MAGNETOTELLURIC VARIATION PROCESSING

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(1) It is known that the magnetotelluric (MT) variation processing problem is to determine the transfer functions describing the linear relationships among the MT-field components at the same point of space or at different ones. The transfer functions serve as the mutual frequency or transient responses, relating the field components and, ideally, must be independent of the field observation time and of the variation type, governed only by the electrical structure of the earth. Until recently, the general formulation of the problem appeared fairly clear and simple. In the frequency domain, for example, the linear relationships of the components of the electromagnetic field at a certain point were presented as

$$\begin{pmatrix} \tilde{E}_x(\omega) \\ \tilde{E}_y(\omega) \end{pmatrix} = \begin{pmatrix} \tilde{Z}_{xx}(\omega) & \tilde{Z}_{xy}(\omega) \\ \tilde{Z}_{yx}(\omega) & \tilde{Z}_{yy}(\omega) \end{pmatrix} \cdot \begin{pmatrix} \tilde{H}_x(\omega) \\ \tilde{H}_y(\omega) \end{pmatrix}, \tag{1}$$

where $\tilde{E}_{x,y}(\omega)$ and $\tilde{H}_{x,y}(\omega)$ are intensities of the respective components of electric and magnetic fields; $\tilde{Z}_{xx}$, $\tilde{Z}_{xy}$, $\tilde{Z}_{yx}$, and $\tilde{Z}_{yy}$ are impedance matrix elements at a given frequency, ω [1]. The relation (1) describes the relation between input and output functions of a linear system with two inputs and two outputs, the input functions being the horizontal magnetic field components (Figure 1a). It is assumed that the matrix nature of the relationship (1) is due to horizontal inhomogeneity within the medium, the matrix itself being a tensor which invariantly expresses the linear relationships as the coordinate system rotates at the observation points.

A wealth of experience in practical determinations of the $\tilde{Z}_{i,j}(\omega)$ values corresponding to the above model has shown that the accuracy attained is much below the accuracy in measuring field variatons proper and does not always suit the geophysicists' requirements for subsequent interpreation of the MT data. In terms of the model (1) the low accuracy of the output data determination seems to be unaccountable in many cases. In considering feasible methods for raising the accuracy in determining the transfer function for the MT-field, it is expedient to revise the prerequisits which underlie model (1), to specify the formulation of the MT-variation processing problem, and, at the same time, to elucidate the key points which, if disregarded, may give rise to errors in processing. It should be remembered that model (1) is based on the assumption that the observed MT-field arises because a uniform plane wave varying in time according to harmonic law is incident on the electrically inhomogeneous Earth. It is assumed that, having at one's disposal the realization of the electric and magnetic field at a certain frequency ω, which correspond to different polarizations of the exciting field, one can find the required $\tilde{Z}_{i,j}$ values from a redundant system of algebraic equations obtained from (1).

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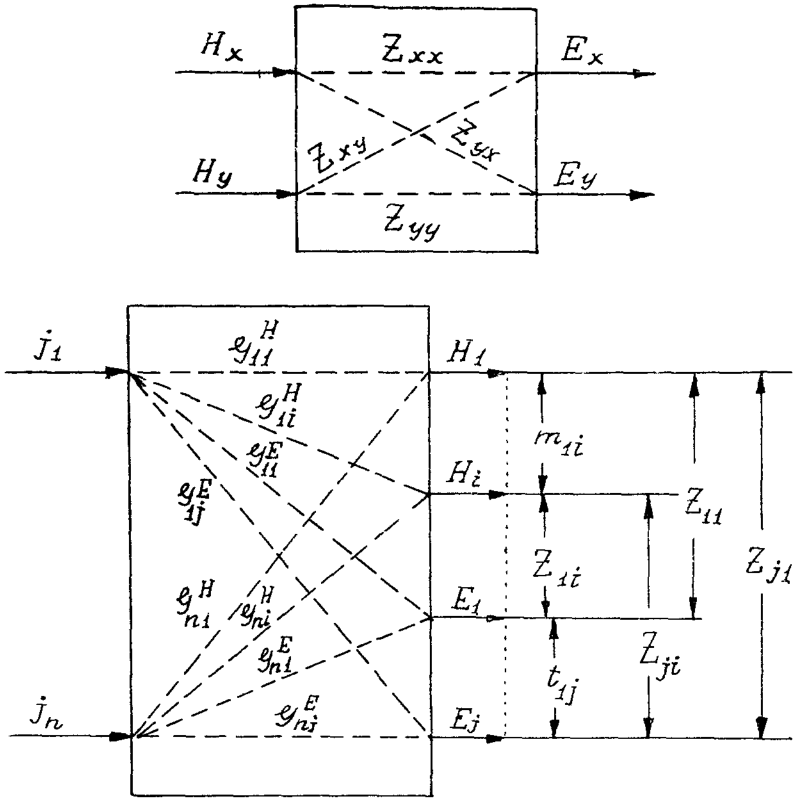

Fig. 1. (a) Traditional equivalent scheme of linear relationships in the MT-field. (b) Revisional system of linear relationships in the MT-field for an electrical current with "n" degrees of freedom.

(2) Let us bring the idealized pattern discussed above closer to the real situation. In reality, we deal with a complicated system of ionospheric, atmospheric and magnetospheric currents which is time-variable in an arbitrary manner. If the primary field of the system in the field observation region can be described with sufficient accuracy by a homogeneous plane wave of variable polarization, then the current system proper may be approximated by two plane currents of different directions, varying in time independently of one another. Berdichevsky and Zhdanov (1984) and the present authors [2] have demonstrated that, with regard to the processing problem, a more accurate approximation of complicated current systems can be obtained if they are treated as a finite set of n individual current systems $j_i(t)$ and if the time variations of currents in each of the systems are governed by an individual, independent law characterized by a certain sufficiently arbitrary function of time $f_i(t)$ (figure 2). In this case the current density variation $j(t)$ within the entire system may be presented as

$$\bar{j}(P_0, t_0) = \hat{a}(P_0, t_0 - t'_0) * \bar{f}(t'_0), \tag{2}$$

where the symbol $*$ means convolution in time, t'_0 is a variable of integration, P_0

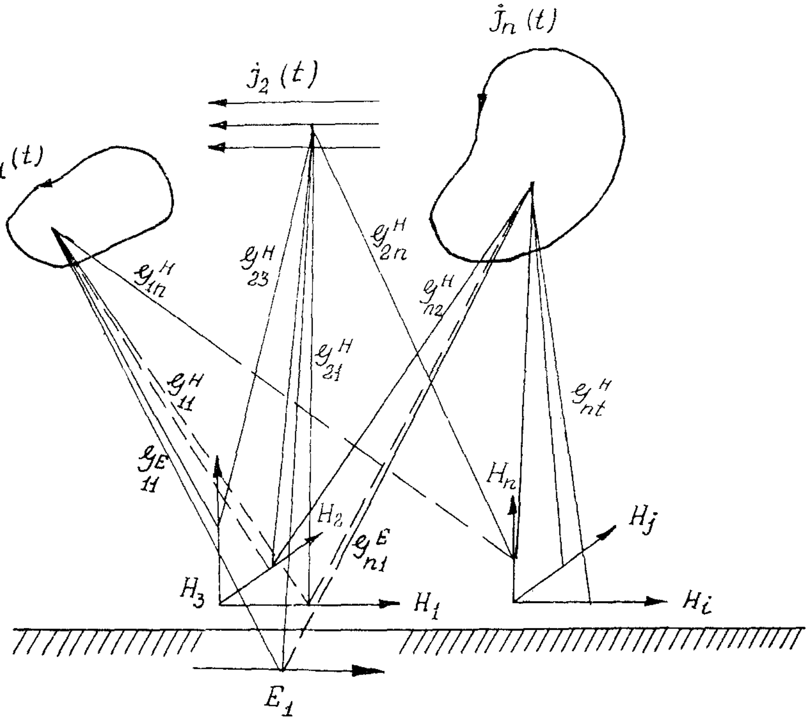

Fig. 2. Schematic model of media exited by a complex current system.

and t_0 denote the coordinates of the point within the current system and the time observation of current in this point; $\overline{f}(t_0')$ is the column-vector composed of the time functions $f_i(t_0')$; $\hat{a}(P_0, t_0) = \{a_{ik}(P_0, t_0)\}$ is the matrix operator of dimension $3 \times n$ which defines the linear temporal relationships within the current system. The linear independence of the time functions is understood in the sense that equality

$$\sum_{i=1}^{n} \alpha_i(t_0 - t_0') * f_i(t_0') = 0 \tag{3}$$

can be satisfied only if all $\alpha_i(t_0) = 0$. The approximation (2) includes the particular cases of a linearly-polarized plane current ($n = 1$) and horizontal plane currents with variable polarization ($n = 2$). In the general case n may be a certain number whose value is to be found in the course of the processing.

