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Three-Dimensionality of the BC87 Magnetotelluric Data Set Studied Using Mohr Circles

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The BC87 magnetotelluric data are examined as Mohr circles. Examples are discussed which show two-dimensionality and three-dimensionality in the recorded data; there are only occasional instances of one-dimensionality, which are not consistent over wide frequency bands. Strong anisotropy and skew are common features of the impedance tensors, and from the Mohr circles it is clear that at some sites, the computed magnetotelluric phase will go outside the $0 \rightarrow 90°$ range of the first quadrant.

1. Introduction

Mohr circles, commonly used in the analysis of mechanical stress and strain (see, for example, the recent paper by PASSCHIER, 1993), may be used to examine the three-dimensional nature of magnetotelluric (MT) data, and their distortion. This application was initially introduced in LILLEY (1976), and developed further in LILLEY (1993); the latter paper was in response to increased interest in the interpretation of complicated magnetotelluric data.

The essence of the application is to take the real and quadrature parts of an MT impedance tensor separately, and to plot the variation of Zxx against Zxy as the observing axes rotate through 180°. In this way series of circles are generated (analogous to the Mohr circles of mechanical stress) which display all information known about the impedance tensor. Various characteristics of the tensor are seen visually, increasing the understanding of its (possible) three-dimensional nature.

The BC87 data set is from a LITHOPROBE traverse in southeastern British Columbia, Canada. The traverse is placed between the Valhalla Gneiss Complex and the Rocky Mountain Trench; some introductory geological information is given in JONES *et al.* (1988). Recordings were made at 27 MT sites, over a distance of 150 km; generally, at each site, the data are MT responses at 40 different frequencies, in the range 384 Hz to 1820 s (period). Geographic positions for the sites are quoted as easting and northing coordinate pairs in metres, relative to a reference point at geographic co-ordinates (50°N, 120°W).

The BC87 data set is from a geologically complex area, and Mohr circle analysis is well-suited to display the three dimensionality of the MT responses. Such sets of circles have been drawn for all the BC87 data sites; examples from four sites are presented and discussed in this paper, demonstrating a range of three-dimensional characteristics.

2. Method

For an observed magnetotelluric impedance tensor with real parts

$$\begin{bmatrix} Zxx_r & Zxy_r \\ Zyx_r & Zyy_r \end{bmatrix}$$

which changes to

$$\begin{bmatrix} Z'xx_r & Z'xy_r \\ Z'yx_r & Z'yy_r \end{bmatrix}$$

upon rotation of the measuring axes θ' clockwise, a circle may be drawn on a plot of $Z'xx_r$ against $Z'xy_r$, as shown in LILLEY (1976, 1993). The circle is of radius R given by

$$R = \frac{1}{2}\left[(Zxx_r - Zyy_r)^2 + (Zxy_r + Zyx_r)^2\right]^{1/2}$$

and is centred on the $Z'xy_r$, $Z'xx_r$ co-ordinate axes at point

$$Z'xy_r = \frac{1}{2}(Zxy_r - Zyx_r),$$

$$Z'xx_r = \frac{1}{2}(Zxx_r + Zyy_r)$$

where the subscript r denotes the real part of the appropriate tensor element.

If a radial arm is drawn from the centre of the circle to the observed pair of values at the point (Zxy_r, Zxx_r), then rotation of that arm by angle $2\theta'$ anticlockwise takes its outer end to the point on the circle corresponding to $(Z'xy_r, Z'xx_r)$.

Figure 1 shows these quantities marked on example circles. Figure 1a is for induction in a two-dimensional (2D) structure, and Fig. 1b is for induction in a general three-dimensional (3D) structure. In the 3D case, the centre of the circle has moved up off the horizontal axis.

For one-dimensional (1D) structure, the circle in Fig. 1a reduces to its central point (and so there is no variation with axis rotation). Diagrams similar to those in Fig. 1 also may be drawn for the quadrature elements of an impedance tensor.

3. Errors

The BC87 data set includes standard errors for the impedance tensor elements. In computing the parameters for a particular circle, these errors may be carried through to give standard errors for the coordinates of the circle centre, and for the radius of the circle.

Taking a real circle first, and adopting notation $eZxx_r$ for the standard error in Zxx_r (and similarly for the other tensor elements), the standard error e_1 in the abscissa of the circle centre is given by

$$e_1 = \frac{1}{2}(eZxy_r + eZyx_r).$$

The standard error in the ordinate for the circle centre is similarly given by

$$e_2 = \frac{1}{2}(eZxx_r + eZyy_r)$$

and the error in circle radius is estimated as

$$e_3 = \frac{1}{2}e_4/R$$

where

$$e_4 = e_1(|Zxy_r + Zyx_r|) + e_2(|Zxx_r - Zyy_r|).$$

Similar expressions apply for the errors of a quadrature circle.

