J. Geomag. Geoelectr., **45**, 1039–1043, 1993

An Automatic Scheme for 2-D Magnetotelluric Modelling, Based on Low-Order Polynomial Fitting

P.-A. SCHNEGG

Geological Institute, Observatoire Cantonal, CH-2000 Neuchâtel, Switzerland

(Received January 13, 1993; Revised August 26, 1993; Accepted September 3, 1993)

Magnetotelluric 2-D modelling can be a daunting task where much of the work consists of stepwise model modifications and mesh design. The scheme proposed here is an entirely automatic strategy in place of the usual operator interaction. Model parameters (resistivities and thicknesses in log space) are regarded as polynomials of the distance along the profile, and the proposed scheme automatically varies the polynomial coefficients to minimize the misfit.

1. Statement of the Problem

In the 1970s several authors (BREWITT-TAYLOR and WEAVER, 1976; RIJO, 1977) proposed finite elements and finite differences methods to solve the Helmholtz equation governing electromagnetic field distribution in conducting structures of 2-D geometry. Later, WANNAMAKER *et al.* (1985), made available a computer program which makes it possible to calculate the responses for both TE and TM modes much faster, and with good numerical accuracy, even with machines handling 32-bit words.

To arrive at the final model however, it is necessary to modify the model manually, until a minimum misfit is reached. This forward modelling method is therefore quite demanding in human resources since the finite-element mesh must be re-designed for every new model.

The basic idea underlying the present modelling scheme is to let the computer build the model and design the mesh. To overcome the difficulty associated with the tremendous number of independent parameters found in real 2-D structures, we constrain both resistivities and layer thicknesses to a representation by low-order (<5) polynomials, thus keeping the number of parameters reasonably small. The program starts with an initial set of polynomial coefficients deduced from an initial set of 1-D models of apparent resistivities and phases in the TE mode. The program then varies the coefficients to minimize the misfit between the data and the model response. A standard minimization routine called MINDEF (BEINER, 1970) automatically handles the set of parameters in the search for the best model. In the present application, the parameters are the set of polynomial coefficients.

MINDEF seeks a vector

$$\{x_1^0, x_2^0, \ldots, x_N^0\}$$

which minimizes the function

$$\epsilon(x_1, x_2, \ldots, x_N)$$

where the x_i are the N parameters.

The search for a minimum of ϵ is accomplished in 3 phases, successively repeated until a minimum is reached:

Phase 1: A descent direction is searched by varying the parameters around the starting point.

Phase 2: Progression into the descent direction.

Phase 3: If the progression fails, the parameters are varied toward the space perpendicular to the progression, trying to get a breakthrough.

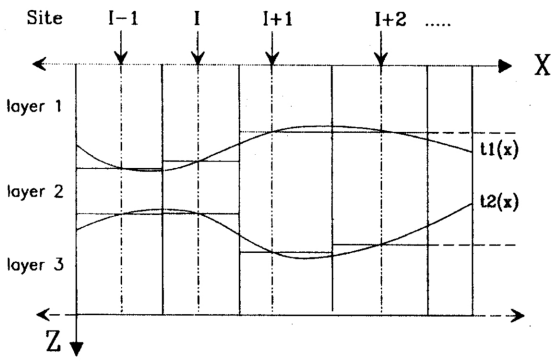

Fig. 1. 2-D model showing a juxtaposition of 1-D models with layer thicknesses described by polynomials t1 and t2. Index I refers to station sites.

2. The Scheme

The general structure of the model used in our scheme does not differ from those designed manually, juxtaposition of 1-D models with limited lateral extension. Figure 1 shows how a 2-D structure is divided into columns, with as many columns as there are measuring stations. The vertical boundaries between columns are located at mid-distance between neighbouring sites and remain fixed during the process. This has the advantage of keeping any irrelevant information to a minimum. Note that the first and last columns extend laterally toward infinity, thus providing the required 1-D quarter-spaces.

To prepare the initial model the layering of each column is derived from 1-D models of the TE data. An identical number of layers are required for every column since the scheme implies that layer i of each column has a resistivity and a thickness described by polynomials of the abscissa x across the profile. Note, however, that neither the resistivity nor the thickness are allowed to vary within a given layer of a given column: they are determined by the polynomial values at the station sites in each column, as shown with arrows in Fig. 1 along the profile axis x. A set of starting polynomials ($2N-1$ for a 1-D model with N layers) can be computed, using the values of as many sites as required for a given polynomial order. In fact the polynomials are computed with the logarithms of the layer resistivities and thicknesses:

Resistivity of layer k:

$$r_k = \exp_{10}\left(\sum_{i=0}^{R_k} c_{ik} x^i\right).$$

Thickness of layer k:

$$t_k = \exp_{10}\left(\sum_{i=0}^{T_k} d_{ik} x^i\right)$$

where R_k and T_k are the orders of the polynomials for the resistivity and the thickness of layer k, and c_{ik}, d_{ik} are the polynomial coefficients to be optimized by MINDEF.