The electromagnetic field generated by such a system as (2) at a certain point P in the observation region and at an arbitrary moment t may be found using the tensorial Green function $\hat{\mathcal{G}}(P, P_0, t - t_0)$. An arbitrary component of electric or magnetic field along some direction (designated as $E(P, t)$) can be expressed as

$$E(P, t) = \int_{v_0} \int_{-\infty}^{t} \overline{\mathcal{G}}(P, P_0, t - t_0) \cdot \overline{j}(P_0, t_0) \, \mathrm{d}V_0 \, \mathrm{d}t_0 =$$

$$= \overline{\mathcal{G}}(P, P_0, t - t_0) \otimes \overline{j}(P_0, t_0), \tag{4}$$

where $\bar{\mathcal{G}}$ is a vectorial Green function (a line of the respective Green function $\hat{\mathcal{G}}$); the symbol $\otimes$ means integration over the exciter coordinates P_0 within the region V_0 where $j \neq 0$, and convolution in time. Substituting (2) in (4) and integrating over intermediate coordinates, we obtain

$$E(P, t) = \bar{\mathcal{G}}(P, P_0, t - t_0') \otimes \hat{a}(P_0, t_0' - t_0) * \bar{f}(t_0) = \\ = \bar{K}(P, t - t_0) * \bar{f}(t_0), \tag{5}$$

where

$$\bar{K}(P, t - t_0) = \bar{\mathcal{G}}(P, P_0, t - t_0') \otimes \hat{a}(P_0, t_0' - t_0)$$

is a line-vector of n elements. From (5) it follows that, like the field of the exciting current system, the set of the functions characterizing the time variations of any component of the field at any observation point is one and the same finite-dimensional functional space with the basic set of the time functions $\bar{f}(t_0)$ in which the multiplication operation is a convolution with the functions $\bar{K}(P, t - t_0)$, dependent on the time difference $t - t_0$. Any set of n independent time functions characterizing the time variations of some rationally selected field components $H_i(P, t_0)$, $i = 1, 2, \ldots, n$ may be taken instead of $f_i(t_0)$ to be the basic set in the space of observed time functions. If such a basic set does exist the mutual one-to-one correspondence:

$$\bar{H}(P', t_0) = \hat{T}(P', t_0 - t_0') * \bar{f}(t_0') \tag{6}$$

$$\bar{f}(t_0') = \hat{T}^{-1}(P', t_0' - t_0) * \bar{H}(P', t_0) \tag{6'}$$

must exist between the basis $\bar{f}(t_0)$ and $\bar{H}(P', t_0)$. Substituting (6′) in (5), we obtain the expression for any of the observed field components through the novel basis functions:

$$E(P, t) = \bar{m}(P, P', t - t_0) * \bar{H}(P', t_0), \tag{7}$$

where $\bar{m}(P, P', t - t_0) = \bar{K}(P, t - t_0') * \hat{T}^{-1}(P', t_0' - t_0)$ is the vectorial operator of convolution which expresses the linear relationships among the components of the examined electromagnetic field. The mathematical meaning of the operator $\bar{m}(P, P', t) * (\cdot)$ will be specified later. In the frequency domain the relation (7) takes the form

$$\tilde{E}(P, \omega) = \tilde{\bar{m}}(P, P', \omega) \cdot \tilde{\bar{H}}(P', \omega), \tag{7'}$$

where $\tilde{\bar{m}}(P, P', \omega) = \tilde{\bar{K}}(P, \omega) \cdot \tilde{\hat{T}}^{-1}(P', \omega)$. As a particular case, the expression (7′) is in correspondence with any of two linear algebraic relations obtainable from the matrix relation (1) if the functions $H_x(\omega)$ and $H_y(\omega)$ are taken to be the basis. The expressions (7) and (7′) must underlie the problem of determining the transfer functions $\bar{m}(P, P', t)$ and $\tilde{\bar{m}}(P, P', \omega)$ in MT studies as well as the other important problem of eliminating MT-variations from the results of observing the main magnetic field or artificial electromagnetic fields.

(3) Let us return from the above discussed simple, but abstract, mathematics to the practical aspect of data processing and find out what must be taken into consideration when formulating and solving the MT-variation processing problem. Firstly, the relations (1) are but a special case of a more general system of linear relationships expressed through the matrix transfer functions of dimension n. Actually, let us assume there is another set of n independent time-variable field components $\bar{E}(P, t)$ along with the already above selected basis time functions $\bar{H}(P, t)$. Then one-to-one correspondence between these sets of the functions can be written on the basis of the relation (7):

$$\bar{E}(P, t) = \hat{m}(P, P', t-t') * \bar{H}(P', t'), \tag{8}$$

where the matrix lines $\hat{m}(P, P, t-t')$ are formed of vector transient functions for the field components $E_i(P, t)$. Similarly, in the frequency domain we have:

$$\tilde{\bar{E}}(P, \omega) = \tilde{\hat{m}}(P, P', \omega) \cdot \tilde{\bar{H}}(P', \omega) \tag{8'}$$

For the uniform plane wave of variable polarization, when $n = 2$, the relation (1) follows from (8′), and $P = P'$,

$$\tilde{\bar{H}}(P, \omega) = \begin{pmatrix} \tilde{H}_x \\ \tilde{H}_y \end{pmatrix}, \quad \tilde{\bar{E}}(P, \omega) = \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} \quad \text{and} \quad \tilde{\hat{m}} = \tilde{\hat{Z}}.$$

At the same time it is necessary to notice that relationship (1) holds true not only in this case but for more general conditions, when time variations of the currents exciting the MT-field may be described by means of two independent functions $f_i(t)$. that is in any case, the current system being of two degrees of freedom. The relationship between the frequency characteristics of the horizontal components of magnetic and electric fields should be described by the same relation (1) in the case, for example, when the MT-field source can be approximated with oscillating horizontal magnetic or electric dipoles of varying orientation. Naturally, as this takes place, the transfer functions $\tilde{Z}_{ij}(\omega)$ differ from the corresponding plane wave impedances. The matrix form of the presentation (1) is convenient when transition is made from one basis (H_x, H_y) to another (E_x, E_y). As applied to the processing problem, a more adequate expression is given by the equalities (7) and (7′) which express the scalar product of two vectorial functions, namely, the transfer function $\bar{m}$ and the basis function $\bar{H}$. In case of (1) the two expressions are of the form

$$E_i(t) = \sum_{j=1}^{2} Z_{ij}(t-t_0) * H_j(t_0) \quad \text{or} \quad \tilde{E}_i(\omega) = \sum_{j=1}^{2} \tilde{Z}_{ij}(\omega) \cdot \tilde{H}_j(\omega). \tag{1'}$$

If the relations (1′) fail to give a satisfactory solution for the processing problem, i.e. when the field calculated from given H_j and from found Z_{ij} (the synthesized E) is substantially different from the observed pattern or the values Z_{ij} vary significantly from one realisation to another, then the system of basis functions H_x, H_y must be complemented (for example, with H_z and, may be, with some other functions). One of the key problems to be resolved in the course of the processing is to decide on the required number and on the concrete selection of the basis and transfer functions. If the subsequent interpretation of the transfer functions is essentially made in terms of the plane-wave model, it is desirable to exclude the realizations described by the transfer functions of dimension $n > 2$ from the set of the MT-field variation realizations to be processed. When this approach is being used the same problem is solved, as is done in the paper [3] by "the remote reference point MT technique", but by other means. More efficiently the problem is worked by means of the technique of the magnetotelluric survey with the remote reference point, specially developed for this purpose and commonly used in the USSR. The stable impedance matrix is determined at the reference point as the result of prolonged measurements. Then the synchronous observations are carried out at the reference and common survey points. If now the impedance values defined using measured variations at the reference point markedly differ from the curves defined earlier, these realizations are excluded from the processing on all other points of observation too.

