligned} e_x &= \alpha\beta\Delta Z h_x + (Z_h - \alpha^2\Delta Z)h_y \\ e_y &= -(Z_h - \beta^2\Delta Z)h_x - \alpha\beta\Delta Z h_y \end{aligned} \tag{25}$$

where

$$\Delta Z = \frac{Z_h - Z_e}{\eta_0^2} \text{ and } \eta_0 = \sqrt{\alpha^2+\beta^2}.$$

On the surface of the Earth

$$\begin{aligned} e_x^0 &= \alpha\beta\Delta Z^0 h_x^0 + (Z_h^0 - \alpha^2\Delta Z^0)\, h_y^0 \\ e_y^0 &= -(Z_h^0 - \beta^2\Delta Z^0)h_x^0 - \alpha\beta\Delta Z^0 h_y^0 \end{aligned} \tag{26}$$

where $Z_h^0 = Z_h(z = 0)$ and $\Delta Z^0 = \Delta Z(z = 0)$. The spectra of the horizontal components of the electric field are linearly transformed spectra of the horizontal components of the magnetic field. The linear transformation coefficients are expressed in terms of the spectral impedances of the magnetic and electric types. The expressions thus derived determine the form of the Tikhonov–Cagniard operator in *the spectral domain*.

To determine the spatial relations it is convenient to cast transformation (26) in the form

$$\begin{aligned} e_x^0 &= Z_h^0 h_y^0 + \Delta Z^0(\alpha\beta h_x^0 - \alpha^2 h_y^0) \\ e_y &= -Z_h^0 h_x^0 + \Delta Z^0(\beta^2 h_x^0 - \alpha\beta h_y^0). \end{aligned} \tag{27}$$

This allows one to consider the coefficients Z_h^0 and ΔZ^0 as frequency characteristics of some *spatial filters* affecting the horizontal components of the magnetic field and their horizontal derivatives. It is easy to show that the functions Z_h^0 and ΔZ admit an inverse Fourier transformation. Indeed, it follows from (20) and (24) that they depend parametrically on η_0 and have no singularities within the range $0 < \eta_0 < \infty$. At the ends of this range their asymptotic values are limited:

$$\left.\begin{aligned} Z_h^0 &= Z^0 + 0(\eta_0^2) \\ Z_e^0 &= Z^0 + 0(\eta_0^2) \\ \Delta Z^0 &= \text{const} + 0(\eta_0^2) \end{aligned}\right\}, \quad \text{for } \eta_0 \longrightarrow 0 \tag{28}$$

and

$$\left.\begin{aligned} Z_h^0 &= 0(1/\eta_0) \\ Z_e^0 &= 0(\eta_0) \\ \Delta Z^0 &= 0(1/\eta_0) \end{aligned}\right\}, \quad \text{for } \eta_0 \longrightarrow \infty \tag{29}$$

where $Z^0 = Z(z = 0)$ is the Tikhonov–Cagniard impedance determined by (2).

Fourier transformation of Z_h^0 and ΔZ^0 provides the *spatial characteristics* of the filters:

$$\begin{aligned} G_Z^0(x,y) &= \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} Z_h^0(\sqrt{\alpha^2+\beta^2}) \exp i(\alpha x + \beta y)\, d\alpha\, d\beta \\ G_{\Delta Z}^0(x,y) &= \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} \Delta Z^0(\sqrt{\alpha^2+\beta^2}) \exp i(\alpha x + \beta y)\, d\alpha\, d\beta. \end{aligned} \tag{30}$$

These filters exhibit two remarkable properties: 1) they are *axially symmetric* and 2) they are *local.*

To prove their axial symmetry, use will be made of the well-known relations between Fourier and Hankel integrals:

$$\frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} f(\sqrt{\alpha^2+\beta^2}) \exp i(\alpha x + \beta y)\, d\alpha\, d\beta = \frac{1}{2\pi}\int_0^\infty f(\eta_0)\, J_0(\eta_0 r)\, \eta_0 d\eta_0$$

where $\eta_0 = \sqrt{\alpha^2+\beta^2}$, $r = \sqrt{x^2+y^2}$, and J_0 is the zeroth-order Bessel function. Then, according to (30),

$$\begin{aligned} G_Z^0(x,y) &= G_Z^0(r) = \frac{1}{2\pi}\int_0^\infty Z_h^0(\eta_0)\, J_0(\eta_0 r)\, \eta_0 d\eta_0 \\ G_{\Delta Z}^0(x,y) &= G_{\Delta Z}^0(r) = \frac{1}{2\pi}\int_0^\infty \Delta Z^0(\eta_0)\, J_0(\eta_0 r)\, \eta_0 d\eta_0. \end{aligned} \tag{31}$$

Locality of the filters follows from the Tikhonov [6] theorem of asymptotic behavior of integrals containing Bessel functions. By applying this theorem to integrals (31) one finds that for $r \to \infty$ the G_Z^0 and $G_{\Delta Z}^0$ functions decrease faster than $1/r^n$, where n is any integer.