In Fig. 2 below, such errors are plotted as crosses at the circle centre, and as an extension of the radial arm of the circle beyond the actual plotted circumference. These error depictions are most obvious in Fig. 2b.

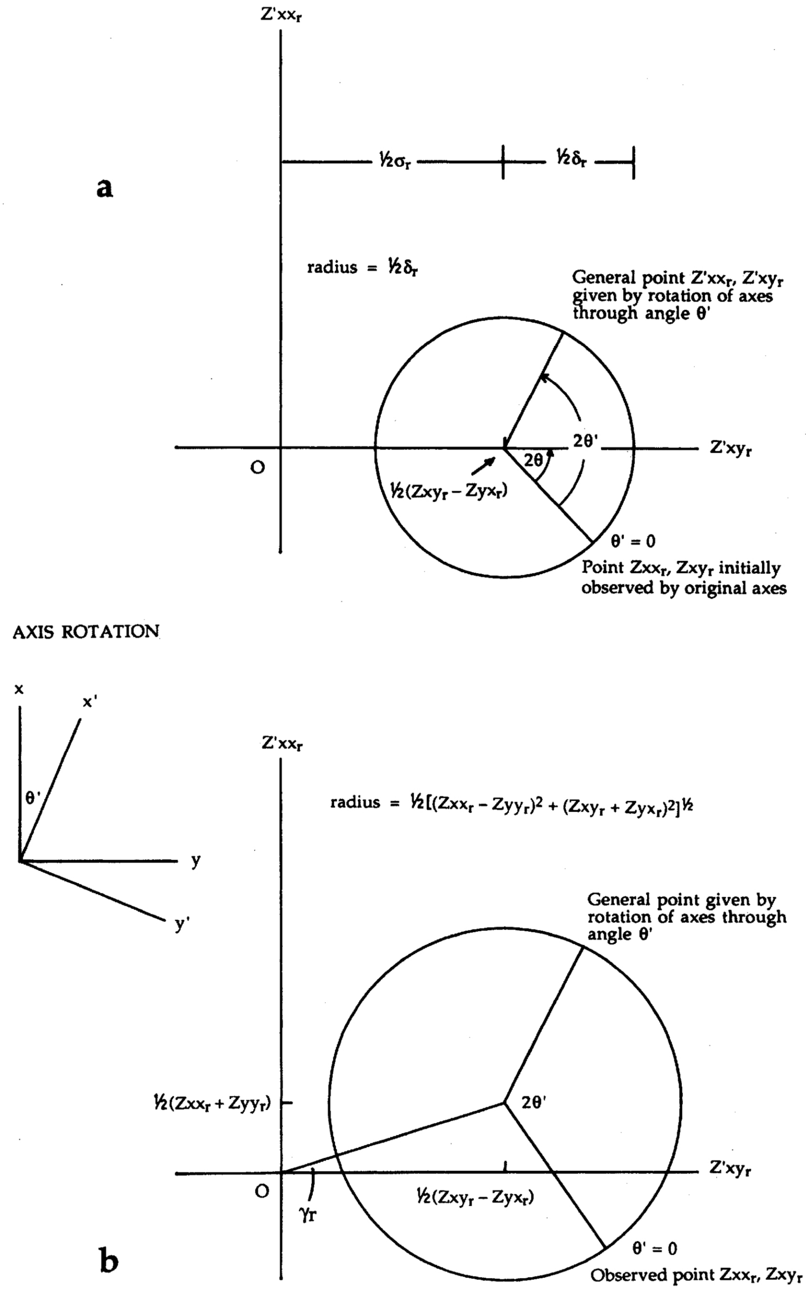

Fig. 1. Mohr circles for the values taken by the elements of a magnetotelluric impedance tensor, upon rotation of the measuring axes through angle θ'. a. Circle for a 2-D structure; b. Circle for a 3-D structure. For a 1-D structure, the circle in Fig. 1a reduces to its central point. Similar diagrams can be drawn for the quadrature elements.

4. The BC87 Magnetotelluric Data

Full sets of Mohr circles have been drawn for all the BC87 sites, with frequency variation shown by changing colour. Representative circles, for high, medium, and low frequencies, from just four of the sites are shown in this paper, in Fig. 2, as examples. The circles in Fig. 2 are a limited representation of the data. They will enable some basic points to be made, but for detailed interpretation full colour sets of the circles should be recreated on a computer screen.

In the simplest case of MT data recorded above a uniform half-space, the tensor elements are proportional to (period)$^{-1/2}$, and so decrease with increasing period. It is therefore helpful in the present exercise to first normalize the impedance tensor data by multiplying each element value by $T^{1/2}$, where T denotes period in seconds.

