MODELLING AND ANALYSIS OF ELECTROMAGNETIC FIELDS IN 3D INHOMOGENEOUS MEDIA

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Abstract. The main progress in the 3D modelling of electromagnetic fields, achieved during the last few years, is summarized. Various 3D numerical modelling techniques are described and compared as to their possibilities and efficiency. Numerical studies are complemented by laboratory scale modelling.

Conventional methods of the analysis of the magnetotelluric and magnetovariational surface characteristics are confronted with new trends in MT tensor analysis and decomposition. Various approaches to the analysis of simplified 3D structures are also presented.

1. Introduction

The interpretation of practical geoelectrical measurements requires a thorough study of various kinds of conductivity inhomogeneities within the earth which allows us to approximate real geoelectrical conditions in the earth. For the understanding of the effects of geoelectrical inhomogeneities, 3D modelling of electromagnetic fields is of crucial importance. 3D modelling techniques and effects have recently been summarized in several excellent review papers – from the methodical point of view by Hohmann (1983), Varentsov (1983), and Kaikkonen (1986) and from the point of view of the 3D effects by Jones (1983), and Menvielle (1988). Because of the large extent of the subject of 3D modelling and analysis of electromagnetic fields and in view of the previous review papers, in this review we confine ourselves particularly to the latest results.

2. Differential Equation Method

The most common approach to the modelling of 2D complex geoelectrical structures is based on the direct solution of field differential equations by numerical methods – either by the finite difference (FD) or the finite elements (FE) method. The success of these methods results from their numerous advantages, of which the following aspects are worth mentioning: a simple formulation of the problem based immediately on Maxwell's equations, the possibility of writing universal computer programs suitable for dealing with a broad variety of different, often very complex geoelectrical structures, and the possibility of assuming real values of internal geoelectrical parameters, including high conductivity contrasts within the structure. These general advantages are accompanied by convenient numerical properties of the approximate formulation of the problem – the resultant matrix is sparse and banded, and in the case of classic boundary conditions (field at infinity) it is

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symmetrical. On the other hand, all these numerical approaches require approximating the problem in a region extending far beyond the limits of the area of interest. This fact adversely affects the computer memory and time requirements. To solve the resultant system of linear algebraic equations, both iteration methods and, particularly lately, different versions of the Gaussian elimination method are used.

In principle, the development of the FE and FD 2D modelling techniques was accomplished in the late 70's. The recent papers have been mainly directed toward further improvements of the modelling technique (Wannamaker *et al.*, 1986; Wannamaker *et al.*, 1987) and also towards the comparison of the FD and FE methods with other numerical and analytical approaches. In this respect, Weaver's 2D reference model (Weaver *et al.*, 1985; Weaver *et al.*, 1986) is valuable. For this model the solution was found analytically for the E- and H-polarization of the electromagnetic field and comparisons were made with FD and FE results. Improved formulae for the derived field characteristics are also presented.

Recently the moving finite element (MFE) method has been used for 2D MT analyses. (Travis and Chave, 1986). MFE method is an adaptive method which includes the location of the mesh nodes in the minimization of the global measure of solution error, and both the base function coefficients a_i and the node locations s_i are solved simultaneously. This leads to the automatic concentration of the mesh points in regions of large gradients in the model parameters. Recent work has revealed a way to modify MFE so that the computation of s_i and a_i is decoupled, resulting in a pair of smaller matrices for solution.

Zhdanov *et al.* (1986) analysed a 3D-axisymmetric medium by transforming the problem into a 2D domain and by solving the transformed problem by the FE method.

2D problems were solved directly in the time domain by the FD method (Oristaglio and Hohmann, 1984) and by the FE method (Goldman *et al.*, 1986).

In the estimates of various numerical methods and computer programs the international COMMEMI project has played an important role. Within this project several 2D and 3D test models have been proposed.

In 3D applications, the FD and FE methods have not been so far able to compete in their efficiency with the integral equation (IE) method, hybrid methods and with some of the thin sheet approaches. Therefore these methods have not been widely used in practice although corresponding computer programs are being developed in several laboratories. In 3D problems, the FD and/or FE solutions lead to serious difficulties – the number of variables increases enormously, the FD/FE matrix is non-symmetrical and even the application of asymptotic boundary conditions does not sufficiently reduce the number of variables. For special types of 3D structures (localized structures in a layered medium) this problem is in fact quite radically solved by hybrid methods. Due to a large number of variables and a large band-width of the FD/FE matric, the Gaussian elimination may be only used in quite exceptional cases although a new generation of extraordinarily efficient computers is available. Iterative methods frequently meet serious convergence difficulties. In addition the results are degraded by the numerical differentiation, which is required to obtain all field components. In the future, approaches based on the multi-grid method might be a suitable tool for the solution of these great modelling problems.

Recent results of a 3D FD modelling algorithm are presented by Spichak (1983), and Zhdanov and Spichak (1980), where a 7-point FD scheme for the electric field components has been developed and used to model the MT field for a conductive rectangular parallelepiped in a resistive host and for the 3D COMMEMI models (Figure 1).

Weaver and Pu (1988) presented first results from a 3D FD modelling scheme which allows the model to approach either E- or H-polarization two-dimensional configurations at infinity and the solution in the electric (magnetic) field is obtained