Secondly, there is one more type of variation which should be processed using extreme care: they are generated with a moving source. Since (4) and (5) comprise integration over the exciter coordinates P_0, these and subsequent expressions in horizontally-inhomogeneous media will be of the convolution type only if the field exciter is immobile. If this condition is not satisfied, the transfer functions become transient. This is quite understandable physically because the excitation of inhomogeneities in a medium varies as the exciter moves with respect to them. Therefore, the variations during which the exciter position relative to observation point varies substantially (for example, solar-diurnal variations) must also be excluded from the processing. The more faithful processing method for variations like these is spherical harmonic analysis resulting in stationary frequency characteristics of impedance spherical harmonics (spectral impedances).

Thirdly, from the above it follows that the sought transfer functions are very different from the transfer functions studied in the theory of linear systems, for, in contrast to the latter, they, owing to their inherent physical sense, establish the relationship only among the output functions rather than between the input and the output in a linear system (Figure 1b). The input functions for the MT-field are understood in a natural way as the currents, exciting it (the function $f_i(t)$ describing time variations of the current). The measured electromagnetic field (both electric and magnetic components) should be considered as the response of the linear system (the Earth) to this excitation. Therefore, the MT transfer functions may not, generally, exhibit the important properties (causality and stability) which characterize the conventional transfer functions of passive linear systems, although from (7) it follows that their stationarity (i.e. dependence on only the difference between the time arguments) is preserved. The causality of the transfer functions ($m(P, P', \tau) = 0$ at $\tau < 0$) appears to hold, but only in quasistationary fields (in the wave fields there can be the equality of more general form: $m(P, P', \tau) = 0$ at $\tau < \tau_0 \geqslant 0$).

However, these functions may not exhibit the stability property, i.e. the finiteness of the integral $\int_0^\infty |m(P, P', \tau)|\, d\tau < \infty$. It is, therefore, more correct to treat them and their Fourier transforms as generalized functions or distributions and to understand in this sense the mutual transitions from $\bar{m}(P, P', \tau)$ to $\tilde{\bar{m}}(P, P', \omega)$ and vice versa and the operation of their convolution with the field variations [4]. As a rule, the transfer functions between the like field components at different points contain δ-functions in the time domain, while the impedance and admittance transfer functions contain nonintegrable features at zero or infinity. The impedance of the homogeneous half-space $Z(\omega) = \sqrt{(i\omega\mu/\sigma)}$ has no, in common functional sense, inverse Fourier transform. However, in a generalized sense, the pulse impedance characteristic does exist and is expressed by means of the generalized function $Z(t) = (1/2\sqrt{\pi})\sqrt{\mu/\sigma}(1/t_+^{3/2})$, being determined with the linear continuous functional of the form

$$\left\langle \frac{1}{t_+^{3/2}}, \varphi(t) \right\rangle = \int_0^\infty \frac{1}{t^{3/2}} [\varphi(t) - \varphi(0)]\, dt$$

under the space S of basic functions $\varphi(t)$, which are infinitely differentiable and speedily decreasing at infinity (speedier than any power of $1/t$). When $t > 0$ this generalized function coincides with the common function $1/t^{3/2}$. If differentiability of the convolution results is not required, this definition can be generalized to arbitrary, bounded, and one time differentiable functions $H(t)$, which can be identified with MT-field variations transformed in a special way. Hereinafter we will discuss this and another approach to the time domain regularization of the generalized transfer functions. Since we are not interested in the accurate values of the transient and frequency characteristics of the transfer functions both at zero and at infinity and, moreover, since they cannot be obtained in practical calculations due to the limited length of the realizations and to the discretization of the continuous values of the field, the transition to the generalized function is quite justified.

Forthly, the concept of causality is extended to the generalized functions. The causality of the transfer functions gives rise to their important analytical properties which have to be used as a priori information, when solving the processing problem making it possible to improve the stability of the solution for the problem. These properties include, for instance, relationships between the real and imaginary parts of the transform function $\tilde{m}(P, P', \omega)$, expressed by the Hilbert transform. In cases when there is the possibility of identification of the basic functions in (7, 7′) with the input functions of the linear system, following from the general physical and mathematical propositions, much more efficient a priori information can be attracted to solve the processing problem. Particularly it can be done when horizontal components of the magnetic field at the Earth's surface coincide with accuracy of constant factors with the primary field of the exciter (the case of the horizontal homogeneous section in the plane wave field, the 2D-medium in the H-polarized plane field). In situations like these one may contend that the frequency characteristics of the MT-transform functions $\tilde{m}(P, P', \omega)$ are analytical on the complex plane $\check{\omega} = \omega + is$ everywhere, except the positive half-axis $s > 0$, similar to the case of the conventional transfer functions of the quasi-stationary field [5, 6]. From this it follows that these transfer functions can be presented as

$$m(P, P', \tau) = \int_0^\infty m(P, P', s)\, e^{-s\tau}\, \mathrm{d}s \tag{9}$$

$$\tilde{m}(P, P', \omega) = \int_0^\infty \tilde{m}(P, P', s) \frac{1}{s + i\omega}\, \mathrm{d}s, \tag{9'}$$

where $\tilde{m}(P, P', s)$ is called the exponential spectrum of a quasistationary transient process [5].

(4) As noted in the very beginning, the processing problem is to determine the transfer functions relating to each other the variations of different field components in the time or frequency domain. Determining and analysing the frequency spectrum of the MT-field components proper falls outside the scope of the problem, but underlie its solution in the frequency domain when the algebraic equation (7) is to be solved. Strictly speaking, the set (7) may only be solved if we know the values of $\tilde{E}(P, \omega)$ and $\tilde{H}(P', \omega)$ at n sufficiently close frequencies where the frequency spectrum of the transfer functions varies little, while the frequency spectra of the field components vary pronouncedly. Since the spatial orientation of a strictly monochromatic field cannot be changed, the field polarization variations imply in the case of the plane-wave model that at least two sufficiently close frequencies exist in the field spectrum. This circumstance predetermines the highly jagged form of the field frequency spectrum. Historically, it has so occured that the processing problem was initially formulated in the frequency domain and the efforts of researchers in the field of MT-data processing were aimed at carrying out the spectral analysis of the field components, i.e. at solving the secondary, rather than principal, problem. The secondary problem proved to be not less complicated than the principal problem because the MT-field variations are highly unspecified. When treated as functions of time, they appear to be neither absolutely integrable nor square integrable and, therefore, the classical Fourier-transform formalism is inapplicable to them. This is why the researchers began paying their attention mainly to the methods of generalized harmonic analysis according to Wiener [7] on the assumption that the application of such analysis is justified by the fact that the observed realizations of the MT-field variations belong to a stationary ergodic process. We are of the opinion that this approach to the MT-variation processing involves essentially a nonverifiable hypothesis because we can never have an ensemble of realizations and must deal with but a single realization of a limited length. Division of the realization into parts is not a way out of the situation and entails only an increased volume of the material to be processed. It should be noted that the application of the methods of generalized harmonic analysis according to Wiener does not at all necessitate such a hypothesis and requires only [8] that the observed field variations should be functions of a finite mean power, i.e. that the limit

$$W = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} A^2(t)\, \mathrm{d}t < \infty \tag{10}$$

should exist and be finite. The existence of this limit gives rise to correlation functions and entails applicability of all the methods of generalized harmonic analysis. Such a hypothesis seems to be physically justified and may be verified using (10) on the intervals of duration 2T which are sufficient to obtain the frequency spectrum of the process. It is probably because the observed MT-field variations belong to the class of functions with finite mean power that the generalized harmonic analysis yields satisfactory results. The statistical approach introduces none other than the respective terminology to the processing operations. This is quite obvious because the detailed analysis of particular algorithms realized in terms of the methods based on the statistical approach has shown that they coincide essentially with the solution of the redundant system of algebraic equations (8) obtained in a deterministic way by the least squares method.

From the present-day point of view the Wiener generalized harmonic analysis is a particular case of the Fourier analysis being a consequence of the generalized function theory. This theory suggests abandonment of the knowledge of the function values $A(t)$ or $\tilde{A}(\omega)$ in every point, physically superfluous and unachievable practically, and to supersede it with the knowledge of certain linear continuous functionals (generalized functions) of the form $\langle A, \varphi\rangle = \int A(t)\, \varphi(t)\, \mathrm{d}t$ on a certain space s of the basic functions $\varphi(t) \in s$ being selected in such a way that the corresponding integrals may converge. The generalized function values $\langle A, \varphi\rangle$ are, roughly, the weighted values of the conventional function $A(t)$. These very integrals not infrequently are of interest when mathematically examining physical phenomena.