The functions G_Z^0 and $G_{\Delta Z}^0$ are the kernels of integral and integro-differential transformations determining the form of the Tikhonov–Cagniard operator in the *space domain*. Indeed, applying an inverse Fourier transformation to (27) and using the theorem of spectral convolution reveals that

$$E_x^0(M_0') = \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) H_y^0(M_0)\, dS_{M_0} + \iint_{-\infty}^{+\infty} G_{\Delta Z}^0(r_{M_0M_0'}) \left(\frac{\partial^2 H_x^0(M_0)}{\partial x \partial y} - \frac{\partial^2 H_y^0(M_0)}{\partial x^2} \right) dS_{M_0}$$

$$E_y^0(M_0') = - \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) H_x^0(M_0)\, dS_{M_0} + \iint_{-\infty}^{+\infty} G_{\Delta Z}^0(r_{M_0M_0'}) \left(\frac{\partial^2 H_x^0(M_0)}{\partial y^2} - \frac{\partial^2 H_y^0(M_0)}{\partial x \partial y} \right) dS_{M_0}.$$

In these formulas $M_0 = \{x, y, 0\}$, $M_0' = \{x', y', 0\}$, and $r_{M_0M_0'} = \sqrt{(x-x')^2 + (y-y')^2}$. It is evident from (32) that the main contribution to $\vec{E}_\tau^0$ arises from a bounded area which has the form of a circle centered at the point M_0'. This is in effect a *filter pass area*. Its radius is a measure of the Tikhonov–Cagniard operator locality.

The Tikhonov–Cagniard operator consists of integral and integro-differential terms. Let us attempt to elucidate their physical sense. The integral terms are associated with the kernel G_Z^0 determined by the spectral impedance Z_h^0 of the magnetic type. They therefore reflect the contribution of the H mode, i.e., the *electromagnetic induction* in the Earth. The integro-differential terms, on the other hand, are associated with the vertical component of the electric currents. Indeed,

$$\iint_{-\infty}^{+\infty} G_{\Delta Z}(r_{M_0M_0'}) \left(\frac{\partial^2 H_x^0(M_0)}{\partial x \partial y} - \frac{\partial^2 H_y^0(M_0)}{\partial x^2} \right) dS_{M_0} = - \iint_{-\infty}^{+\infty} G_{\Delta Z}^0(r_{M_0M_0'}) \frac{\partial j_z(M_0)}{\partial x}\, dS_{M_0}$$

$$\iint_{-\infty}^{+\infty} G_{\Delta Z}^0(r_{M_0M_0'}) \left(\frac{\partial^2 H_x^0(M_0)}{\partial y^2} - \frac{\partial^2 H_y^0(M_0)}{\partial x \partial y} \right) dS_{M_0} = - \iint_{-\infty}^{+\infty} G_{\Delta Z}^0(r_{M_0M_0'}) \frac{\partial j_z(M_0)}{\partial y}\, dS_{M_0}.$$

They evidently reflect the contribution of the E mode, i.e., the *galvanic mechanism* of Earth excitation. Numerous evaluations performed by different authors for many types of MT variations have shown that the contribution of the E mode to the horizontal components of the MT field is negligible and that the telluric currents have an *inductive* origin (Chapman and Bartels [7]; Eckhardt *et al.* [8]; Swift [9]; Berdichevsky *et al.* [10]; Berdichevsky and Feinberg [11]; Vanyan and Berdichevsky [12]; Hermance [13]; Dmitriev [14]). Therefore, without any noticeable loss of accuracy, it is possible to disregard galvanic phenomena and to consider only electromagnetic induction. Mathematically this means that instead of transformations (27) and (32), consideration is given to the transformations

$$e_x^0 = Z_h^0 h_y^0 \qquad e_y = -Z_h^0 h_x^0 \tag{33}$$

and

$$E_x^0(M_0') = \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) H_y^0(M_0)\, dS_{M_0}$$

$$E_y^0(M_0') = - \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) H_x^0(M_0)\, dS_{M_0} \tag{34}$$

connected with the E mode (Wait [4]; Price [5]). Chetaev *et al.* [15] object rather vigorously, though without sufficient grounds, to such an approximation. The question of neglecting the E mode is, however, not a critical point in magnetotellurics since, over the range of frequencies used for practical MT soundings, the asymptotic behavior (28) holds good and, hence, $Z_e^0 \approx Z_h^0 \approx Z^0$ (Madden and Swift [16]). Over this frequency range, the lower limit of which depends on a degree of field nonuniformity, *both modes yield the Tikhonov–Cagniard impedance.*

IV. Spectral and Local Magnetotelluric Sounding

The spectral and spatial Tikhonov–Cagniard transformations make it possible to construct two schemes of MT sounding. The first scheme makes use of spectral transformations (26) effected by the matrix operator

$$\hat{Z}_{\sigma,\omega}^0 = \begin{Vmatrix} \alpha\beta\Delta Z^0 & Z_h^0 - \alpha^2 \Delta Z^0 \\ -Z_h^0 + \beta^2 \Delta Z^0 & -\alpha\beta\Delta Z^0 \end{Vmatrix} \tag{35}$$

which, with the E mode discarded, takes the form

$$\hat{Z}_{\sigma,\omega}^0 = \begin{Vmatrix} 0 & Z_h^0 \\ -Z_h^0 & 0 \end{Vmatrix}. \tag{36}$$

This scheme assumes synchronous measurements of the horizontal components $\vec{E}_\tau^0$, $\vec{H}_\tau^0$ over a rather vast area, and reconstruction of the impedance $Z_h^0 \cong Z^0$ from the Fourier spectra $\vec{e}_\tau^0$ and $\vec{h}_\tau^0$. This type of MT sounding will be called *spectral*. Spectral MT sounding gives a distribution $\sigma(z)$ averaged over the whole area covered by the Fourier transformation. Evidently, this procedure is appropriate only if one wants to study rather large-scale or global effects. To the authors' knowledge there has never been any practical application of the spectral scheme of MT sounding. It is, however, very attractive from a theoretical viewpoint as it deals with simple formulas and facilitates analysis of the MT sounding resolution. Excellent examples of such analysis can be found in the work by Wait [3], [4], Price [5], Srivastava [17], and Weidelt [18].