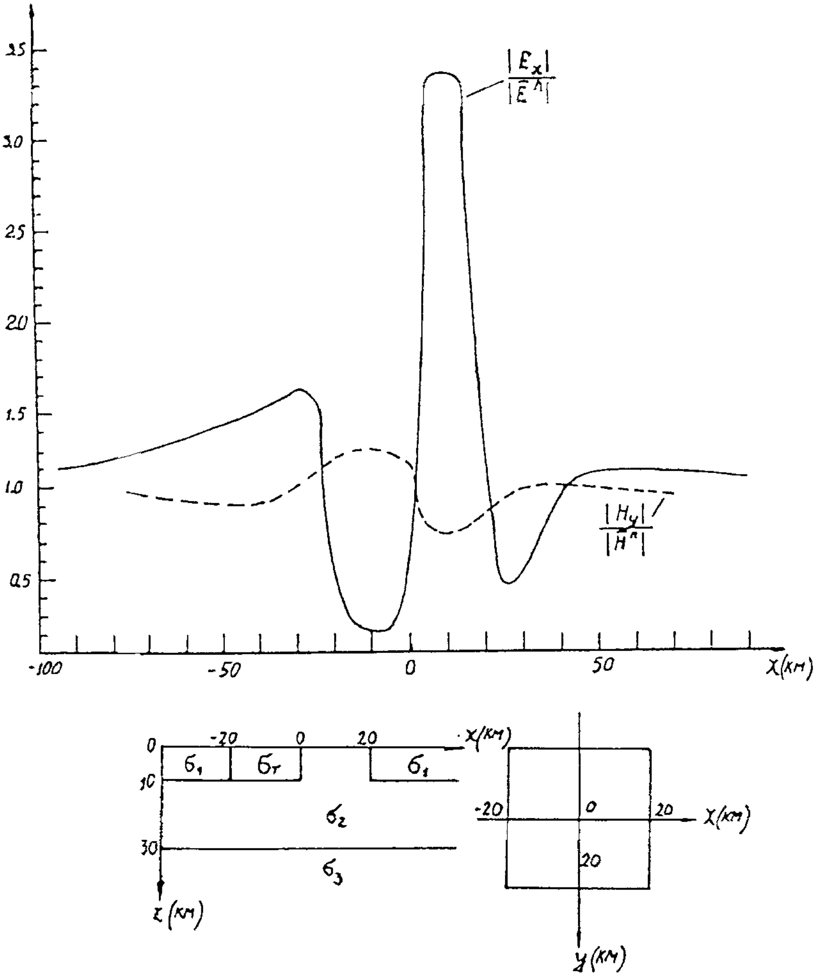

Fig. 1. 3D-2 COMMEMI model (left – front view, right – plan view), conductivities $\sigma_1 = 0.1\ \mathrm{Sm}^{-1}$, $\sigma_2 = 0.01\ \mathrm{Sm}^{-1}$, $\sigma_3 = 10\ \mathrm{Sm}^{-1}$, $\sigma_T = 1\ \mathrm{Sm}^{-1}$. Taken from Zhdanov and Spichak (1988).

by representing the conductivity (resistivity) at each node by the weighted mean of the values in adjacent cells. Integral boundary conditions are used at the surface and base of the model.

Quite recently a spectral difference method of solving the 3D electromagnetic problem with a localized source in the time domain has been published (Druskin and Knizhnerman, 1988). The 3D electromagnetic problem is at first approximated by FD in space and, after several transformations, the resulting system of the ordinary differential equations in the time domain is solved by Galerkin method. The Lanczos process is applied to express the solution in the form of a series of orthogonal functions. This procedure is demonstrated by Druskin and Knizhnerman (1987). Numerical errors of the algorithm are estimated and comparison with the results of SanFilipo and Hohmann (1985) is presented.

Further numerical methods, based on the direct solution of the field differential equations, have been developed to deal with special types of geoelectrical structures:

(a) To model structures with smoothly varying interfaces, the slopes of which do not exceed about 20 deg, the Rayleigh-FFT method was successfully used (Danian and Peeples, 1986; Boersma and Jiracek, 1987). This technique derives expressions for the electric and magnetic field components directly from Maxwell's equations via the use of Fourier transforms. Spectral amplitudes of the field components are obtained by solving the system of equations that result from the boundary conditions. The results are affected by the number of scattering orders included, which, on the other hand, limits the maximum slope of the interface, the electrical parameters of the medium and the frequency of the incident waves. The Rayleigh-FFT technique does not require numerical differentiation.

(b) Barashkov (1988a, b) solved Maxwell's equations for a three-layer medium in which the conductivity of the first surface layer is variable. For gently horizontal conductivity variations he used quasi-1D equations to obtain an approximate solution of the electromagnetic problem for low and medium frequencies.

3. Integral Equation Method

The most frequently used approach to the 3D modelling of electromagnetic fields is the method of integral equations (IE). The principal ideas of this approach are based on the fundamental works of Raiche (1974), Hohmann (1975), Weidelt (1975). The advantage of the IE methods consists in computing the electromagnetic field by the integration over the anomalous region only, not over the entire space. Owing to this fact, the matrix of the linear system of equations, resulting from the numerical approximation of the integral equations, is usually not so large as in FE or FD methods, but, compared with those numerical techniques, it is full.

(a) FREQUENCY DOMAIN IE MODELLING

Major progress has been recently achieved by Wannamaker *et al.* (1984a, b). They presented an IE algorithm for a layered model involving a 3D localized inhomo-

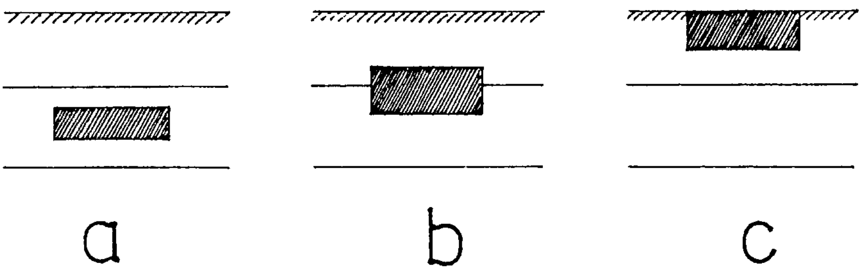

Fig. 2. Schematic models of a 3D localized inhomogeneity embedded in a layered medium dealt with by the IE modelling approach. (a) inhomogeneity within one of the layers, (b) inhomogeneity intersecting a layer interface, (c) an outcropping inhomogeneity.

geneity embedded within one of the layers (Figure 2a). A similar technique was also developed in the Computer Centre of the State University, Moscow. The theoretical background of this approach is summarized by Dmitriev and Zakharov (1987). The primary field may be modelled as a plane wave or excited by an artificial source of a more general character. At the Moscow University various models of axisymmetrical inhomogeneities within the layered earth were also solved (Eremin *et al.*, 1985; Dmitriev and Nesmyanova, 1985).

Wannamaker (1986) presented an algorithm which implied the ability to simulate 3D structures intersecting layer interfaces of the layered earth host (Figure 2b) and the ability to model structures which outcrop (Figure 2c).

Xiong *et al.* (1986) further generalized the IE method to model a localized 3D inhomogeneity in a two-layer anisotropic (horizontally isotropic) earth. The integral equation obtained relates the exciting electric field and the scattering currents in the inhomogeneity through the electric tensor Green's function deduced from the vector potentials in the lower layer of the earth. The effect of anisotropy on apparent resistivity is expressed by a reduction of the size of the anomalies, by a shift of the anomaly position downwards, and by the enhancement of the effect of the overburden.

Tabbagh (1985) studied 3D localized conductivity inhomogeneities in a two-layer earth by the moment method. For the particular purpose of shallow sounding he considered the spatial variability of both the electrical conductivity and magnetic susceptibility within the model. He analysed his electromagnetic modelling results from the point of view of the possibilities of electromagnetic sounding and electric and magnetic prospection methods.

The results of one of the 3D models proposed as a test model within the COMMEMI project (conductive prism embedded in a resistive halfspace) are presented by Hvoždara *et al.* (1987).

West and Ward (1988) used the IE technique (Wannamaker *et al.*, 1984a) to analyse a surface-to-borehole CSAMT response of 3D inhomogeneities.

Doherty (1988) presented a theory of solving 3D electromagnetic transmission boundary value problem by the surface IE method.