When the continuously differentiable functions, decreasing together with all the derivatives at $|t| \to \infty$ quicker than any power $1/|t|$, are selected as the basic functions, the integral $\int_{-\infty}^{\infty} A(t)\, \varphi(t)\, \mathrm{d}t$ converges for any $A(t)$ increasing as a power function at $|t| \to \infty$ and, in particular, for the arbitrary bounded functions $A(t)$, describing real MT-field variations. The Fourier transform of the generalized functions is defined on the basis of one of the modifications of the Parceval equality:

$$\langle A(t), \tilde{\varphi}(t)\rangle = \int_{-\infty}^{+\infty} A(t)\, \tilde{\varphi}(t)\, \mathrm{d}t = \int_{-\infty}^{+\infty} \tilde{A}(\omega)\, \varphi(\omega)\, \mathrm{d}\omega = \\ = \langle \tilde{A}(\omega), \varphi(\omega)\rangle, \tag{11}$$

where

$$\tilde{\varphi}(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varphi(x)\, e^{-ixy}\, \mathrm{d}x, \quad \varphi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\varphi}(y)\, e^{ixy}\, \mathrm{d}y.$$

If the functions $A(t)$ are non-integrable over $-\infty < t < \infty$, the spectrum $\tilde{A}(\omega)$ is the generalized function being determined from the functional equation (11), the basic functions $\varphi(t)$ and their Fourier transforms in (11) belonging to the space S of rapidly decreasing functions respectively in the axis t and ω. An important feature of the space of the generalized functions themselves S' (the linear continuous functionals $A_\varphi = \langle A(t), \varphi(t)\rangle \in S'$) is its completeness being understood in the sense that if the function sequence $\varphi_n(t) \in S$ converges to any function $\varphi(t)$ then the corresponding sequence of the generalized functions $A_\varphi = \langle A(t), \varphi_n(t)\rangle$, being determined by $\varphi_n(t)$, converges to A_φ. In the case of the regular generalized functions being determined by means of the integrals $\langle A(t), \varphi(t)\rangle = \int_{-\infty}^{\infty} A(t)\, \varphi(t)\, \mathrm{d}t$ the convergence of the generalized functions is the weak one of the conventional functions. The generalized function theory permits one to understand properly the periodogram technique, used to define the Fourier transform spectral evaluations of the realizations, as corresponding to functions $A(t)$ which do not decrease when $t \to \infty$, multiplied by the corresponding windows $K_n(t)$. It is known that the spectral evaluations of the functions $A(t) \cdot K_n(t)$ do not converge when the window is extended at $n \to \infty$ neither pointwise nor in the least squares sense. However they converge weakly in the space of the functions $\varphi(t) \in S$; that is, as the generalized functions do.

The generalized function theory entails a number of new spectral value determination techniques. In particular the spectrum of the bounded function $A(t)$, not decreasing at $t \to \infty$, may be defined in the generalized sense as the generalized second-order derivative of the function [9]:

$$\tilde{\Psi}_A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t) \frac{1}{t^2} \left[\frac{1 - i\omega t}{1 + \varepsilon t^2} - e^{-i\omega t}\right] \mathrm{d}t, \tag{12}$$

where $\varepsilon > 0$. $\tilde{\Psi}_A''(\omega)$ is a conventional Fourier transform for absolutely integrable functions $A(t) \in L_1$. For bounded functions $A(t)$, which do not belong to L_1, $\tilde{\Psi}_A(\omega)$ defines the spectrum $\tilde{A}(\omega)$ in the generalized sense:

$$\begin{aligned}\langle \tilde{A}(\omega), \varphi(\omega)\rangle &= \langle \tilde{\Psi}_A''(\omega), \varphi(\omega)\rangle = \langle \tilde{\Psi}_A(\omega), \varphi''(\omega)\rangle = \\ &= \int_{-\infty}^{\infty} \tilde{\Psi}_A(\omega)\, \varphi''(\omega)\, \mathrm{d}\omega = \int_{-\infty}^{\infty} A(t)\, \tilde{\varphi}(t)\, \mathrm{d}t = \\ &= \langle A(t), \tilde{\varphi}(t)\rangle. \end{aligned} \tag{13}$$

Supposing the weak convergence of the spectra is sufficient to solve practical problems, one may get the spectral value of the function $A(t)$ by means of the smoothing of the function $\tilde{\Psi}(\omega)$ and following double differentiation with respect to ω. For bounded functions, belonging to the special function type with finite mean power (10), it may be possible to define the generalized Fourier transform by means of the single differentiation in the sense of the distributions of the spectrum $\tilde{S}_A(\omega)$, derived by [7]:

$$\tilde{S}_A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t)\, K(\omega, t)\, \mathrm{d}t, \quad K(\omega, t) = \frac{e^{-i\omega t} - 1}{-it}, \tag{14}$$

but not by means of the second-order derivative of $\tilde{\Psi}_A(\omega)$ (12).

Thus, the generalized function theory permits one to define spectral values of the bounded functions on a deterministic basis without using their special statistical features. However, irrespective of a particular hypothesis underlying the generalized harmonic analysis and of a particular modification of the method used, the solution for the processing problem in the frequency domain seems to us to be faced with additional difficulties arising from the necessity that the variation spectrum should be determined beforehand. We do not consider this approach on the whole as having no prospects, but, nevertheless, prefer the temporal approach which makes it possible to find the transfer functions without making spectral analysis of the variations proper.

(5)Let us examine the problem of determining the transfer functions in the time domain firstly using for example, the scalar impedance relationship. In the frequency domain the relationship is of the form

$$\tilde{H}(\omega)\cdot\tilde{Z}(\omega)=\tilde{E}(\omega). \tag{15}$$

This pointwise equality in the frequency domain expresses the algebraic equation with respect to $\tilde{Z}(\omega)$. Even in the case where the functions $\tilde{H}(\omega)$ and $\tilde{E}(\omega)$ are the Fourier transforms of the functions $H(t)$, $E(t)\in L_1$, we cannot go directly over to the time-domain because $\tilde{Z}(\omega)$ rises at $\omega\to\infty$ as $\sqrt{\omega}$. Let us multiply both sides of (15) by the function $\tilde{K}(\omega)$, which belongs to the class of physically realizable functions, decreasing rapidly enough at infinity (for instance $\tilde{K}(\omega)\in S$) and is close to 1 in the frequency band of interest to us:

$$\tilde{H}(\omega)\cdot\tilde{Z}(\omega)\cdot\tilde{K}(\omega) = \tilde{E}(\omega)\cdot\tilde{K}(\omega). \tag{16}$$

Since the impedance $\tilde{Z}^*(\omega)=\tilde{Z}(\omega)\cdot\tilde{K}(\omega)$ decreases rapidly enough at $\omega\to\infty$, one may go over to convolutions in the time domain:

$$H(t)*Z^*(t) = \int_0^\infty Z^*(\tau)\cdot H(t-\tau)\,\mathrm{d}\tau = E^*(t), \tag{17}$$

where

$$Z^*(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{Z}(\omega)\cdot\tilde{K}(\omega)\;e^{i\omega t}\mathrm{d}\omega \qquad \text{and}$$

$$E^*(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{E}(\omega)\cdot\tilde{K}(\omega)\;e^{i\omega t}\mathrm{d}\omega = \int_0^\infty K(\tau)\,E(t-\tau)\,\mathrm{d}\tau.$$

Since $K(t)$ and $Z^*(t)$ rapidly decrease at $t\to\infty$, when $\tilde{K}(\omega)$ is suitably selected (it is sufficient that $K(t)$, $Z^*(t)=0(t^{-1-\alpha})$, $\alpha>0$) the convolution integral equation (17) should exist for any bounded functions. Let us select $\tilde{K}(\omega)$ as a sufficiently narrow-band function, then

$$\tilde{Z}(\omega)\,\tilde{K}(\omega-\omega_0) \approx \tilde{Z}(\omega_0)\cdot\tilde{K}(\omega-\omega_0)$$

and

$$K(t) = K_0(t)\, e^{i\omega_0 t}, \quad K_0(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{K}(\omega)\, e^{i\omega t} \mathrm{d}\omega$$

and now on the basis of (17) we shall obtain:

$$\tilde{Z}(\omega_0) \int_0^{\infty} K_0(\tau)\, e^{i\omega_0 \tau} H(t-\tau)\, \mathrm{d}\tau = \int_0^{\infty} K_0(\tau)\, e^{i\omega_0 \tau} E(t-\tau)\, \mathrm{d}\tau. \quad (18)$$

That is, the values of the variations $H_{\omega_0}(t)$ and $E_{\omega_0}(t)$ are filtered by means of $K(\omega - \omega_0)$ and at any instance of time are related by

$$\tilde{Z}(\omega_0) \cdot H_{\omega_0}(t) = E_{\omega_0}(t). \quad (18')$$

This equality is the basis of the technique of the variation narrow-band filtration.