Choosing the exponential function for both the resistivity and thickness insures that the minimizing routine MINDEF cannot feed non-physical negative values of these parameters to the forward modelling routine of WANNAMAKER *et al.* (1985), which could occur since MINDEF varies parameters by adding to, or subtracting from, them small fixed quantities (steps). Furthermore the exponential function is appropriate in that it can change its value very fast, even for small

increments of the independent variable x. Moreover the logarithmic scale is known to be the natural scale for magnetotelluric parameters, for both resistivities and layer thicknesses.

A favourable property of polynomials is their suitability for representing a large subset of different, continuously variable shapes. With such polynomials it is possible to describe many geological features, like trenches, dykes, faults, etc.

In terms of economy of computer time in particular, the choice of polynomials turns out to be a useful one, since the total number of parameters is independent of the number of measuring stations, and amounts to

$$2N - 1 + R_N + \sum_{k=1}^{N-1} (R_k + T_k)$$

with N the number of layers.

In some models, where the true earth is partly known in advance, it is advisable to allow for laterally discontinuous resistivities. Although the scheme is not able to locate such discontinuities by itself, provision is made to force any block of any layer to keep its original resistivity and/or thickness during modelling.

The step size for the various polynomial coefficients is given different values according to the degree of the exponent of the position x; otherwise some steps would lead to excessive variations of the high-order parts of the polynomials. The step is therefore weighted by the inverse of the length raised to the corresponding power.

Choosing the order of the polynomials is somewhat subjective, although a clue can be obtained from the initial resistivity distribution of the 1-D model. Generally, it proves advisable to start modelling with second-order polynomials for the layer thicknesses, and keep the layer resistivities constant. Further attempts with higher-order polynomials, using the previous result as starting model, invariably succeed in reducing the misfit, at the price of a longer processing time. Of course, any parameter step can be set to zero if it is found desirable to keep a particular model feature constant.

In order to make the scheme fully autonomous, an automatic mesh generator is used at every iteration of the minimizing routine, since the shape and resistivity of the blocks evolve gradually. The mesh size is much smaller than the blocks and depends on the period and the smallest resistivity in the corresponding horizontal layer or vertical column.

3. Inversion of Real Data

The real MT data set used for testing our scheme is the "reduced" COPROD2R data set, which has been made available to interested groups by the Geological Survey of Canada, and described by JONES (1993). This data set consists of apparent resistivity and phase in both TE and TM modes at 20 sites distributed along a profile across the Phanerozoic Williston Basin in South Saskatchewan, Canada. The data have been corrected for undesirable static shift effects. Due to hardware limits (memory space of 4 Mbytes on a Mercury MC3200AT accelerator board hosted by a PC), only the 10 central sites showing the largest MT anisotropy have been used in our test, namely sites PCS001 to PC5011, located over the NACP conducting anomaly.

1-D modelling was performed to devise the initial model. As the data showed perfect one-dimensionality over the top 5 kilometers and did not display significant lateral variations at shorter periods, the polynomials corresponding to the resistivity and thickness of the top layer were fixed to their 1-D model values. Accordingly, only periods longer than 30 seconds were considered in the modelling.

Simultaneous TE and TM mode apparent resistivity and phase were used at 3 periods: 100, 300 and 1000 seconds. The 1-D model suggested that 4 layers, including the low-resistivity top layer, would be required, leading to a total of 7 polynomials. The one which represented the

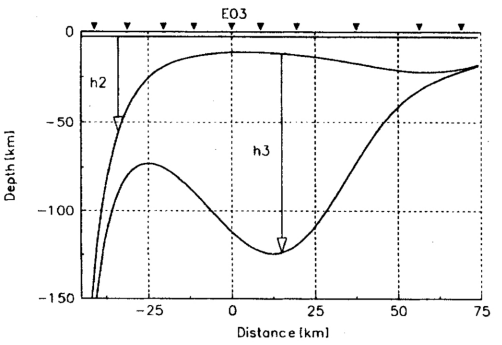

Fig. 2. Final 4-layer 2-D model with station sites (arrows). Thicknesses h2 and h3 are computed with the final polynomials. Resistivities of layers 1, 3 and 4 are 2.8, 3.2 and 260 Ωm (Zero-order polynomials). Resistivity of layer 2 decreases eastward between 200 and 10 kΩm (1st order polynomial).