(b) TIME DOMAIN IE MODELLING

The most evident progress in 3D IE modelling has been made in studying the transient electromagnetic (TEM) responses of 3D bodies. In Hohmann (1983) and Newman *et al.* (1986) the TEM responses are computed by transforming the frequency responses, obtained by using earlier developed IE modelling algorithm of frequency domain, into the time domain. A direct time-domain numerical solution of 3D Maxwell's equations is presented by SanFilipo and Hohmann (1985) for a symmetrical target in a uniform resistive host with no overburden. In this case analytical expressions for the time-domain Green's tensor can be found. In Newman *et al.* (1986) and SanFilipo *et al.* (1985) a detailed comparison of 2D and 3D TEM interpretations is made and also the effect of non-zero conductivity of the host medium is analysed. Gunderson *et al.* (1986) studied the 3D TEM soundings excited by the abrupt termination of the currents in a grounded wire. Newman *et al.* (1987) analysed TEM interpretation possibilities for the central-loop configuration. The effectivity of different kinds of the electromagnetic responses is studied by Eaton and Hohmann (1987). Newman and Hohmann (1988) solved the electromagnetic problem for multiple prisms in a layered halfspace (Figure 3). Approximating the scattering currents within each of the prisms by a system of pulse and divergence-free base functions they obtained stable solution for a host of any resistivity. Their results may be used for both frequency and time domain modelling.

(c) APPROXIMATE IE APPROACHES

In addition to the 3D modelling techniques, which intent to determine the electromagnetic responses of the medium as accurately as possible, there are also several approximate approaches allowing us to study in a simplified manner some characteristic 3D electromagnetic effects. These approaches are usually simpler from the computational point of view and they often make it possible to obtain a deeper insight into the physical processes which control the distribution of the electromagnetic field. West and Edwards (1985) derive the spectral response of a 3D small conductive target in a conductive environment. The representation of the response

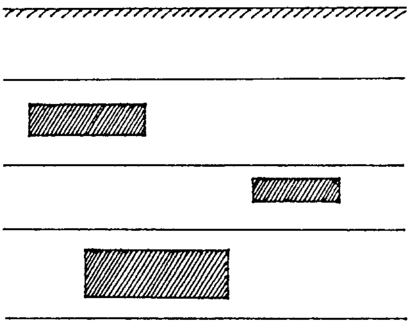

Fig. 3. Model of multiple prisms in a layered halfspace studied in Newman and Hohmann (1988) by the IE modelling approach

contains the direct response from the layered host medium and the first-order effects of eddy current induction, current channelling, magnetic induction, and the coupling between eddy current and magnetic induction in the anomalous body as modified by the host medium.

From the theory of the generalized potential of a double-layer, Hvoždara (1990a, b) derived integral expressions for the stationary approximation of the magnetotelluric field in a model consisting of a vertical contact with a local 3D prismatic body. The joint effect of the vertical contact and of the local inhomogeneity results, e.g., in a significant shift of the inversion line of the induction vectors off the axis of symmetry of the prism.

4. Hybrid Methods

In modelling 3D structures by the IE method, we are confronted with difficulties in models with large conductivity contrasts as well as in models with targets near the surface. The principal disadvantage of the FD and FE methods is that they require defining a mesh large enough to model the entire half-space effect, leading generally to an extremely large number of variables. A hybrid scheme, which combines the positive qualities of the IE and FE techniques, was introduced by Scheen (1978).

The basis of the hydrid method consists in limiting the boundary of the FE mesh. A mesh is defined which incorporates the inhomogeneity plus the surrounding layer of cells. Outside the inhomogeneity the electromagnetic field is determined by using the IE method. In the hybrid method, the field inside the inhomogeneities determined by using the FE method. This makes it possible to subdivide the inhomogeneity into arbitrary homogeneous subregions. The boundary values are obtained by requiring equality between the FE and IE solutions on the mesh boundary. There are two principal approaches within the hybrid schemes. The iteration hybrid scheme uses the iterative technique to enforce consistency between the IE and FE solutions on the boundary of the region. The direct hybrid scheme solves the coupled IE/FE system directly. The size of the resulting matrix is slightly larger than the one associated with the IE technique, but the scheme does not need Green's functions between elements within the finite element mesh to be evaulated, thus avoiding the problem of singular cell integration.

In Lee *et al.* (1981) the problem was solved by the iteration hybrid method for the electric field; in Best *et al.* (1985) a similar hybrid approach was used for evaluating the magnetic field. In the latter paper attention is paid to low frequency FE results, where convergence problems arise if the total magnetic field is solved. Therefore, the hybrid algorithm is modified to solve for the amount by which the total magnetic field in the conductive body differs from the static field. This approach gives much better convergence. The method can cope with infinitely large conductivity contrasts, at least if the body is conductive and the surrounding medium highly resistive.

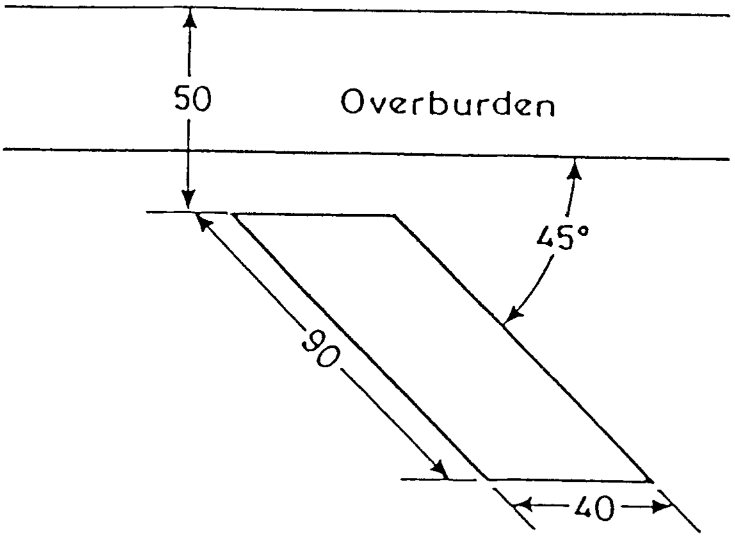

Fig. 4. Model of a dipping prism in a two-layered earth analyzed in Gupta *et al.* (1988) by the 3D time-domain electromagnetic modeling using a compact finite element frequency-stepping method.

Further progress in developing the hybrid algorithms was achieved by Gupta *et al.* (1987). With respect to the most time-consuming part of the hybrid computations, which consists in evaluating a large number (~ 10^5) of convolution integrals of Green's function, the original 2D interpolation scheme (Lee *et al.*, 1981) was replaced by a 1D interpolation and reading of the convolution integrals from a reference table. Computation time was reduced by up to 70% and accuracy was improved. To avoid convergence problems of the iterative approach, a direct hybrid scheme was proposed, which again reduced the computation time. Computation time was cut by up to 90% and the process was made more stable numerically. A comparison of the hybrid results with the IE solution (Hohmann, 1983) shows excellent agreement.

In Gupta *et al.* (1988) the direct hybrid scheme was used for time-domain 3D electromagnetic modelling. The vector diffusion equation is Fourier-transformed into the vector Helmholtz equation. Frequency-domain solutions for 20–30 logarithmically spaced frequencies are obtained and, by transforming these back in the time domain, the time-domain response can be constructed rapidly. The particular model solved consists of a heterogeneous target embedded in the bottom layer of a two-layer halfspace (Figure 4).