One may not require the band of the filter $\tilde{K}(\omega - \omega_0)$ to be narrow to high degree. In this case it should approximate the impedance $\tilde{Z}(\omega)$ in the transmission band of the filter and with the desired accuracy by the truncated Taylor series, using the impedance analyticity in the vicinity of ω_0, $(\omega_0 - \omega > 0)$:

$$\tilde{Z}(\omega) = \sum_{n=0}^{\mathcal{N}} \frac{\tilde{Z}^{(n)}(\omega_0)}{n!\, i^n} (i\omega - i\omega_0)^n. \quad (19)$$

As a result the integral equation (17) goes again over to the algebraic one of the more general form:

$$\sum_{n=0}^{\mathcal{N}} \frac{\tilde{Z}^{(n)}(\omega_0)}{n!} H_{n\omega_0}(t) = E_{\omega_0}(t), \quad (19')$$

where $H_{n\omega_0}(t) = H(t) * K^{(n)}(t)$ is the variation $H(t)$, filtered by the filter $K^{(n)}(t) = \mathrm{d}^n/\mathrm{d}t^n K_0(t)\, e^{i\omega_0 t}$ with the frequency characteristic $(i\omega - i\omega_0)^n \cdot \tilde{K}(\omega - \omega_0)$. The equality (19′) should exist at any instant of time t and therefore permits one to obtain a redundant system of equations to determine $\tilde{Z}^{(n)}(\omega_0)$. The technique based on the solving of Equation (19′), is called "the generalized mathematical filtration method". As compared with the technique of the narrow-band filtration (18′), the more general many-term approximation of impedance is more attractive because it permits application of wider filters in the frequency domain and, hence, of shorter filters in the time domain. This circumstance leads to a more time-localized determination of the impedance characteristics and to a reduction of the total length of the variations necessary for processing. Another advantage of the method of generalized mathematical filtration is that it permits effective application of a priori information when approximating the impedance. The approximation (19′) allows for the impedance analyticity within a circle of radius ω_0 centered at the point ω_0 of the real axis ω. However this method permits application of other ways of approximating $\tilde{Z}(\omega)$ within the transmission band of the filter, for example (9′). In this case one may take into account that the impedance, in the conditions discussed above, belongs to an even narrower class of functions which are analytical on the entire complex plane $\check{\omega}$, except the negative imaginary half-axis.

The methods of the same kind result from the Parceval equality for the generalized functions (11). When integrating with respect to ω (16) we obtain:

$$\langle \tilde{H}(\omega) \cdot \tilde{Z}(\omega), K(\omega) \rangle = \langle \tilde{E}(\omega), K(\omega) \rangle \tag{20}$$

Let us suppose, for example, that $K(\omega)$ belongs to the space of integer functions, being finite in the time domain. If $\tilde{K}(\omega)$ is selected, as in the case (18), in the form of a function of sufficiently narrow bandwidth then, on the basis of (20) we have:

$$\begin{aligned}\langle \tilde{H}(\omega)\, \tilde{Z}(\omega), K(\omega - \omega_0) \rangle &= Z(\omega_0) \langle \tilde{H}(\omega), K(\omega - \omega_0) \rangle = \\ &= \langle \tilde{E}(\omega), K(\omega - \omega_0) \rangle\end{aligned} \tag{21}$$

according to (11) we have in the time domain:

$$Z(\omega_0) \cdot \int_T H(t)\, \tilde{K}_0(t)\; e^{-i\omega_0 t} \mathrm{d}t = \int_T E(t)\, \tilde{K}_0(t)\; e^{-i\omega_0 t} \mathrm{d}t, \tag{21'}$$

where T is the domain of the finite function $K(t)$. This equality is the basis for determining $\tilde{Z}(\omega)$ by means of the Fourier transform of the functions $H(t)$ and $E(t)$, multiplied by a time-window. In the U.S.S.R. the method is named "the method of momentary spectra". It is a complete analogue of the narrow-band filtration technique, in which the time window moves continuously along the axis t. Now it is not difficult to form, on the basis of (20) and (11), a corresponding analogue of the method of generalized mathematical filtration. The transfer function values in the frequency domain are directly determined by means of all these methods using transforms in the time domain.

(6) Now let us discuss the problem of direct determination of the pulse transient characteristics of the transform functions, a sufficiently wide-band filter $\tilde{K}(\omega)$ in (17) being selected for this aim. In this case one may consider $Z^*(t)$ as a regularized pulse impedance characteristic, the relationship (17) being considered as an integral equation for its determination. Now $Z^*(t)$ does not contain the unintegrable singularity at zero. It decreases sufficiently rapidly (not slower than $1/t^{3/2}$) at $t \to \infty$ that one may go over to the finite upper limit T:

$$\int_0^T Z^*(\tau)\, H(t-\tau)\, \mathrm{d}\tau = E^*(t) \tag{22}$$

This form of the integral equation with respect to $Z^*(t)$ is suited to solving the problem numerically. The transient functions $Z^*(t)$ being derived as a result may, without any difficulties, be transformed to corresponding frequency characteristics. A similar problem is investigated in [10]. However, from our point of view, they may properly become the basis of the following interpretation of MT-data in its transient version. The relevant calculations of such characteristics over horizontally layered models have demonstrated their higher resolving power than one with the frequency characteristics $\tilde{Z}(\omega)$. It may be expected they should have some advantages inherent to other electromagnetic methods, using transient fields, in the conditions of horizontally inhomogeneous media too.

Distortions of $Z^*(t)$ because of low-frequency filtration may be easily taken in to account in the frequency domain by means of the known frequency characteristic of the filter $\tilde{Z}(\omega) = \tilde{Z}^*(\omega)/\tilde{K}(\omega)$; however in the time domain they $(Z^*(t))$ may essentially be distinguished from $Z(t)$ at small time-values. In view of this circumstance it is more expedient to use here another method of the regularization of $Z(t)$ based on the formation of the space of basis functions which become zero at $t = 0$. Such functions may be formed simply on the basis of observed MT-variations. Actually, owing to both the linear relationship between field variations and its invariancy under an operation of time-shifting, one may use, as the kernel of the convolution operator and the right-hand side of the equation (7), the difference in the values between directly observed field variations and shifted ones in an arbitrary time interval T, rather than these variations proper. One may select these shifts so that the values $H(t)$ at the shifted points are equal and therefore $H(t) - H(t + T) = 0$. Then the convolution equation may be written:

$$\int_0^T Z(\tau)[H(t_j - \tau) - H(t_j + T_j^i - \tau] \, \mathrm{d}\tau = E(t_j) - E(t_j + T_j) \tag{23}$$

for all points t_j and shifts T_j^i, obeying these requirements (Figure 3). The singularity of the function $Z(\tau)$ at $\tau \to 0$ becomes integrable and the integrals (23) exist if the continuous differentiability of the variation $H(t)$ may be assumed. To solve equation (23) numerically it is sufficient to collect the required number of points t_j and shifts T_j^i; that seems to be always possible. Reducing the problem of determination of $Z(t)$ to (23) is an alternative way of regularizating the transfer functions as compared to (22).