The second scheme is aimed at the local determination of $\sigma(z)$. It is connected with the use of the Tikhonov–Cagniard impedance. This scheme assumes measurements of the horizontal components $\vec{E}_\tau^0$, $\vec{H}_\tau^0$ at a single site and reconstruction of the matrix operator

$$\hat{Z}_{\sigma,\omega}^0 = \begin{Vmatrix} 0 & Z^0 \\ -Z^0 & 0 \end{Vmatrix} \tag{37}$$

effecting transformation (1). This type of MT sounding will be called *local*. Local MT sounding is one of the most popular methods employed in modern geoelectrics. It is applicable in the class of fields for which the Tikhonov-Cagniard model is valid.

V. On the Practical Applicability of the Tikhonov-Cagniard Model

This question is usually treated in terms of spectral representations (Wait [3], [4]; Price [5]; Madden and Swift [16]; Berdichevsky *et al.* [19]). The deciding criteria have been derived from the asymptotic behavior of the spectral impedances Z_h^0, Z_e^0 of the magnetic and electric types for $\eta_0 \to 0$. According to (28),

$$Z_h^0 \sim Z^0 \text{ and } Z_e^0 \sim Z^0, \text{ for } \eta_0 = \sqrt{\alpha^2 + \beta^2} \longrightarrow 0$$

where Z^0 is the Tikhonov-Cagniard impedance. These asymptotic formulas define a range of low spatial frequencies where the spectral impedances practically coincide with the Tikhonov-Cagniard impedance. Consider, as an example, a homogeneous Earth (σ = const) for which, according to (2), (20), and (24),

$$Z^0 = \sqrt{-\frac{i\omega\mu_0}{\sigma}} \tag{38}$$

$$Z_h^0 = -\frac{i\omega\mu_0}{\eta} = -\frac{i\omega\mu_0}{\sqrt{\eta_0^2 - k^2}} = Z^0 \frac{1}{\sqrt{1 + i(\eta_0^2 d^2/2)}} \tag{39}$$

and

$$Z_e^0 = \frac{\eta}{\sigma} = \frac{\sqrt{\eta_0^2 - k^2}}{\sigma} = Z^0 \sqrt{1 + i\frac{\eta_0^2 d^2}{2}} \tag{40}$$

where

$$d = \sqrt{\frac{2}{\omega\mu_0\sigma}} \tag{41}$$

is the field penetration depth. If

$$\eta_0 d \ll 1 \tag{42}$$

then $Z_h^0 \approx Z_e^0 \approx Z^0$. Condition (42) relates the field penetration depth and the spatial frequencies for which application of the Tikhonov-Cagniard model is permitted. This is the well-known Wait-Price criterion. Theoretically it is an ideal criterion, but it proves difficult to use in practice, since we know too little about the spatial spectrum of pulsations and bays which provide the basic excitation sources for MT sounding. Price tries to avoid this difficulty by approximating the field with single spatial harmonics whose parameter η_0 reflects the source size. But such a model approximation is rather speculative and hardly improves the accuracy of the results. Thus the spectral approach is more of cognitive than of practical importance.

The Wait-Price criterion has played a significant role in the development of MT's. But it gave rise to a false impression that the Tikhonov-Cagniard model can be applied only in the cases where the field is "uniform over horizontal distances which are large compared to the penetration depth" (Niblett [20]).

Looking for more realistic practical criteria we turn to the Tikhonov-Cagniard spatial transformations. We shall prove the following statement: *if the horizontal components of the magnetic field vary linearly, i.e., if*

$$\vec{H}_\tau^0(M_0) = \vec{H}_\tau^0(M_0') + (x - x')\frac{\partial \vec{H}_\tau^0(M_0')}{\partial x'} + (y - y')\frac{\partial \vec{H}_\tau^0(M_0')}{\partial y'}$$

then *the Tikhonov-Cagniard integro-differential operator determined by (32) degenerates into matrix operator (37), whose effect is reduced to multiplying H_x and H_y by the Tikhonov-Cagniard impedance.* To prove this we introduce (43) into (32):

$$E_x^0(M_0') = H_y^0(M_0') \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})\, dS_{M_0} + \frac{\partial H_y^0(M_0')}{\partial x'} \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})\, dS_{M_0} + \frac{\partial H_y^0(M_0')}{\partial y'} \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})(y - y')\, dS_{M_0}$$

$$E_y^0(M_0') = -H_x^0(M_0') \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})\, dS_{M_0} - \frac{\partial H_x^0(M_0')}{\partial x'} \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})(x - x')\, dS_{M_0} - \frac{\partial H_x^0(M_0')}{\partial y'} \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})(y - y')\, dS_{M_0}.$$

According to (28) and (30) the first integrals on the right-hand sides of these formulas simply yield

$$\iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})\, dS_{M_0} = \lim_{\substack{d \to 0 \\ \beta \to 0}} \iint_{-\infty}^{+\infty} G_Z^0(x, y) \exp -i(\alpha x + \beta y)\, dx\, dy = \lim_{\eta_0 \to 0} Z_h^0(\eta_0) = Z^0.$$