5. Thin Sheet Approximation

An important direction in the modelling of electromagnetic fields in non-homogeneous structures is represented by thin sheet approximation approaches. The thin sheet approaches have found their principal field of application in modelling and analysing the influence of complex near-surface inhomogeneities (including the effect of oceans) upon low-frequency electromagnetic fields. For long enough periods of the electromagnetic field, the 3D conductivity distribution $\sigma(x, y, z)$ reduces to a 2D plane distribution of conductance $S(x, y)$. Consequently, a general 3D problem is reduced to a 2D plane problem in long period asymptotics (quasi-3D modelling).

For the solution of the thin sheet problem various approaches were adopted. The most fundamental development is connected with the following works: Vasseur and Weidelt (1977) use the integral equation approach for modelling a bounded anomalous region embedded in a uniform thin sheet at the surface of a layered halfspace; Dawson and Weaver (1979) apply a similar approach to a thin sheet, which approaches 2D limits at infinity, underlain by a uniformly conductive halfspace; Debabov (1982) develops the spectral-iteration method for the solution of Price's equation; Fainberg and Zinger (1980) devise the iteration-dissipative solution for Price's equation; Yegorov (1982) presents an improved hybrid algorithm to solve the classic Price's equation. Computer programs based on these fundamental papers have been recently applied to many practical interpretations – Zinger *et al.* (1984) for the Soviet Middle Asia region, Mareschal and Vasseur (1984) for Scotland (in this paper a thorough comparison is performed of Vasseur and Weidelt's (1977) and Dawson and Weaver's (1979) thin sheet algorithm leading to the conclusion that there is very little qualitative difference between the results which the corresponding programs produce), Menvielle and Tarits (1986) for the Rhine-Graben conductivity anomaly, Józwiak and Beamish (1986) for North England and South Scotland, Mareschal *et al.* (1987) for the South India region and Červ *et al.* (1987) for the territory of Czechoslovakia.

McKirdy *et al.* (1985) hydridized their original algorithm (Dawson and Weaver, 1979) with that of Vasseur and Weidelt (1977) in order to deal with a layered halfspace underlying the thin sheet with a generally 2D conductance distribution at infinity (Figure 5). The computer program was tested on 2D situations and further applied to model geoelectrical conditions in northern Scandinavia.

The model of a thin sheet which covers a radially symmetrical conductive sphere, considering the galvanic coupling between the thin sheet and the underlying conductive sphere, is solved by Zinger *et al.* (1986) and is used for the analysis of global geomagnetic data (Zinger *et al.*, 1986, Zinger *et al.*, 1987).

Tarits and Menvielle (1983) used the original algorithm of Vasseur and Weidelt (1977) to model the effect of two mutually isolated non-uniform thin sheets (Figure 6). They proved that, for most geophysical cases, the separation of the anomalous magnetic field of superficial origin and that excited by deep-seated inhomogeneities might be easily performed for periods longer than 1800 s; the anomalous field of an intra-litospheric source is the difference between the observed anomalous field and that related to the lateral contrasts of conductivity in the superficial part of the crust. In this case, the problem of two isolated thin sheets can be reduced to solving twice the single thin sheet problem. The efficiency of this approach was

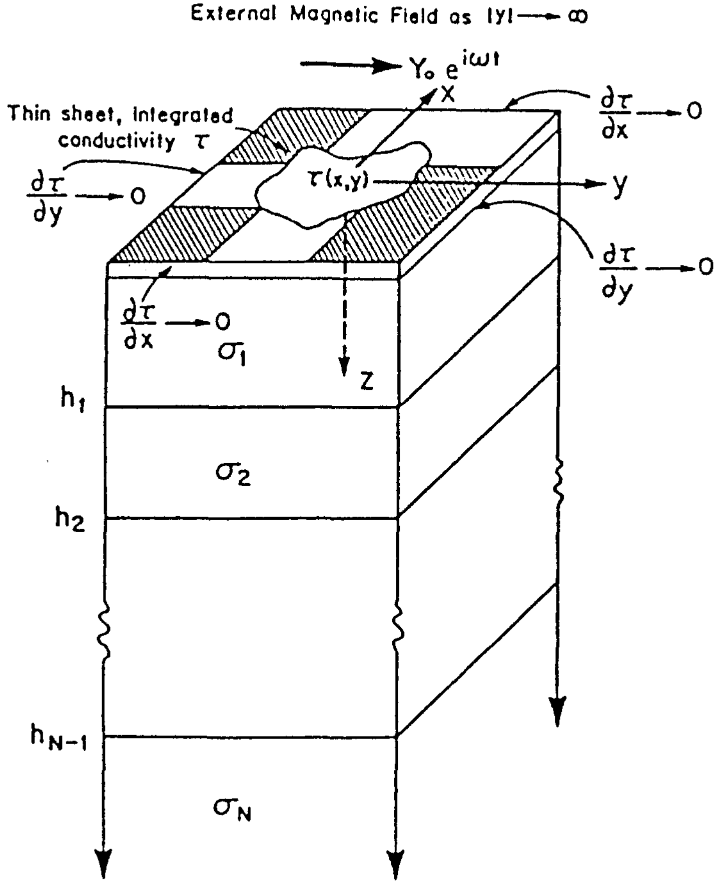

Fig. 5. Schematic situation of a layered halfspace covered by a thin sheet with a generally 2D conductance distribution at infinity. Taken from McKirdy *et al.* (1985).

demonstrated on modelling the geoelectrical structure in tectonically active areas in California, Japan and Peru (Tarits and Menvielle, 1983; Tarits and Menvielle, 1986).

Robertson (1987) generalized the algorithm of Vasseur and Weidelt (1977) in order to deal with a thin layer buried in a conductive uniform halfspace (Figure 7). Also presented is the extension of the method to several buried thin sheets within the geoelectrical section, where the inter-layer inductive coupling, has to be taken into account. In this more general case the mathematical formulation of the problem results in several mutually coupled integral equations, which must be sovled simultaneously at each level occupied by a thin sheet.

A further logical generalization of the thin sheet modelling is represented by the 'layer stacking'. Park (1983) modified and extended Ranganayaki and Madden's (1980) approach, this extension making it possible to stack heterogeneous layers to build up 3D structures. Park's algorithm is based on the differential form of

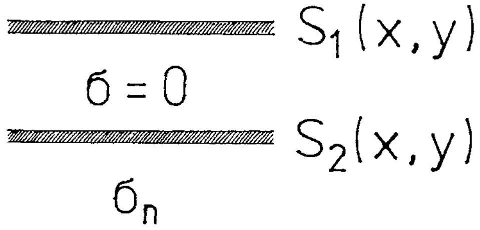

Fig. 6. Schematic model of two mutually isolated non-uniform thin sheets.