It is necessary to note the particular features which reveal themselves as a result of sampling the initial data in equation (22). After sampling $H(t)$ and $E(t)$ in a step Δt their spectrum is known to be the periodic function (Figure 4) with the period $2f_m = 1/\Delta t$. As a result the spectrum of the sought transfer function (for example $\tilde{Z}(\omega)$) becomes periodic too with the same period, the transfer function proper

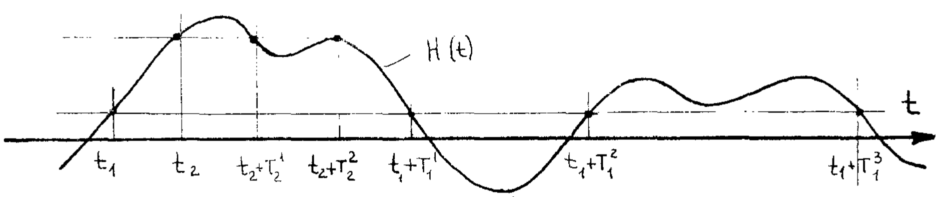

Fig. 3. The scheme for the formation of the basic functions, which have reverted to zero in any given time moment setting $t = t_k$.

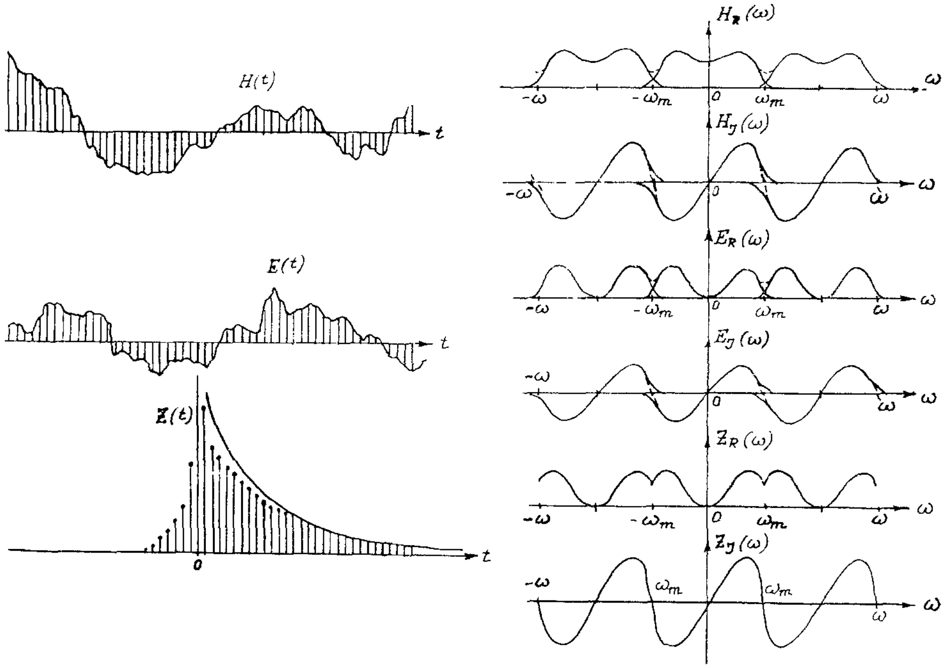

Fig. 4. Sampled field components $H(t)$ and their spectra together with the transfer functions, which correspond to sampled field variations in the frequency and time domains.

($Z(t)$) becoming also sampled. Though in this case non-integrable singularities of the function $Z(t)$ vanish, at small steps $Z(t)$ increases sharply and the values of the sampled function $Z_d(t)$ in these steps may not be equal to those of the initial continuous function in these points. As the spectral frequency band of the functions $H(t)$ and $E(t)$ is not bounded rigorously, the same differences arise between $\tilde{Z}_d(\omega)$ and $\tilde{Z}(\omega)$ at the frequencies in the vicinity of f_m and therefore bounding prefiltration of analog data by means of the filter $K(t)$ becomes additionally important. On the other hand one should take into account all these distinctions between initial and sampled values of the transform functions in interpretation. There is a special, fundamental, distinction for the function $Z^*(t)$ in the time domain. If bounding prefiltration is correct the values $Z_d(t)$ are equal:

$$Z_d(t) = \frac{1}{2\,\omega_m}\int_{-\omega_m}^{\omega_m} \tilde{Z}(\omega)\, e^{i\omega t}\,\mathrm{d}\omega = \frac{1}{2\,\omega_m}\int_{-\omega_m}^{\omega_m} (\tilde{Z}_R \cos\,\omega t - \tilde{Z}_J \sin\,\omega t)\,\mathrm{d}\omega \qquad (25)$$

where $\tilde{Z}_R = \mathrm{Re}\,\tilde{Z}(\omega)$, $\tilde{Z}_J = \mathrm{Im}\,\tilde{Z}(\omega)$, $\omega_m = 2\pi f_m$. At $t < 0$ this integral is not equal to zero in the general case because of the equality:

$$\int_{-\omega}^{\omega} (\tilde{Z}_R \cos\,\omega t + \tilde{Z}_J \sin\,\omega t)\,\mathrm{d}\omega = 0$$

for the initial function exists only when being integrated over $-\infty < \omega < \infty$. This denotes the sampled function $Z_d(t)$ is not causal. It must be taken into account when one solves the system of algebraic equations arising from sampling (22′).

(7) The Equations (22) and (23) too is an integral equation of the first kind and, therefore, may have no accurate solution and may prove to be non-unique or unstable. An accurate solution may not exist because of the experimental nature of the input data and due to the presence of noise in the kernel and in the righthand side $E^*(t)$. In such a case, it is necessary to seek a so-called quasisolution, i.e. such a function $Z^*(t)$ which, having been subjected to convolution with $H(t)$, gives a synthesized $E^*(t)$ value showing the minimum deviation (for example, in a quadratic metric) from the known right-hand side. The solution may be non-unique if the spectrum of its kernel is zero in some region. The non-uniqueness of the problem may be excluded if the solution allows for the availability of a priori information about the $\tilde{Z}^*(\omega)$ analyticity in the upper half-plane of the complex frequency $\check{\omega}$. The instability of the problem means that the small variations (i.e. noise) in the right-hand side of (22) ($E^*(t)$) or in the kernel of the equation $H^*(t)$ may give rise to substantial changes of the solution. Such a situation may arise when the kernel spectrum appears to be comparable in some region with the noise spectrum. In such cases the use of certain a priori information about the solution is the only cardinal way of improving its stability. From this viewpoint, if specific information about the examined geoelectrical profile is absent, our best knowledge concerning the sought function is that it can be represented, in the cases qualified above, through an exponential spectrum of the form (9). A less effective, but more general, method for regularizing the solution is to reduce the initial problem (22) to the problem of minimizing the smoothening functional according to Tikhonov [11]:

$$M(Z^*) = \|H(t) * Z^*(t) - E^*(t)\|_{L_2} + \alpha\Omega(Z^*). \tag{25}$$

The first term in the right-hand side of (25) designates the quadratic norm of the deviation of the electric field synthesized using a certain transfer function $Z^*(t)$ from the measured field. $\Omega(Z^*)$ is the stabilizing functional which in the simplest case is a norm of the deviation of the solution $Z^*(t)$ from a certain hypothesis substantiated by a priori information in the space of L_2 or W_2^1 (of the square integrable functions together with their first derivatives). The factor α designates parameter of regularization: as α increases, the solution is rendered more and more stable and, simultaneously, more smoothed and artificially forced towards the hypothesis adopted. The correct choice of the parameter α is of great importance when solving (25). The parameter must be sufficiently small in order that the discrepancy of the synthesis is comparable with the measurement error and must be sufficiently great in order that noise in the right-hand side of the equation does not affect the solution too strongly. By virtue of the broad dynamic range of the sought solution, a stable and, at the same time, non-smoothed solution may be obtained if the parameter α is a function of (t). The solution stability depends to a great extent on the form and spectral completeness of the function $H(t)$. The 'quality' of this function may be verified, for example, by the method of 'standard example'. The resultant approximate solution $Z(t)$ is taken to be an accurate solution and is used to synthesize, on the basis of a given $H(t)$ value, an electric field which is afterwards disturbed by an artificial noise comparable with measurement errors. Again, the integral equation is solved by the regularization method and a novel $Z(t)$ is found. If the latter differs from the initial function within admissible limits, the found solution is considered as suitable; otherwise, the solution is to be sought using another realization.