Because of the axial symmetry of the G_Z^0 filter it follows that

$$\iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})(x - x')\, dS_{M_0} = \int_0^\infty \int_0^{2\pi} G_Z^0(r) \cos\varphi r^2\, dr\, d\varphi = \int_0^\infty G_Z^0(r) r^2\, dr \int_0^{2\pi} \cos\varphi\, d\varphi = 0$$

and

$$\iint_{-\infty}^{+\infty} G_Z^0(r_{M_0 M_0'})(y - y')\, dS_{M_0} = \int_0^\infty \int_0^{2\pi} G_Z^0(r) \sin\varphi r^2\, dr\, d\varphi = \int_0^\infty G_Z^0(r) r^2\, dr \int_0^{2\pi} \sin\varphi\, d\varphi = 0.$$

Consequently,

$$E_x^0(M_0') = Z^0 H_y^0(M_0') \text{ and } E^0(M_0') = -Z^0 H_x^0(M_0')$$

or

$$\begin{Vmatrix} E_x^0(M_0') \\ E_y^0(M_0') \end{Vmatrix} = \begin{Vmatrix} 0 & Z^0 \\ -Z^0 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} H_x^0(M_0') \\ H_y^0(M_0') \end{Vmatrix}$$

which was to be proven. In this model the Tikhonov-Cagniard impedance can be derived from the horizontal components of the fields measured at any point of the Earth's surface. We have, therefore, shown that the *model with an arbitrary linear variation in $\vec{H}_\tau^0$ still allows the formal determination of the Tikhonov-Cagniard impedance.*

To obtain the Tikhonov-Cagniard impedance in practice, it will suffice if the horizontal components of the magnetic field change linearly over the pass area of the G_Z^0 and $G_{\Delta Z}^0$ filters. We shall now estimate the radius of this area (filter radius). The simplest case is a homogeneous Earth, where σ = constant. According to (31), (39), and (40),

$$G_Z^0 = -\frac{i\omega\mu_0}{2\pi}\int_0^\infty \frac{J_0(\eta_0 r)\eta_0 d\eta_0}{\sqrt{\eta_0^2 - k^2}} = -\frac{i\omega\mu_0}{2\pi}\frac{\exp ikr}{r}$$

$$G_{\Delta Z}^0 = -\frac{1}{2\pi\sigma}\int_0^\infty \frac{J_0(\eta_0 r)\eta_0 d\eta_0}{\sqrt{\eta_0^2 - k^2}} = -\frac{1}{2\pi\sigma}\frac{\exp ikr}{r}. \tag{44}$$

The G_Z^0 and $G_{\Delta Z}^0$ filters are two-dimensional. Their effect is characterized by the functions

$$\tilde{G}_Z^0 = 2\pi r G_Z^0 \text{ and } \tilde{G}_{\Delta Z}^0 = 2\pi r G_{\Delta Z}^0. \tag{45}$$

Indeed, if the two-dimensional axially symmetric filter G affects F, then

$$\iint_{-\infty}^{+\infty} G(r_{M_0M_0'})F(M_0)\, dS_{M_0} = \int_0^\infty G(r) r dr \int_0^{2\pi} F(r,\varphi)\, d\varphi = \int_0^\infty \tilde{G}(r)\tilde{F}(r)\, dr$$

where $\tilde{F}$ is the mean value of F on the circle of radius r,

$$\tilde{F}(r) = \frac{1}{2\pi}\int_0^{2\pi} F(r,\varphi)\, d\varphi$$

and $\tilde{G}$ is the *effective* filter characteristic

$$\tilde{G}(r) = 2\pi r G(r).$$

The effective characteristics of the G_Z^0 and $G_{\Delta Z}^0$ filters, according to (44) and (45), have the form

$$\tilde{G}_Z^0 = -i\omega\mu_0 \exp ikr \quad\text{and}\quad \tilde{G}_{\Delta Z}^0 = -\frac{1}{\sigma}\exp ikr.$$

Upon normalization, one obtains

$$\left|\frac{\tilde{G}_Z^0}{\omega\mu_0}\right| = e^{-r/d} \quad\text{and}\quad \left|\sigma\tilde{G}_{\Delta Z}^0\right| = e^{-r/d} \tag{46}$$

where d is the field penetration depth calculated by (41). Clearly the two characteristics coincide. They are shown in Fig. 3. Having limited the pass area by a level of 0.2 one obtains the filter radius $r_G = 1.6d$. The maximum size of the pass area is $3.2d$. Thus the following statement can be made: *If on the surface of a homogeneous Earth H_x and H_y vary linearly over distances of the order of three times the field penetration depth d, the ratios E_x/H_y and E_y/H_x determine the Tikhonov-Cagniard impedance irrespective of the relations between the H and the E modes.* It is interesting to compare this criterion with the spectral criterion (42) written in the form $L \gg 2\pi d$, where L is the harmonic spatial period. The harmonic comprising a few tens of d admits of a linear approximation over a distance of $3d$. The two criteria are therefore consistent.