Maxwell's equations for a generally anisotropic medium from which the vertical components of the electromagnetic field are eliminated. In the resultant differential equation for the horizontal field components the vertical derivatives are approximated by finite differences, which limits the size of the steps in the vertical direction – hence the term 'thin sheet'. Horizontal derivatives are computed exactly in the wave number domain. The operators of the field continuation from the bottom to the top of a thin sheet are obtained, and by employing the boundary conditions at the interfaces of the individual layers and by prescribing the source field, the electromagnetic field can be carried up through the heterogeneous structure to the earth's surface. This approach enabled them to model localized 3D structures and to obtain valuable results from the point of view of practical interpretations (Park *et al.*, 1983; Park 1985).

McKirdy (1986) presented a modification of a 2D thin sheet modelling providing acceptable results with layers up to 0.5 skin depths thick ('thick' sheet modelling). His approach allows multiple stacking of layers and 3D generalization.

Smith and West (1987), using Price's analysis, formulated numerical techniques for calculating the response, to an arbitrary 3D source field, of a laterally varying overburden which is thin and flat, underlain by a perfectly resistive halfspace. The algorithm was developed in two versions – frequency-domain method and time-domain method. The advantages of the particular versions are demonstrated for various geoelectrical prospecting methods.

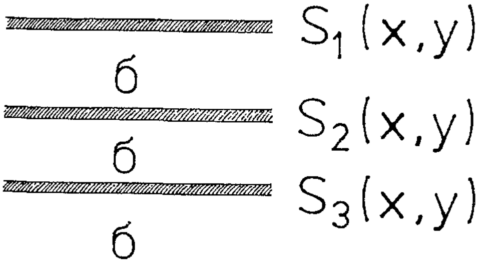

Fig. 7. Schematic model of a system of several non-uniform thin sheets buried in a homogeneous conducting halfspace.

6. Laboratory Analogue Scale Modelling

A large volume of results has been recently obtained by the laboratory analogue scale modelling. This approach, based on the electromagnetic similarity principle $\mu\omega\sigma L^2 = \text{const.}$ (μ: magnetic permeability, ω: angular frequency, σ: conductivity, L: characteristic dimension), allows models approximating real geoelectrical structures of regional extent to be studied on a small laboratory scale. In comparison with mathematical modelling there are no problems with constructing 3D models. Also large conductivity contrasts, which often impose serious limitations on mathematical modelling methods, can be easily coped with in laboratory conditions. Difficulties arise in modelling small conductivity contrasts due to the lack of suitable materials. Obtaining geoelectrical characteristics in a broad frequency band is sometimes difficult too. Therefore the laboratory scale modelling is of particular advantage for estimating the effects of various characteristic 3D structures with extreme electrical parameters. Nevertheless attempts to fill the conductivity gap between the classical laboratory materials are made e.g., by Chakridi and Chouteau (1988), where a polyester composite filled with aluminium fillers is used to design the required materials (Figure 8).

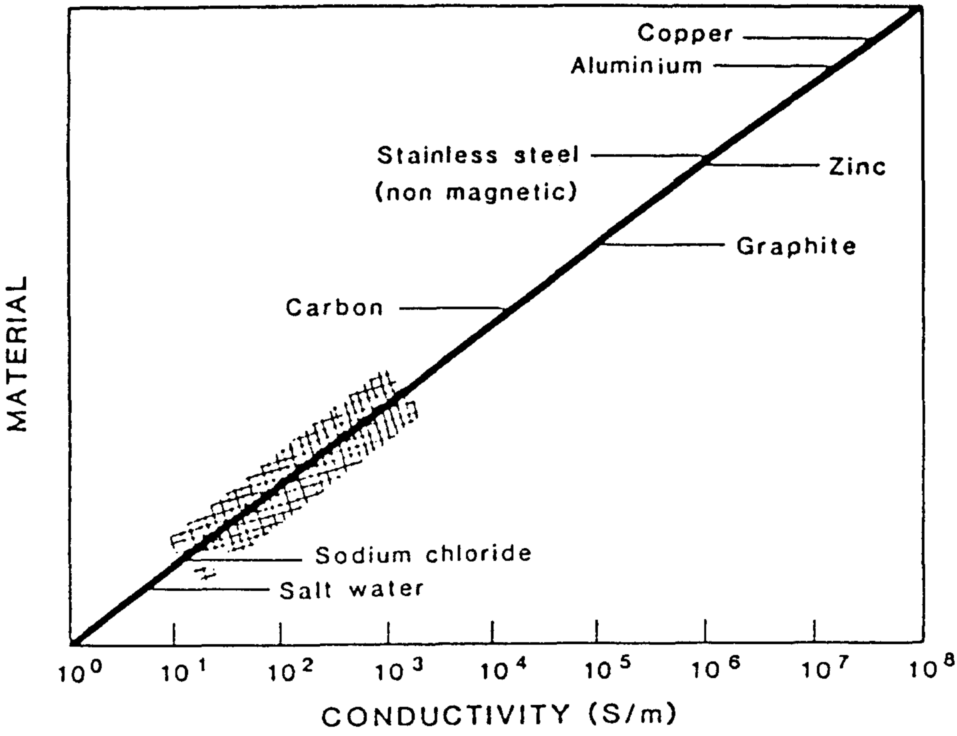

Fig. 8. Conductivity of the available materials for the electromagnetic scale modelling; the hatched area shows the conductivity gap to be filled with a new modelling material. Taken from Chakridi and Chouteau (1988).

Moroz (1984) described a physical modelling system applied in the Lvov (U.S.S.R) laboratory to model magnetotelluric fields. A great number of modelling results for various geoelectrical structures were published by Berdichevsky *et al.* (1984c), Kobzova *et al.* (1985), and Kobzova *et al.* (1987), and particularly in the monograph by Berdichevsky *et al.* (1987). In this book a series of near-surface inhomogeneities are modelled, e.g., models of horst, graben, quasi-sinusoidal perturbation of a horizontal interface, the screening effect of a high-resistivity interlayer etc., and the coast effect, the island effect (e.g., for the region of the Hawaiian islands) etc. For deep geoelectrical structures MT sounding curves are constructed and distortion effects are studied in some specific cases which are difficult to cope with by mathematical modelling techniques. As an example of practical importance, a significant influence of deep fractures is demonstrated, through which deep conducting zones become identifiable for MT soundings. Whereas the formal (1D) interpretation usually leads to rough errors, with the use of this interpretation the deep conducting zones can be detected at least qualitatively.

In Leningrad, on analysing their physical modelling results, Vagin and Kovtun (1986) arrived at semi-empirical formulae to determine characteristic parameters of horst-like 3D structures from low-frequency asymptotic values of MT impedances.

The physical modelling laboratory in Sopron, Hungary, described in detail by Märcz *et al.* (1986) is used to model various direct current and induction methods (Szarka, 1987). Electric and magnetic dipoles are used to generate the electromagnetic field and in some special cases electromagnetic plane waves can be also approximated by a horizontal electric dipole for magnotetelluric modelling.