Additional and essential causes of the instability in the obtained solutions arise from the vectorial character of equation (7). The main problem which has to be solved in this case is to find the necessary set of basis functions in (7). First of all, the basis functions should be independent in the sense of (3), i.e. none of them can be found from others using a convolution with some transfer functions. More strictly, if such transfer functions have nevertheless been found in a certain realization, their values will be substantially different in another realization. In the frequency domain the equality (3) is equivalent to a low coefficient of multiple coherence of one of the functions with respect to the rest of them. The second requirement is that their number should be such that the transfer functions found stably from (7) would cause a small discrepancy in the righthand side of the equation. In practice, the concrete choice and determination of the number of basis functions are made when solving (7). The solution for (7) is sought by successively raising the number of basis functions from $n = 1$. In this case the synthesis discrepancy must decrease monotonically, while the solution stability can deteriorate. The addition of a linearly-dependent function to the basis leads only to a rapid rise of instability without reducing the synthesis error. The basis is regarded as chosen when a stable solution can be obtained with a synthesis discrepancy comparable with the measurement error of the right-hand side.

(8) A brief description of algorithms of the integral equation and the generalized mathematical filtration methods is given hereafter. We do not describe an algorithm of generalized harmonic analysis method because different modifications of this method are presented in [12]. We suppose that selection of field realizations satisfying a two-component system of linear relationships among the MT-field components was carried out beforehand. As an example we consider the problem of impedance tensor components determination. The processing algorithm for the integral equation method involves the steps:

(a) MT-field component variations are filtered by a rather wide-band filter $K(t) \div \tilde{K}(\omega)$

$$E_{ij}^{*}(t) = E_{ij}(\tau) * K(t-\tau), \quad i=j=1,2=x,y.$$

After this the problem is reduced to one of seeking the filtered pulse transfer function impedance tensor components $Z_{ij}^{*}(\tau)$ from a vector integral equation of the form:

$$\int_{0}^{T} \sum_{i=1}^{2} Z_{ij}^{*}(\tau) H_j(t-\tau)\, \mathrm{d}\tau = E_j^{*}(t), \quad j=x,y=1,2, \quad t_1 \leqslant t \leqslant t_2 \qquad (26)$$

(b) After replacing the integrals with the integral sums the system of linear algebraic equations for values of sought functions $Z_{ij}(\tau_K)$ on a given time set $\tau_1, \tau_2, \ldots \tau_{LS}$

$$\sum_{K=1}^{LS} \sum_{i=1}^{2} Z^*_{jiK} H_{Kli} = E^*_{jl}, \quad i = x, y = 1, 2, \quad l = 1, M \tag{27}$$

is established. Here $\{H_{Kli}\}^{LS\,M}_{K=1, l=1}$, $i = x, y$ is matrix with the elements being determined by sampled magnetic field variations $H_i(t)$, $i = x, y$; E^*_{jl} is a column vector with elements being determined by sampled electric field variations $E_j(t)$ filtered by means of a filter $K(t)$; LS and M are the lengths of a sought solution and field realization expressed in sampling intervals accordingly.

(c) The algebraic equation system (27) of dimensionality $M \times 2LS$ for $j = x, y$ is solved by the regularization technique. The solution is thought to be acceptable if the discrepancy of the system (27) solution does not exceed a given threshold value.

(d) The Fourier transforms of found functions $Z^*_{ij}(t)$ are calculated and normalized by the filter frequency characteristic $\tilde{K}(\omega)$.

The MT-data processing algorithm based on the generalized mathematical filtration method is:

(a) In accord with (19′) one may form the redundant linear algebraic equation system for coefficients of a Taylor series of scalar impedance. In the tensor case one sets up a similar algebraic equation system to the following

$$\sum_{K=1}^{\mathcal{N}} \sum_{i=1}^{2} a_{jiK} P_{Kli} = E^*_{jl}, \quad j = x, \quad y = 1, 2, \quad l = 1, M, \tag{28}$$

where

$$\{P_{Kli}\}^{\mathcal{N}}_{K=1}, {}^{M}_{l=1}, \quad j = x, y$$

is a matrix with elements determined by sampled values of magnetic field variations $H_i(t)$, $i = x, y$ filtered by means of the derivatives of the bandpass filter $K_\Delta(t)$ of K-order; E^*_{jl}, $l = 1, M$, $j = x, y$ is a column vector with elements determined by sampled electric field variations $E_j(t)$, $j = x, y$ being filtered by means of the bandpass filter $K_\Delta(t)$, a_{jiK} are the sought coefficients of a Taylor series of impedance tensor elements Z_{ji}, M — the length of realization, expressed in sampling intervals, N — the number of terms in a truncated Taylor series.

(b) The equation system (28), of dimensionality $M \times 2N$ for $j = x, y$ is solved by the regularization method. The solution is acceptable if the discrepancy of the system (28) solution does not exceed a given threshold value σ.

(c) The coefficients a_{jiK}, which have been found from the equation system are used to determine the elements of the impedance tensor Z_{ji} in the range $[\omega_0 - \Delta, \omega_0 + \Delta]$ in accord with the expression:

$$Z_{ji}(\omega) = \sum_{K=1}^{\mathcal{N}} a_{jiK}(i\omega - i\omega_0)^K, \quad i, j = x, y. \tag{29}$$

The solution is repeated in a given set of the frequencies ω_0.

(9) For an estimation of the processing errors one sets some noise in the right sides of equations (27) or (28) and solves them several times.

The power of the noise adopted is approximately equal to the power of the solution discrepancy for the assumed problem without noise. For the noise the function of the form

$$S(t) = \sum_{i=1}^{I} a_i \sin(\omega_i t + \varphi_i) \tag{30}$$

is used. Here a_i are given coefficients (for instance $a_i = 1$, $i = 1, 2, \ldots, T$), ω_i — is a set of frequencies within the frequency bound of the variations, φ_ι is the set of random numbers distributed in accordance with some law (for instance, uniformly). Specifically ω_i may be distributed uniformly in a linear scale (white noise) or in a logarithmic one (logarithmic noise). The latter maintains an equal perturbation of observed MT-field variations in relatively equal intervals of the frequency band.

The envelope of the found family solutions determines the allowable interval of solution variation.

To estimate accuracy, robustness and resolving power of the different processing programs a system of tests are designed, using a set of calculated field variations for a given media with a known impedance. As a model for time variations of the magnetic field the function

$$H(t) = \sum_i \alpha_i \varphi_i(t) + S(t) \tag{31}$$

is used. Here $\varphi_i(t) = (t/q_i)^2\, e^{-t/q_i}$; $S(t)$ — is the function, determined by (30); α_i; q_i — are given numbers. The electric field variations are calculated as a convolution of the media transfer function with the magnetic field.

Next we shall demonstrate two tests to analyse the integral equations method. In the first test (test B) the impedance of a 1-D medium is used. The model consists of two S-planes (the planes with a given longitudinal conductivity S_1, S_2), the separation S_1, from S_2 is d. This test is used for scalar problems. The second test (test D) is applied to tensor problems.

The impedances of two different 1D-media as well as 2D-media derived from numerical modelling of some standard geoelectric profiles are used as impedance tensor elements.

(10) Now we present some results of processing model and field MT-data. In Figure 5 the model variations of $H(t)$ and $E(t)$ (test B) and the realization of a noise signal (of the type (30)) are shown. The noise signal is selected so that the spectral properties of the noise and of the observed field $E(t)$ are close to each other. Families of solutions of the scalar problem of test B processing for 20%