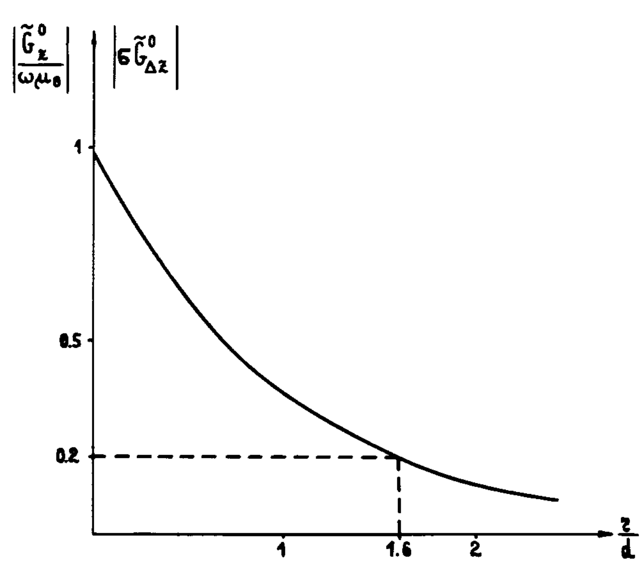

Fig. 3. Effective characteristics of the G_Z^0 and $G_{\Delta Z}^0$ filters for a homogeneous Earth.

In the case of a layered Earth, calculations become more complicated. The best estimate is derived after splitting up the $G_{\Delta Z}^0$ filter. We introduce the wave number $k_1 = \sqrt{i\omega\mu_0\sigma}$, of the first layer and represent ΔZ^0 in the form

$$\Delta Z^0 = \frac{Z_h^0 - Z_e^0}{\eta_0^2} = \frac{Z_h^0}{k_1^2} - \frac{V^0}{k_1^2} \tag{47}$$

where

$$V^0 = \frac{(\eta_0^2 - k_1^2)Z_h^0 + k_1^2 Z_e^0}{\eta_0^2}. \tag{48}$$

Then, according to (31),

$$G_{\Delta Z}^0(r) = \frac{G_Z^0(r)}{k_1^2} - \frac{G_V(r)}{k_1^2} \tag{49}$$

where

$$G_V^0(r) = \frac{1}{2\pi}\int_0^\infty V^0(\eta_0) J_0(\eta_0 r)\eta_0 d\eta_0. \tag{50}$$

Consequently,

$$E_x^0(M_0') = \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'})\left\{H_y^0(M_0) - \frac{1}{k_1^2}\frac{\partial j_z^0(M_0)}{\partial x}\right\} dS_{M_0} + \frac{1}{k_1^2}\iint_{-\infty}^{+\infty} G_V^0(r_{M_0M_0'})\frac{\partial j_z^0(M_0)}{\partial x} dS_{M_0}.$$

The second term can be integrated by parts:

$$E_x^0(M_0') = \iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) \left\{ H_y^0(M_0) - \frac{1}{k_1^2} \frac{\partial j_z^0(M_0)}{\partial x} \right\} dS_{M_0}$$

$$+ \iint_{-\infty}^{+\infty} G_W^0(r_{M_0M_0'}) \frac{j_z^0(M_0)}{k_1} \frac{x - x'}{r_{M_0M_0'}} dS_{M_0} \qquad (51)$$

where

$$G_W^0(r) = -\frac{1}{k_1} \frac{dG_V^0(r)}{dr}$$

$$= \frac{1}{2\pi k_1} \int_0^\infty V^0(\eta_0) J_1(\eta_0 r) \eta_0 d\eta_0. \qquad (52)$$

In a similar way,

$$E_y^0(M_0') = -\iint_{-\infty}^{+\infty} G_Z^0(r_{M_0M_0'}) \left\{ H_y^0(M_0) + \frac{1}{k_1^2} \frac{\partial j_z^0(M_0)}{\partial y} \right\} dS_{M_0}$$

$$+ \iint_{-\infty}^{+\infty} G_W^0(r_{M_0M_0'}) \frac{j_z^0(M_0)}{k_1} \frac{y - y'}{r_{M_0M_0'}} dS_{M_0}. \qquad (53)$$

In these transformations the E mode splits into two parts. The first part, expressed by the horizontal derivatives of j_z^0, joins to the G_Z^0 filter. The second part, connected directly with j_z^0, remains in the pass area of the G_W^0 filter. The G_Z^0 and G_W^0 filters have the same dimension and affect $H_{y,x}^0 \mp (1/k_1^2)(\partial j_z^0/\partial x, y)$ and j_z^0/k_1. If the Earth is homogeneous, then, according to (39), (40), (48), and (52), $G_W^0 = 0$. Thus this filter reflects the influence of the earth's vertical inhomogeneity ("layering effect").

Consider two rather typical models, imitating the geoelectrical structure of regions with, respectively, a higher and a poorer conducting sedimentary cover and with a fairly well conducting zone in the upper mantle. These are four-layer models with the parameters:

The G_Z^0 filter just slightly depends on ρ_1. Let us find its range r_{G_Z}, as radius of the pass area limited by a level of 0.2. The value of r_{G_Z} increases with the variation period. Geophysicists engaged in studying sedimentary covers are usually quite satisfied to observe variations whose periods reach T_1 and T_2. Over these periods $r_{G_Z} = 15$ and 75 km. Upper mantle investigations, on the other hand, would involve variations with periods T_3 and T_4. Here $r_{G_Z} = 150$ and 175 km.

The G_W^0 filter depends to a large extent upon ρ_1. Defining level 0.2 off a maximum of the filter effective characteristic one finds the radii r_{G_W}.