For many years, the research group at the University of Victoria, Canada, has been one of the most active physical modelling teams. Dosso *et al.* (1986) examined the problem, how the addition of a nearby elliptical island to an otherwise complex continental coastline, involving bays and capes, affects electromagnetic induction in coastal regions. Most of their recent works have analysed the island and coast effects in particular regions, e.g., Hainan Island region (Hu *et al.*, 1986), Tasmania region (Dosso *et al.*, 1985). A unique model of the tectonically active Vancouver Island area is studied by Dosso and Nienaber (1986) with emphasis on examining the effects of the subducting ocean plates. The model attempts to include the effects of the ocean and coastlines, the subducting sloping sections of the oceanic crust and the upper mantle, and the sediment wedge at the continental shelf. This is the first laboratory model study to include a simulated subducting oceanic plate.

The laboratory scale modelling approach is limited to some extent by relatively high costs connected with building up an appropriately equipped laboratory. Once the laboratory has been established, it is relatively simple to obtain modelling results for a vast variety of 3D models. This fact explains that so far only the laboratory scale modelling, and to some extent also the thin sheet modelling, have been used to solving practically important more complex 3D models.

7. Analysis of 3D Electromagnetic Fields

(a) CONVENTIONAL ANALYSIS

Since the late 60's much effort has been devoted to the analysis of electromagnetic fields and effects produced by generally 3D inhomogeneities and observed on the earth's surface. The conventional analysis of structural dimensionality of the medium is based on the rotational properties of the impedance tensor

$$\mathbf{Z} = \begin{pmatrix} Z_{xx}, & Z_{xy} \\ Z_{xy}, & Z_{yy} \end{pmatrix},$$

where

$$E_x = Z_{xx}H_x + Z_{xy}H_y, \quad E_y = Z_{yx}H_x + Z_{yy}H_y,$$

and tipper

$$\mathbf{W} = (W_x, W_y),$$

where

$$H_z = W_xH_x + W_yH_y,$$

in the frequency domain. The rotational properties of the impedance tensor and tipper in the horizontal plane (by angle ϑ) are given by the following matrix formulae

$$\mathbf{Z}(\vartheta) = \mathbf{R}(\vartheta)\mathbf{Z}\mathbf{R}(-\vartheta), \quad \mathbf{W}(\vartheta) = \mathbf{R}(\vartheta)\mathbf{W},$$

respectively, where

$$\mathbf{R}(\vartheta) = \begin{pmatrix} \cos\vartheta, & \sin\vartheta \\ -\sin\vartheta, & \cos\vartheta \end{pmatrix}.$$

The rotational properties of particular structures can be sumarized as follows:

1D structures: $Z_{xx}(\vartheta) = Z_{yy}(\vartheta) = 0$, $Z_{xy}(\vartheta) + Z_{yx}(\vartheta) = 0$, $W_x(\vartheta) = W_y(\vartheta) = 0$ for all horizontal rotations of the coordinate system, for all points at the earth's surface and for all frequencies.

2D structures: $Z_{xx}(\vartheta) + Z_{yy}(\vartheta) = 0$, $Z_{xy}(\vartheta) + Z_{yx}(\vartheta) \neq 0$, $W_x(\vartheta)$, $W_y(\vartheta) \neq 0$ in general, but an angle ϑ_s of rotation of the coordinate system exists, for which $Z_{xx}(\vartheta_s) = Z_{yy}(\vartheta_s) = 0$, $W_x(\vartheta_s) = 0$, $W_y(\vartheta_s) \neq 0$ for all frequencies. In this case the x axis coincides with the strike direction of the structure.

3D structures: There are no similar constraints although particular points may exist, in which some of the above formulae hold.

To estimate practically the degree of 3D distortion of the structure studied, further characteristic parameters were introduced

$$\text{skew } \alpha = |Z_{xx} + Z_{yy}|/|Z_{xy} - Z_{yx}|$$

(rotational invariant) and eccentricity of the rotational impedance ellipse

$$\beta(\vartheta) = |Z_{xx}(\vartheta) - Z_{yy}(\vartheta)|/|Z_{xy}(\vartheta) + Z_{yx}(\vartheta)|.$$

which particularly for

$$\vartheta_0 = \frac{1}{4}\operatorname{arctg}\frac{2\,\mathrm{Re}[(Z_{xy} + Z_{yx})(Z_{xx} - Z_{yy})]}{|Z_{xx} - Z_{yy}|^2 - |Z_{xy} + Z_{yx}|^2}$$

defines the ellipticity $\beta(\vartheta_0)$ as a major/minor semiaxes ratio of the rotational impedance ellipse. ϑ_0 maximizes $|Z_{xy}(\vartheta) + Z_{yx}(\vartheta)|$ and minimizes $|Z_{xx}(\vartheta) - Z_{yy}(\vartheta)|$ and allows two mutually perpendicular directions of the structure to be defined. In a 2D case one of these directions is parallel to the strike direction.

The magnetotelluric analysis alone does not allow us to decide which of the two principal directions defines the axis of homogeneity of a 2D structure. To eliminate this non-uniqueness the tipper criterion must be applied, i.e. $W_x = 0$, $W_y \neq 0$ if the axis of homogeneity is parallel to the x axis of the coordinate system. Skew and ellipticity vanish in 2D situations.

Ting and Hohmann (1981), Hermance (1982), and Park (1983) show, however, that the particular non-zero values of skew and ellipticity do not clearly indicate the degree of three-dimensionality of the structure. E.g. in a 3D model published by Park (1983) the maximum skew was 0.15 and maximum ellipticity 0.21, in Hermance's (1982) 3D model the skew varied between 0.001 and 0.72, but the ellipticity was 10^{-7} by the order of magnitude, and in Ting and Hohmann (1981) both the skew and ellipticity were $10^{-2} - 10^{-1}$. Thus, studying solely these two structural parameters it is difficult for us to decide between a 2D or 3D interpretation. In practice most of the researchers would rather prefer a simpler 2D than 3D interpretation based on these values only. For this reason Kao and Orr (1982) and Beamish (1986) introduced three normalized dimensional weights to estimate the relative importance of 1D, 2D and 3D structural contributions simultaneously. Other parameters for estimating the dimensionality of the structure studied are defined, e.g., by Fluche (1983) and Honkura *et al.* (1989). Both these authors use magnetic transfer functions with respect to a reference station as a basis for their structural studies.