The frequency dependence of such normalized data can then more directly be related to electrical conductivity increasing or decreasing with depth in the Earth. Also, of immediate practical value, the different circles in a set will be closer to the same size and so easier to portray on the same diagram. (A similar situation, concerning the depiction of a set of Mohr circles on one figure, is discussed by PASSCHIER, 1993).

This procedure has been followed in the present case. Thus the normalised elements represented in Fig. 2 have the units of S.I. impedance multiplied by (seconds)$^{1/2}$.

The examples in Fig. 2, in the context of their full sets of circles, show the following characteristics.

Set 1. LIT001.DAT; site co-ordinates (469 591, 5 505 143)

At high frequencies the circle centres are on or near the horizontal axes, indicating that the data are closely 2D (see circle 1, both real and quadrature). Only at the lower frequencies, for which the circle centres shift towards the origin, does three-dimensionality set in; and then strongly so, at those frequencies for which the circles cut the vertical axes (see, for example, real circle 3). Points on the circles left of the vertical axes represent negative values of $Z'xy_r$ or $Z'xy_q$, and such points clearly will cause phase values outside the usual range.

Where the circles go near (even through) the origin, very high anisotropy of the impedance tensor is indicated. This condition appears to be the case for all data at the long period end of the frequency band, and is shown by circle 3 in both real and quadrature cases.

The interpretation from Fig. 2a is then that very locally the electrical conductivity structure is quite reasonably 2D; but more regionally, and more deeply, 3D effects are apparent, and dominate the data.

Set 2. LIT003.DAT; site coordinates (478 365, 5 505 101)

The full data set for this site shows scattered circles and indicates the use of circles in alerting the interpreter, immediately, to noisy and scattered data. The extent of the scatter suggests that some errors may be greater than their formal estimates, shown plotted. In both real and quadrature sets of circles, the circle centres shift left with increasing period.

In the present example, the most scattered data are in the real part of the tensor, at the high frequency end of the frequency range (see, for example, real circle 1). At the low frequency end, the circles are 3D and cut the vertical axes; in this pattern they show consistency with the low frequency data of Fig. 2a (site LIT001). This consistency can be seen in the similarity of circle 3 between Fig. 2a and Fig. 2b, both real and quadrature.

Set 3. LIT015.DAT; site co-ordinates (524 091, 5 493 067)

In this data set, the progression from high to low frequencies corresponds to the circle centres moving from right to left in each part of Fig. 2c.