	T_1	T_2	T_3	T_4
Model I	$r_{G_W} = 50$	225	850	1000 km
Model II	$r_{G_W} = 40$	125	270	300 km

As can be seen, the G_W^0 filter is much less local than the G_Z^0 filter. However, these filters affect fields of quite different intensities. To variations in H_τ^0 of order $1\gamma \cong 10^{-3}$ A/m correspond variations in j_z^0 not exceeding, according to the most rigorous estimates by Morgunov *et al.* [15], 10^{-11} to 10^{-12} A/m^2. If $\rho_1 = 3\ \Omega$ m and $T \leqslant 10^4$ s, then over the whole period range concerned in Model I, $|j_z^0/k_1| < 5 \times 10^{-7}$ to 5×10^{-8} A/m, which is three to four orders of magnitude lower than the H_τ^0 intensity. The same relations are obtained for Model II. It is hardly necessary to prove that the contribution of the G_W^0 filter to E_τ^0 is negligible and almost does not affect the degree of locality of the Tikhonov-Cagniard operator.

It seems evident that locality of the Tikhonov-Cagniard operator is largely governed by the G_Z^0 filter. The diameter of this filter is under 100-200 km when a sedimental cover with a thickness of up to 3 km is studied and increases to 300-500 km for investigations of fairly well conducting formations of the upper mantle lying as deep as 100 km. *If, over such distances, the H_x and H_y components change linearly and the Earth layers are horizontal, the ratios E_x/H_y and*

Model I	$\rho_1 = 3\ \Omega$ m	$\rho_2 = 3 \times 10^3\ \Omega$ m	$\rho_3 = 3 \times 10^2\ \Omega$ m	$\rho_4 = 3 \times 10^{-2}\ \Omega$ m
	$h_1 = 3$ km	$h_2 = 50$ km	$h_3 = 50$ km	
Model II	$\rho_1 = 30\ \Omega$ m	$\rho_2 = 3 \times 10^3\ \Omega$ m	$\rho_3 = 3 \times 10^2\ \Omega$	$\rho_4 = 3 \times 10^{-2}\ \Omega$
	$h_1 = 3$ km	$h_2 = 50$ km	$h_3 = 50$ km	

The M_T sounding curves calculated for these models are shown in Figs. 4 and 5. Apparent resistivities were calculated using the traditional Tikhonov-Cagniard formula

$$\rho_T = \frac{|Z^0|^2}{\omega\mu_0}.$$

In the same figures one can see the effective characteristics of the filters

$$\tilde{G}_Z^0 = 2\pi r G_Z^0 \text{ and } \tilde{G}_W^0 = 2\pi r G_W^0$$

normalized by the factor $1/\omega\mu_0$. They have been calculated for characteristic periods T referring to the ascending branches (T_1, T_2), maxima (T_3), and descending branches (T_4) of the ρ_T curves.

E_y/H_x determine the Tikhonov-Cagniard impedance. Surprising as it seems, the Tikhonov-Cagniard model can be applied to fields with fast horizontal variations!

Judging by the results of modern geomagnetic measurements over distances of hundreds of kilometers, the horizontal components of the magnetic field can almost always be approximated by linear functions. The only exceptions arise for zones with local field maxima (say, for the auroral zone). This speaks in favor of an *almost general applicability of the Tikhonov-Cagniard model* for the interpretation of MT sounding carried out in regions characterized by a horizontally homogeneous ground structure. Now it becomes clear why MT soundings in polar latitudes (for instance, in the Arctic ocean) agree fairly well with the Tikhonov-Cagniard model