(b) APPROXIMATION OF 3D ELONGATED STRUCTURES BY 2D STRUCTURES

A strictly 2D earth with a given horizontal strike provides two orthogonal, uncoupled response modes referred to as E-polarization ($E_{\parallel}$) and H-polarization mode ($E_{\perp}$). In each of the polarization modes the corresponding electric and magnetic vectors are mutually perpendicular. The question of the validity of approximating elongated 3D structures by 2D structures was investigated by Berdichevsky and Dmitriev (1976), Berdichevsky *et al.* (1984a, b) Berdichevsky *et al.* (1986) and Berdichevsky *et al.* (1987). For near-surface inhomogeneities it may be concluded that, particularly in the centre of the inhomogeneity, for wave lengths exceeding substantially the characteristic dimensions of the inhomogeneity, the

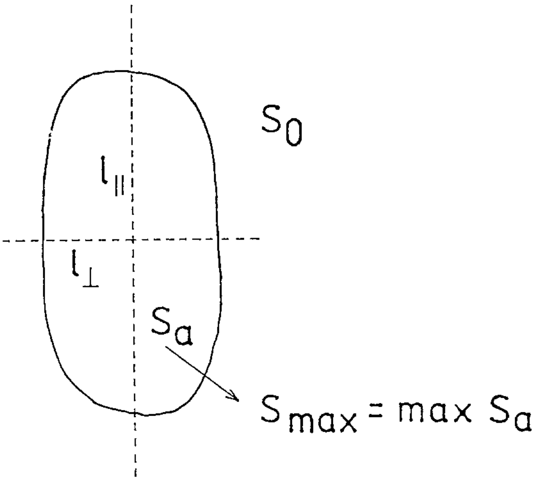

Fig. 9. Criteria of 2D approximability increase of wave length $\lambda \gg$ characteristic dimension 1:
E-polarization: :if $S_a < S_0$ then $l_{\parallel}/l_{\perp} \gtrsim 8$,
:if $S_a < S_0$ then $l_{\parallel}/l_{\perp} \gtrsim 8\ S_{max}/S_0$,
H-Polarization: :$l_{\parallel} \gtrsim 10\, l_{\perp}$

decrease of conductance values S only slightly affects the low frequency branches of the quasi-E-polarization curves if the length of the inhomogeneity exceeds 8–10 times its width. In the case of an increased anomalous conductance S the length/width ratio must be about 8–10 S_{max}/S_0 where S_{max} is the maximum conductance within the anomalous region and S_0 its normal value outside the inhomogeneity. For the quasi-H-polarization mode it is sufficient that the strike extent of the anomaly be about 10 times greater than its width (Figure 9).

For wave lengths comparable with the characteristic size of the anomaly, which is more conductive than its normal environment (Figure 10), Vesselovski and Yudin (1988) concluded from a statistical processing of different numerical modelling results that the length of the anomaly should exceed about 8 times its width for a satisfactory 2D behaviour of the inhomogeneity in the quasi-H-polarization case to be assumed. for the quasi-E-polarization mode the strike extent must be at least

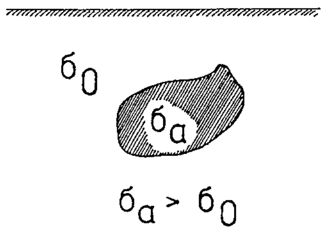

Fig. 10. Criteria of 2D approximability in case of $\lambda \approx 1$:
E-polarization: :$l_{\parallel} \gtrsim 0.5\,\lambda_0$,
H-polarization: :$l_{\parallel}/l_{\perp} \gtrsim 8$.

half the wave length in the surrounding medium. The ratio length/width and σ_a/σ_0 are not very important in this case.

A recommendation on how to approximate 3D elongated structures by 2D models follows from Ting and Hohmann (1981). They made a comparison between results for elongated 3D prisms and those for a 2D model with the same cross-section. For lower frequencies, a significant difference still exists between the longest prism and the 2D model for the E-polarization mode and the results are very sensitive to the strike extent. For the H-polarization mode a better agreement was observed between the apparent resistivities of the 2D and 3D models. In accordance with these facts, the authors suggest that the 2D cross-section be obtained from higher frequency 2D H-polarization data modelling, and that the strike extent be derived by matching with lower frequency E-polarization results due to corresponding 3D models. These authors further studied the frequency dependence of various MT characteristics. It was shown that the phase and skew expressed the three-dimensionality better at high frequencies, while at lower frequencies of the field the 3D character of the structure was better reflected by the apparent resistivity and the ellipticity. These conclusions are in good agreement with laboratory scale modelling results obtained by Vagin and Kovtun (1982).

In studying the possibilities of 2D or 1D interpretation of general 3D structures, Berdichevsky and Dmitriev (1976) proposed two effective impedances

$$Z^{I}_{ef} = \sqrt{Z_{xx}Z_{yy} - Z_{xy}Z_{yx}} \quad \text{and} \quad Z^{II}_{ef} = (Z_{xy} - Z_{yx})/2$$

which are rotational invariants of the impedance tensor. As shown by Berdichevsky *et al.* (1986), near to round-shaped 3D inhomogeneities, due to current channelling, the effective apparent resistivity curves does not significantly differ from the corresponding normal curve. Positive experience from using the effective impedances was obtained by Ranganayaki (1984) and especially by Ingham (1988), where the 3D model of Wannamaker (1984b) was re-interpreted by a 1D model on using Z^{II}_{ef}. Nevertheless, above shallow conductive bodies interpretation of the deeper structure must be regarded with care.

The dimensionality parameters introduced so far represent formal indicators of the symmetry of the medium. Their relation to a particular conductivity distribution within the earth was studied by a number of authors on various 3D models. All these studies resulted in extracting certain classes of characteristic 3D distortions of the electromagnetic field. From various points of view, these allow us to classify 3D effects and their manifestation in the geoelectrical characteristics on the earth's surface. Berdichevsky (Berdichevsky and Dmitriev, 1976; Berdichevsky *et al.*, 1984b; Berdichevsky *et al.*, 1986), e.g., sorts out galvanic effects – the flow-around and concentration effect (current channelling), S effect, screening effect and edge effect, and an induction effect – horizontal skin effect, as diagnostics indicators of near-surface electromagnetic distribution. Park (1985) identifies three distortion mechanisms in 3D structures. The first is resistive coupling between the upper crust and the mantle across the resistive lower crust. The second mechanism is resistive coupling of conductive features in the upper crust. The third mechanism is local induction of current cells within good conductors at finite dimensions. For the individual distortion mechanisms Park gives the order of magnitude estimates in DC approximation.

A complex qualitative and quantitative analysis of 3D distortion effects, both galvanic and inductive, is presented by Menvielle (1988). Among other things he discusses an important question of the adjustment distance, summarizing the individual results obtained by Ranganayaki and Madden (1980), Berdichevsky and Dmitriev (1976), Dawson and Weaver (1982), and Fainberg and Zinger (1987). The same problem was studied by Ádám and Szarka (1985) on using laboratory scale modelling. The value of adjustment distance measured in MT fields are near to the numerical values of Ranganayaki and Madden (1980) in the case of thin dykes.