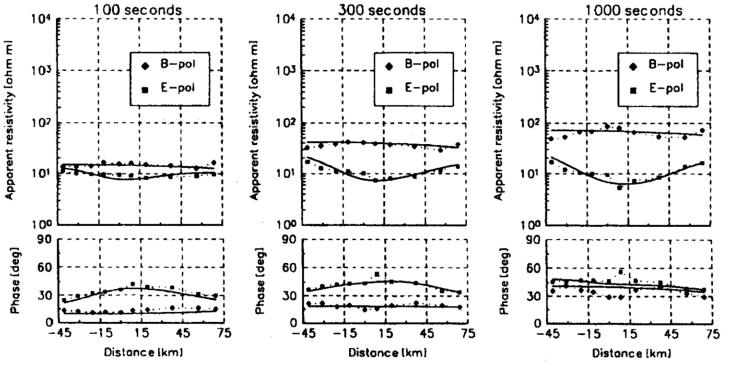

Fig. 3. Interpolated measured (dots) and modelled (line) apparent resistivity and phase as functions of the distance on the profile for 3 periods.

resistivity of layer 2 was of first order. Those associated with the thicknesses of layer 2 and 3 were of order 3, whereas all others were of order zero. Further attempts to increase these orders did not lead to significantly smaller misfit. The number of free coefficients then totalled 14.

The forward modelling routine was called about 300 times. A minimum was not found after that time, probably because of the rather noisy phase data at 1000 seconds. The final misfit value was 0.085.

The resulting model is shown in Fig. 2. The resistivities of layers 1, 3 and 4 are 2.8, 3.2 and 260 Ωm. The resistivity of layer 2 decreases eastward between 200,000 Ωm and 10,000 Ωm. The highly conductive anomaly seems to extend to the base of the crust and even deeper. An important observation, which however only applies to this particular data set, was made during the few modelling trials: if long-period data (>300 s) are discarded from the data set the program invariably leads to a much thinner, more conductive anomaly (0.5 Ωm). Furthermore, this conductor apparently cannot be in a single block, but should be split into several parts, requiring insulating material between the parts. Without provision for these insulating parts, it seems we cannot obtain the correct TM response.

Figure 3 shows a comparison of apparent resistivity and phase values of real and model data

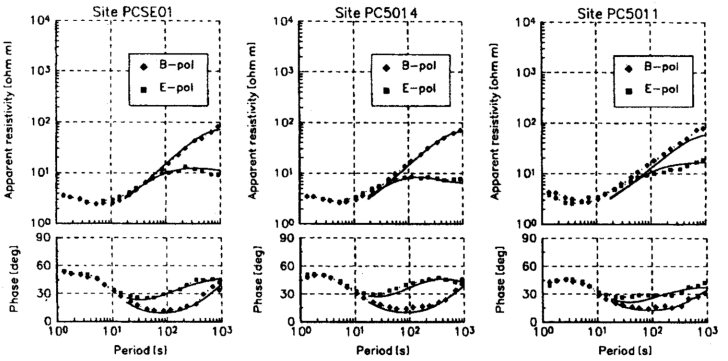

Fig. 4. Interpolated measured (dots) and modelled (line) apparent resistivity and phase as functions of period for 3 stations.

at 3 periods used in the modelling, whereas Fig. 4 gives the same information at 3 sites as a function of the period.

The final model does not fit the phase data very well at a period of 1000 seconds. This may be a consequence of excessive model smoothing by too low-order polynomials, revealing perhaps a weakness of the method which tends to miss faint structural features.

Although the modelling carried out only made use of 3 periods in the 100 to 1000 seconds decade, Fig. 4 spans 20–1000 seconds. The rather small misfit observed at 20 seconds is a consequence of the initial 1D modelling, and its final 1D model is still valid at that period.

4. Conclusion

The modelling scheme presented here does not claim to be either universal or foolproof, but certainly can relieve the operator of repetitive tasks. The final model can undergo further refinement by assigning fixed values to some of its elements.

The author would like to express his gratitude to the Geological Survey of Canada, Ottawa, especially to P. J. Savage of PanCanadian Petroleum Ltd., Calgary, Alberta, for having kindly made available the COPROD2 data set, as well as to Dr. Alan Jones of the Geological Survey for supplying these data after static shift removal. The author is also grateful to the Swiss National Science Foundation for supporting this research project.

REFERENCES

BEINER, J., FORTRAN routine MINDEF for function minimization, Institut de Physique, Université de Neuchâtel, Switzerland, 1970.

BREWITT-TAYLOR, C. R. and J. T. WEAVER, On the finite difference solution of two-dimensional induction problems, *Geophys. J. Roy. Astr. Soc.*, **47**, 375–396, 1976.

JONES, A. G., The COPROD2 dataset: Tectonic setting, recorded MT data and comparison of models, *J. Geomag. Geoelectr.*, this issue, 933–955, 1993.

RIJO, L., Modelling of electric and electromagnetic data, Ph.D. Thesis, University of Utah, 1977.

WANNAMAKER, P. E., J. A. STODT, and L. RIJO, PW2D—finite element program for solution of magnetotelluric responses of two-dimensional earth resistivity structure, Earth Science Laboratory, University of Utah, Salt Lake City, 1985.