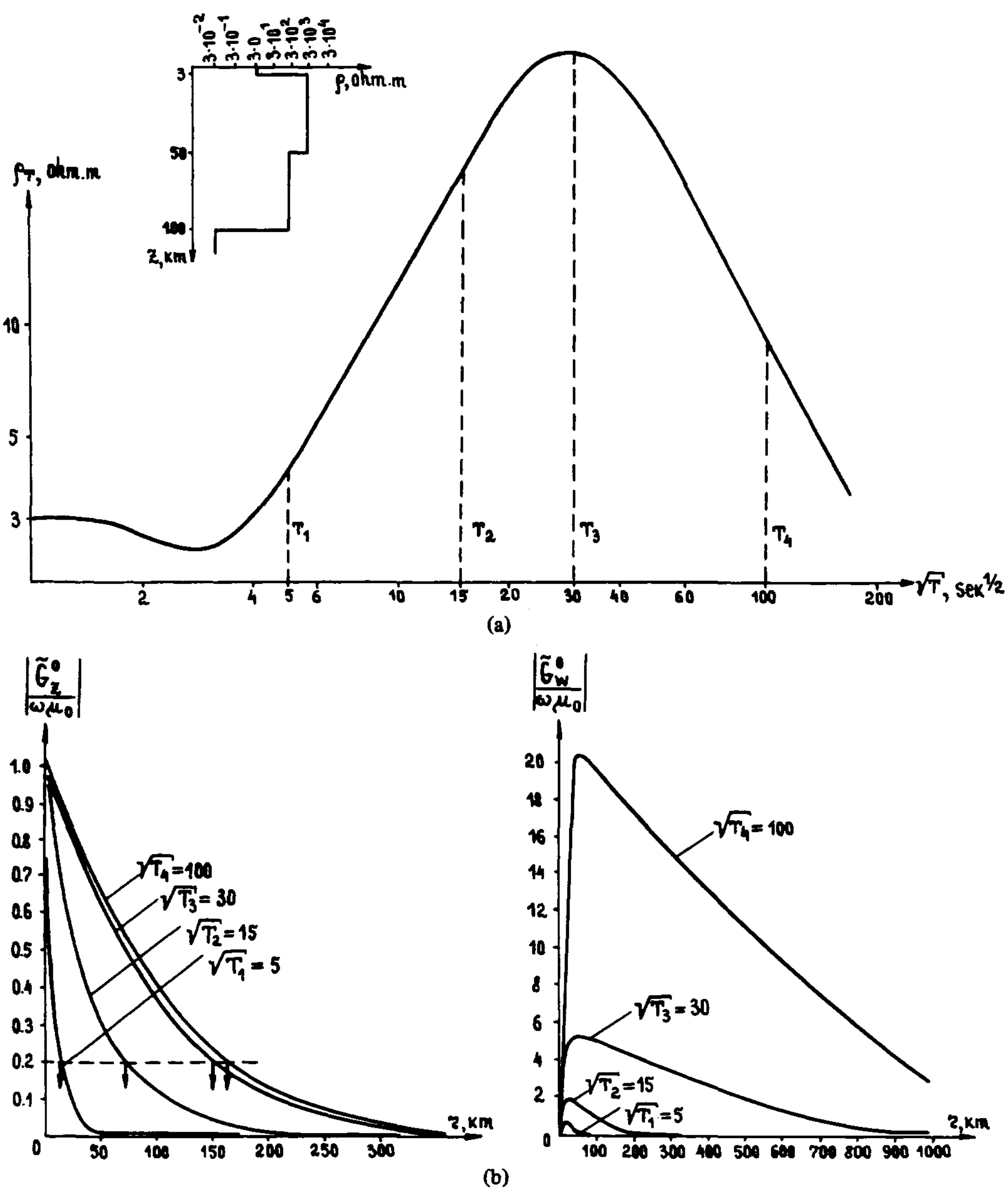

Fig. 4. (a) MT sounding curve. (b) Effective characteristics of the G_Z^0 and G_W^0 filters for Model I.

in spite of rapid horizontal changes of the field (Trofimov and Fonarev [21]).

VI. Conclusion

The conclusion can be drawn that the condition determining applicability of the one-dimensional Tikhonov–Cagniard model is *linearity* of the horizontal field variations rather than the slowness of these variations. It is worthwhile to remember an early result obtained by Wait [3] regarding high-frequency asymptotics of the electric field. In the case of a homogeneous Earth

$$-E_y^0 = Z^0 H_x^0 + \frac{Z^0}{2k^2}\left(\frac{\partial^2 H_x^0}{\partial x^2} - \frac{\partial^2 H_x^0}{\partial y^2} + 2\frac{\partial^2 H_x^0}{\partial x \partial y}\right) + 0\left(\frac{1}{k^4}\right).$$

It is evident here that over a high-frequency range the deviation of E_y^0/H_x^0 from the Tikhonov–Cagniard impedance is associated with the second but not with the first derivatives of $\vec{H}_\tau^0$. This peculiar feature of high-frequency asymptotics is in full accord with our theory.

It has been believed for years that the magnetotelluric fields should be uniform over distances of the order of hundreds of kilometers. This imposed too severe restrictions on the structure of the electromagnetic field under consideration and sometimes gave rise to doubts as to the reality of the Tikhonov–Cagniard model. Now the necessary conditions can be relaxed; all that is required is that *the fields vary only linearly* at the surface of the sounding area. This considerably extends the boundaries of practical applicability of the Tikhonov–Cagniard model.

Madden and Nelson [22] titled their paper "A defence of Cagniard's magnetotelluric method." Our paper could have been titled "A justification of Tikhonov and Cagniard's magnetotelluric method."