(c) NEW APPROACHES TO THE MT TENSOR ANALYSIS

Conventional analysis of MT data is based on utilizing the principal impedances $Z_{xy}(\vartheta_0)$ and $Z_{yx}(\vartheta_0)$, the principal direction ϑ_0, the skew and ellipticity. A disturbing aspect of the conventional analysis is that the principal impedances are independent of the trace of the impedance tensor **Z**. Hence, the set of parameters extracted by the conventional analysis for an impedance tensor corresponding to a 3D structure is incomplete. Eggers (1982) recognized this important fact and, consequently, proposed the eigenstate formulation of the impedance tensor as a technique for the extraction of a complete set of physically meaningful scalar parameters from **Z**. This trend in the analysis of MT impedance was further followed by Spitz (1985), Counil *et al.* (1986), and LaTorraca *et al.* (1986). The most profound analysis of this problem was given by Yee and Paulson (1987). Their canonical decomposition of **Z** parametrizes the impedance tensor in terms of eight physically relevant structural parameters that are sutiable for qualitative interpretation. Four of these structural parameters (two moduli and two phases) determine the two principal impedances and, hence, serve to characterize the transfer properties of the earth system. The remaining four parameters are polarization parameters which resolve the principal coordinate system for **Z**; two of these parameters specify the principal electric field polarization states, whereas the remaining two describe the principal magnetic field polarization states. The association of the structural parameters that emerge from the canonical decomposition of **Z** with particular physical characteristics of the earth system now depend on performing a series of 3D modelling experiments. Only in this way can the exact physical content of these parameters be revealed. An initial step in this direction has already been taken by Counil *et al.* (1986) and LaTorraca *et al.* (1987).

The analyses performed allowed new important characteristics to be defined. For a general 3D case with non-orthogonal electric and magnetic bases Counil *et al.* (1986) introduced the direction of maximum and minimum current

$$\operatorname{tg} 2\psi_c = \frac{2\,\mathrm{Re}(Z_{xx}^{*}Z_{yx} + Z_{yy}Z_{xy}^{*})}{|Z_{xy}|^2 + |Z_{xx}|^2 - |Z_{yx}|^2 - |Z_{yy}|^2},$$

the direction of maximum and minimum induction

$$\operatorname{tg} 2\omega_i = \frac{2\,\mathrm{Re}(Z_{xx}Z_{xy}^{*} + Z_{yx}Z_{yy}^{*})}{|Z_{xx}|^2 + |Z_{yx}|^2 - |Z_{xy}|^2 - |Z_{yy}|^2},$$

and the δ indicator, which is a measure of the two-dimensionality of the structure.

LaTorraca *et al.* (1986) used a variation of singular value decomposition analysis by incorporating phases in the singular values, which are then called characteristic values, and thoroughly analysed the behaviour of these characteristic values for a particular 3D model of a conductive prismatic body with a local elevation. They demonstrate that the phase is a better parameter for modelling deep structures.

From the theoretical point of view progress has been made in studying analytical properties of the impedance tensor. Adopting a purely systems – theoretic viewpoint, Yee and Paulson (1988) showed how a small set of well-established general physical principles such as causality, stability, and passivity, applied to the earth system, determines the analytical properties of the associated impedance tensor in the complex frequency plane. The fundamental dispersion relations for the scaler response function, developed by Weidelt (1972) and Fischer and Schnegg (1980) in the context of 1D and 2D earth systems, respectively, have been extended to the tensor response function corresponding to general 3D earth system. For the general 3D earth, the dispersion relations have direct application in the formulation of data consistency tests, the construction of consistent estimates of the impedance tensor, and the identification of non-linearities, caused, e.g., by external source field effects.

(d) ANALYSIS OF SIMPLIFIED 3D STRUCTURES

Most of the approaches characterized above attempt to determine the general 3D conductivity distribution from known components of the impedance tensor. From the point of view of practical interpretations, also approximate approaches are valuable which analyse simplified 3D structures. Zhang *et al.* (1987) study a two-level model, which consists of a 2D local structure and 2D regional structure with non-parallel strikes. They show that in the long period limit the MT impedance tensor can be written as a sum of the regional undistorted impedance and perturbed impedance generated by the local inhomogeneities. The latter can be written as a product of a local distortion in thin-sheet approximation and the regional impedance. The local strike ϑ_1 is characterized by that direction where diagonal elements of **Z** are proportional. The proportionality constant $\alpha = Z_{xx}/Z_{yy}$ is real, negative, and independent of the period. The regional strike ϑ_r is characterized by that direction where elements of the column of the impedance tensor are proportional. The proportionality constants $\beta = \mathrm{Z}_{\mathrm{xx}}/\mathrm{Z}_{\mathrm{yx}}$ and $\gamma = Z_{yy}/Z_{xy}$ are real and independent of period.

Bahr (1988) discusses a practically more realistic situation of a 2D regional and 3D local geoelectrical structure. He shows that in this case, a complete separation of local and regional anomalies is possible if additional information from geomagnetic depth sounding is used. His method of telluric vectors decomposes the impedance tensor into 10 parameters – two telluric vectors, i.e., 4 parameters, the regional strike (1 parameter), the regional skew (1 parameter) and the distortion matrix (4 parameters). The impedance tensor provides only 8 degrees of freedom, the additional information besides the MT tensor is used. By introducing a new, rotationally invariant skew the author suggests a test whether the regional structure is 2D. This new skew is derived from the phase alone, without using the amplitudes of the impedance tensor elements as in the conventional analysis.

Groom and Bailey (1988) presented a decomposition method which, similarly as the Bahr's approach, assumes static electric field distortion of fields which are introduced in a structure which is at most two-dimensional. They factor the real electric scattering matrix by means of three matrices – twist, shear, and anisotropy matrix. The method allows estimates of the 3D distortion parameters (twist and shear), the regional strike and the modified 2D regional impedances to be determined simultaneously. The authors then use a special model of a small conducting hemisphere embedded in the 2D regional structure to analyse how well both their and Bahr's method perform in realistic conditions, i.e. in the presence of the magnetic effects of current distortion and the added noise. For this model various current parametrizations are studied and an optimal set of structural parameters is proposed.

8. Conclusion

Great progress has been recently achieved in the development of various 3D modelling techniques. To analyse 3D localized inhomogeneities in a layered earth, the IE and hybrid methods seem to be the most suitable approach at present. These have also been successfully applied to time-domain electromagnetic modelling problems. For regional studies various variants of the thin-sheet approximation are most frequently used which can cope with rather complex 3D structures by applying the layer stacking idea. In addition to the mathematical methods, significant results as to the behaviour of 3D electromagnetic fields have been also obtained by the laboratory scale modelling. Mathematical methods based on the direct numerical solution of the differential equations in 3D media are still waiting for a broader exploitation.

The analysis of 3D electromagnetic fields is based on several new approaches whose physical meaning, however, is to be intensively investigated, especially on the basis of a large amount of 3D modelling experiments.

The exploitation of the 3D modelling has not so far penetrated deep enough into the practice of geoelectrical interpretations. 3D modelling programs are exceptionally complex from both the methodological and computational point of view. For this reason, broad international cooperation, especially based on the exchange of 3D modelling and interpretation methods and algorithms, seems to be the inevitable factor of the future progress in this field.

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