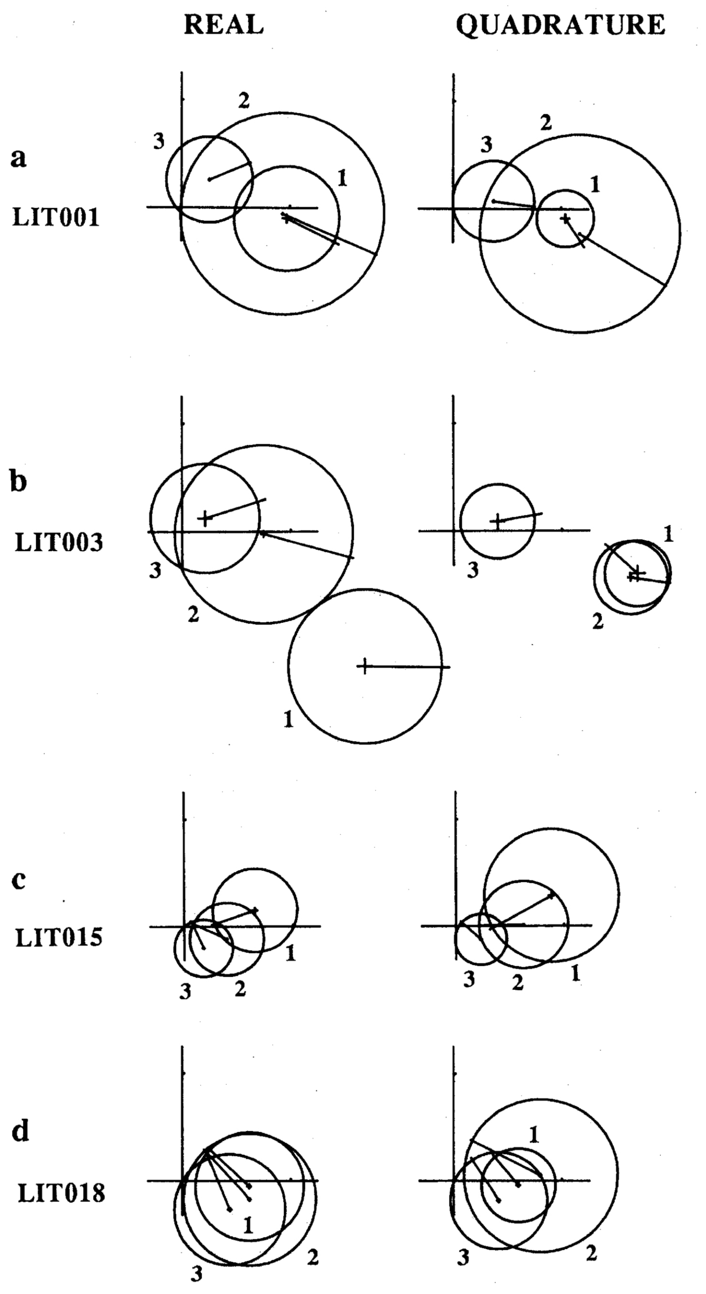

Fig. 2. Examples of Mohr circles for four sites of the BC87 magnetotelluric data set. A description of the circle characteristics is given in the text. The markings 1, 2 and 3 denote periods of 0.0104 s, 1.778 s and 341.3 s respectively. The scale of the figure is given by each pair of real and quadrature axes being 300 units apart, where the units are those of normalized impedance described in the text. a. Site LIT001; b. Site LIT003; c. Site LIT015; d. Site LIT018.

At high frequencies, the data set exhibits three-dimensionality, with circle centres above the horizontal axes (see circle 1, both real and quadrature). As frequency decreases the data move through a 2D form (a good example is quadrature circle 2), with circle centres on the horizontal axes, before returning to 3D again, now with circle centres below the horizontal axes (see circle 3, both real and quadrature).

Also, at the lowest frequencies, the circles cut the vertical axes where they will cause anomalous phase. Strong anisotropy is again indicated at long periods, with circles going near (or through) the axes origins (see circle 3, both real and quadrature).

Set 4. LIT018.DAT; site co-ordinates (535 189, 5 499 612)

These data are quite closely 2D for much of their frequency range (see circle 1, both real and quadrature). Only at low frequencies do they develop 3D characteristics, where the circle centres move progressively below the horizontal axes. This latter characteristic is shown by circle 3 in both real and quadrature cases.

The similarity of the real and quadrature sets of circles means that generally the magnetotelluric phase will be comfortably in the first quadrant, with real and quadrature parts of comparable magnitude. Note the difficulty which would be caused in phase calculations, however, where the lower frequency circles (marked 3) cut the vertical axes, and go to the left hand side indicating negative $Z'xy_r$ and $Z'xy_q$ values.

The circles shown for this site are also good examples to examine for evidence of geologic strike direction, which is indicated by the orientations of the plotted radial arms. These radial arms are typically at an angle of 45° to the horizontal axis, between "10 o'clock" and "11 o'clock". Rotating such radial arms either 45° anticlockwise (to 9 o'clock) or 135° clockwise (to 3 o'clock) will thus give near zero $Z'xx$ values, and minimum or maximum $Z'xy$ values. These rotations are thus those required to align the measuring axes parallel and perpendicular to geologic strike.

From Fig. 1, a rotation of the radial arm 45° anticlockwise corresponds to rotation of the measuring axes $22\frac{1}{2}^\circ$ clockwise; and a rotation of the radial arm 135° clockwise corresponds to rotation of the measuring axes $67\frac{1}{2}^\circ$ anticlockwise. Thus geologic strike is either $22\frac{1}{2}^\circ$ east or $67\frac{1}{2}^\circ$ west of the north axis orientation for the original data.

The lower frequency circles, where they go through or close to the origins of the axes, again show very strong anisotropy in the data.

5. Conclusions

The interpretation of 3D magnetotelluric information is a challenging task, requiring as much insight as possible into the nature of the data. This paper has used Mohr circles to examine the BC87 magnetotelluric data. The four BC87 sites discussed as examples all show, at the long period end of the data range, strong anisotropy and strong three-dimensionality in the basic MT data.

The complete sets of diagrams for the full range of BC87 sites show in many places consistent behaviour patterns between sites which are near each other. Such consistency implies regional two-dimensional and three-dimensional structural effects, knowledge of which may be important in modelling the data. An example, in Fig. 2, is the consistent low-frequency behaviour between sites LIT001 and LIT003, shown in the similarity of the circle patterns for the two sites at low frequencies (for which the circles cut the vertical axes). JONES and GROOM (1993) recently analysed a comparison between two other similarly adjacent sites, LIT007 and LIT008.

The examples in Fig. 2 also emphasize that in the interpretation of such data using 2D models an interpreter is, in effect, transforming each circle to one which has its centre on the horizontal axis: because the forward response of a 2D section will have this characteristic.

Similarly, in carrying out 1D modelling of data such as displayed in Fig. 2, the patterns of circles are being taken to be represented by series of points on the horizontal axes.

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