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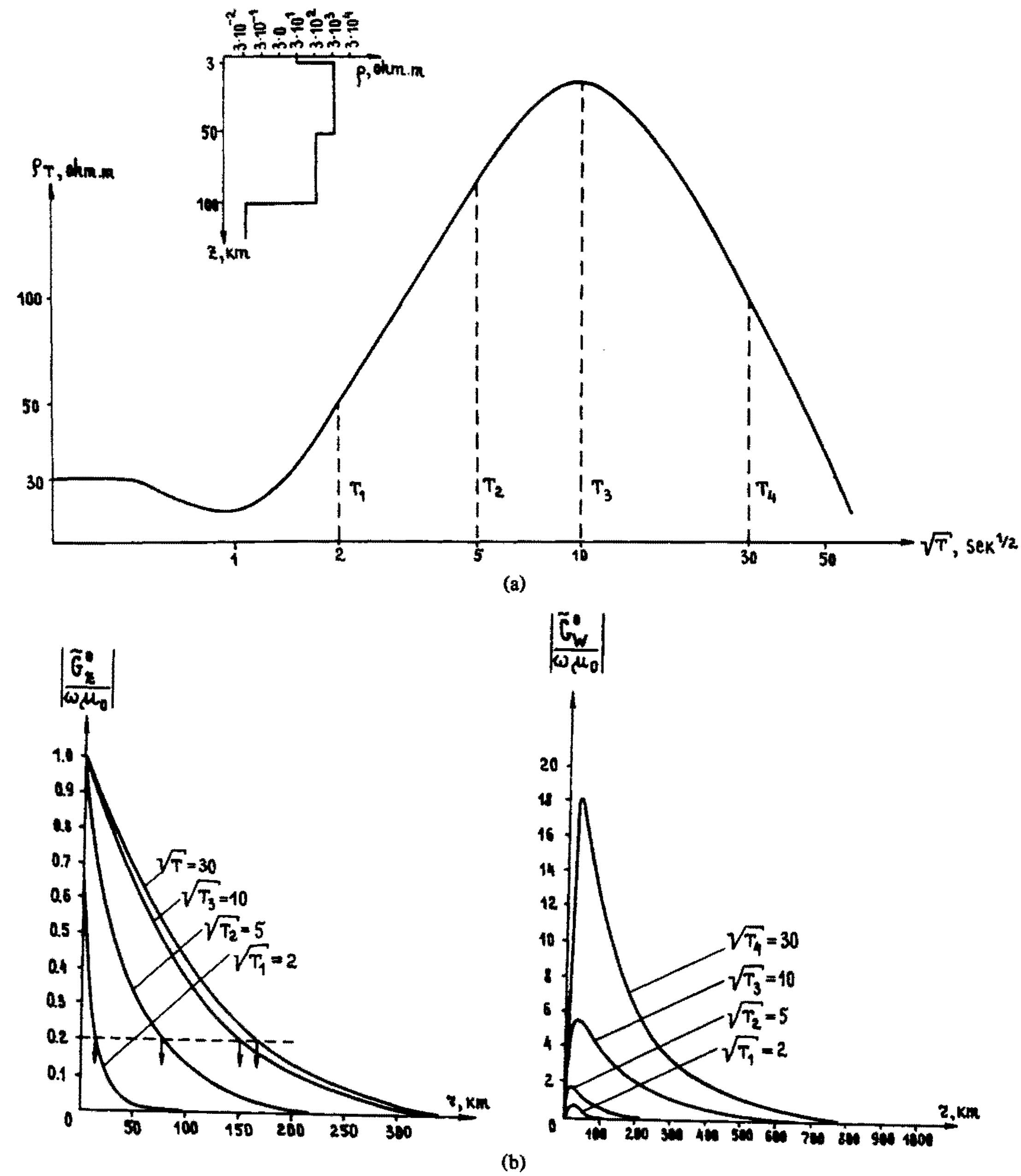

Fig. 5. (a) MT sounding curve. (b) Effective characteristics of the G_Z^0 and G_W^0 filters for Model II.

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Electromagnetic Induction in Thin Sheet Conductivity Anomalies at the Surface of the Earth

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Abstract—Lateral variations in the Earth's conductivity complicate considerably the calculation of the electromagnetic response of the Earth to an external inducing field which is uniform and horizontal. Although analytic solutions have been found for a few simple two-dimensional models in which the conductivity varies in one horizontal direction only, it is necessary, in general, to resort to numerical methods. If the conductivity variations of interest are confined to a surface layer it is often possible to represent the Earth mathematically as a uniform conducting half-space covered by an infinitely thin sheet of variable surface conductance. This simplification effectively reduces by one the number of dimensions over which the field equations need to be integrated numerically. It is shown that for a two-dimensional model the horizontal component of the electric field satisfies an integral equation on the surface of the thin sheet, which can be solved numerically for arbitrary sheet conductance. The accuracy of the numerical procedure is confirmed by applying it to E- and B-polarization induction in two adjacent half-sheets and then comparing the solution obtained with known analytic solutions of the same problem. In three-dimensions the two horizontal components of the surface electric field satisfy a coupled pair of double integral equations which can also be solved numerically for an arbitrarily varying conductance of the surface sheet.

I. Introduction

An ELECTROMAGNETIC problem of current interest in geophysics concerns the effect of lateral changes in the Earth's conductivity on telluric currents induced by an external magnetic source of either natural or artificial origin. Analytical solutions of even the simplest model structures are notoriously difficult, if not impossible, to obtain. In general, numerical methods must be used, but for three-dimensional conductivity structures they can become unwieldy both in storage requirements and in the time required for the iterative procedure to converge.

In some structures of practical interest, the conductivity anomalies are confined to a thin layer near the surface of the Earth. Obvious examples are shallow lying ore bodies, sedimentary layers, and, on a larger scale, the oceans whose electrical conductivity is orders of magnitude greater than that of the surrounding land masses. In such models it is often possible to consider a simplified mathematical model of the Earth in which the surface layer containing the conductivity anomaly is replaced by an infinitely thin sheet of variable conductance (integrated conductivity) underlain by a uniform or horizontally layered conducting half-space. By restricting the lateral variations in conductivity in this way we are able to limit the numerical solution of the induction problem to the surface plane; analytic solutions can be found in the regions above and below the surface. The fact that a numerical grid is required only on the surface plane not only reduces by one the number of dimensions over which the relevant equations have to be solved numerically, but also eliminates the numerical difficulties which would otherwise arise as a result of the need for very small grid spacings in the vertical direction to model the conductivity anomalies adequately.

Across a thin sheet of variable conductance the tangential electric field is continuous, but the tangential magnetic field is discontinuous by an amount proportional to the surface current density in the sheet. These familiar boundary conditions were first applied to problems involving electromagnetic induction in thin sheets by Price [1] and have been used by several other authors since [2]-[5], in obtaining both analytical and numerical solutions.

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