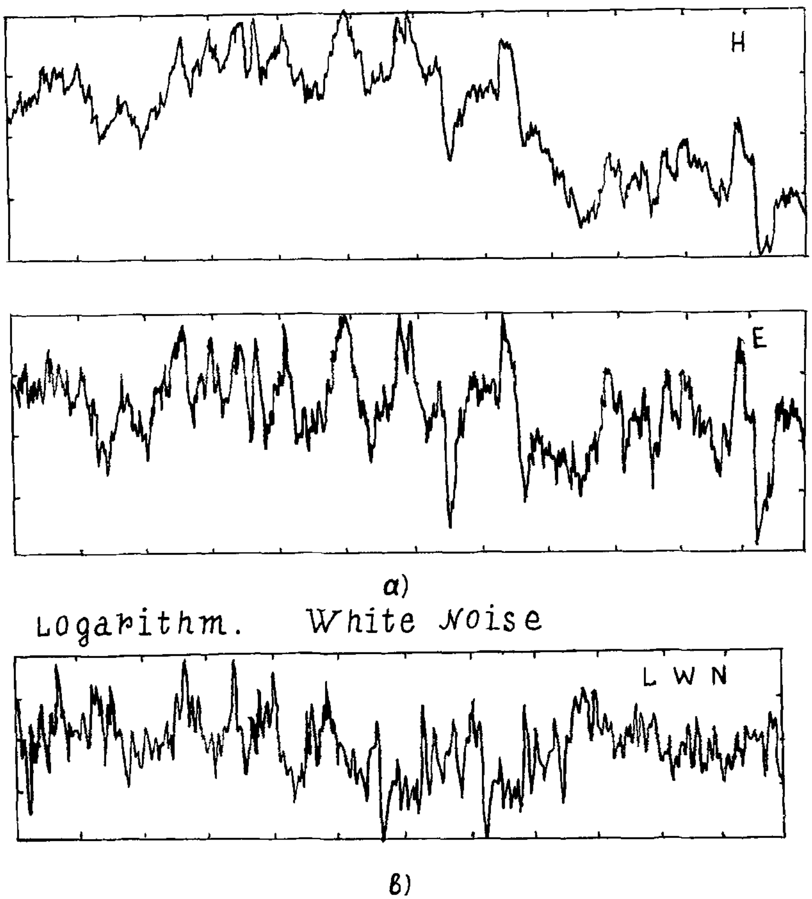

Fig. 5. The test B (the scalar problem) (a) MT-model variations $H(t)$ in the form of (30) and $E(t)$, (b) a realization of a logarithmic white noise.

noise as well as the theoretical impedance curve are shown in Figure 6. It is seen from Figures 5 and 6, that consistent results (for the modulus) are obtained over the range of the periods from $T_{min} = 30\ dt$ to $T_{max} = LR$, where $LR = 1024\ dt$ is the realization duration and dt is a sampling interval. The appropriate range for the phase is $T_{min} = 60\ dt$, $T_{max} = LR$. Beyond the ranges the processing errors rapidly rise.

In figure 7 the model variations of test D are presented. The impedances of two different 1D-models built up from two S-planes are used as the impedance tensor elements. The results of the processing of the model variations of test D are shown in Figure 8. The case (a) is without noise and (b) is with 15% noise in the field $E(t)$. For comparison the results of processing the same data by means of the generalized harmonic analysis method are also given. The worse determination of the impedances Z_{xx} (especially when noise is added) is explained by the fact that the values Z_{xx} are an order of magnitude below Z_{xy}.

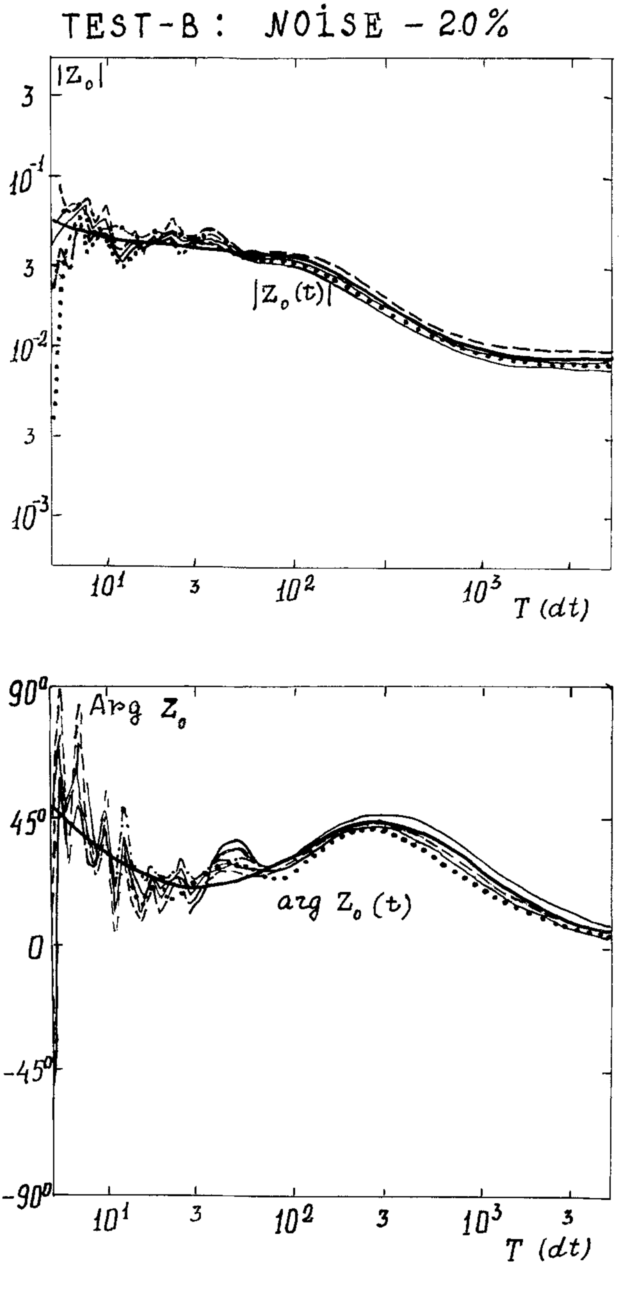

Fig. 6. The family of curves resulting from integral equation method procesisng of the model variations (the test B) in cases of different 20% noise in the field $E(t)$. The sampling interval is $dt = 1$; the duration of the realization $LR = 1024$ dt, the length of the sought transfer characteristic is $LS = 80$ dt. $|Z_0(T)|$, $\arg Z_0(T)$ are the exact theoretical curves.

FILE : TEST - D3

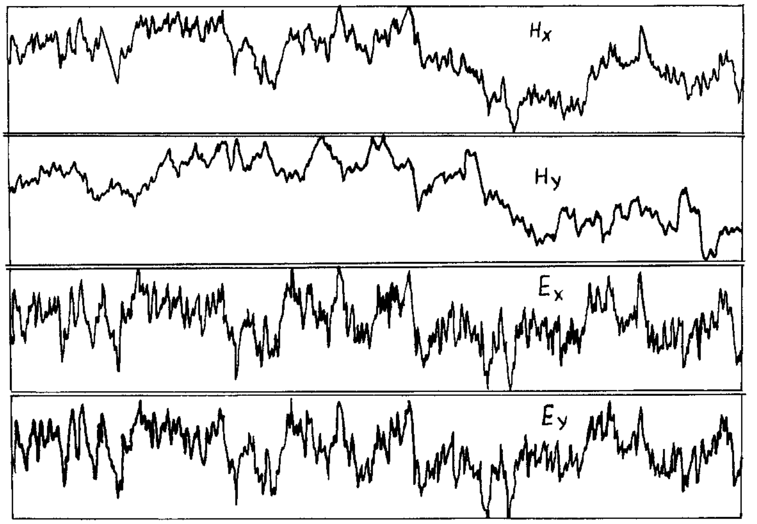

Fig. 7. The test D-3 (the tensor problem) MT-model variations $H(t)$ in the form of (30) and ($E(t)$, $dt = 61\ s$; $LR = 1024\ dt$.

The presented results show that the integral equation method allows the processing for significantly lower frequences than traditional spectral techniques, the maximal determined period reaching $(0.5–1.0)\,LR$. Within the range $T = 10\ dt \div LR/10$ the spectral and the integral methods are in good agreement.

Realization of experimental field variations at a point in Bulgaria is shown in Figure 9a. In Figure 9b impedance curves, which are the result of the processing of this single realization by the integral equation method, are presented. The interval of possible discrepancy is shown by dotted lines. It is obtained by means of disturbance of electric field variations by 25% noise (see previous point). This interval is characterized by variable reliability of the different transfer functions over different ranges.

In Figure 10 the results of the processing of some realizations of experimental field variations by the integral equation and generalized harmonic techniques are given. They confirm the model results: the integral equation method supplies reliable results within the range of periods $T_{\min} = 15\ dt$, $T_{\max} = (0.5 \div 1.0)\,LR$, for the generalized harmonic method this range is $T_{\min} = 10\ dt$ and $T_{\max} = LR/10$. It is of interest that beyond these ranges discrepancy of the curves, determined from different realizations, rises (see Figure 10a). In Figure 11 are the averaged impedance curves, which are obtained by three different processing methods on the same ensemble of realizations of equal durations (17 h). Curve 1 corresponds to the integral equations method, the curve 2 was derived by the method of generalized mathematical filtration and curve 3 was obtained by the generalized harmonic analysis. Comparison of the curves shows that on the one hand the results of

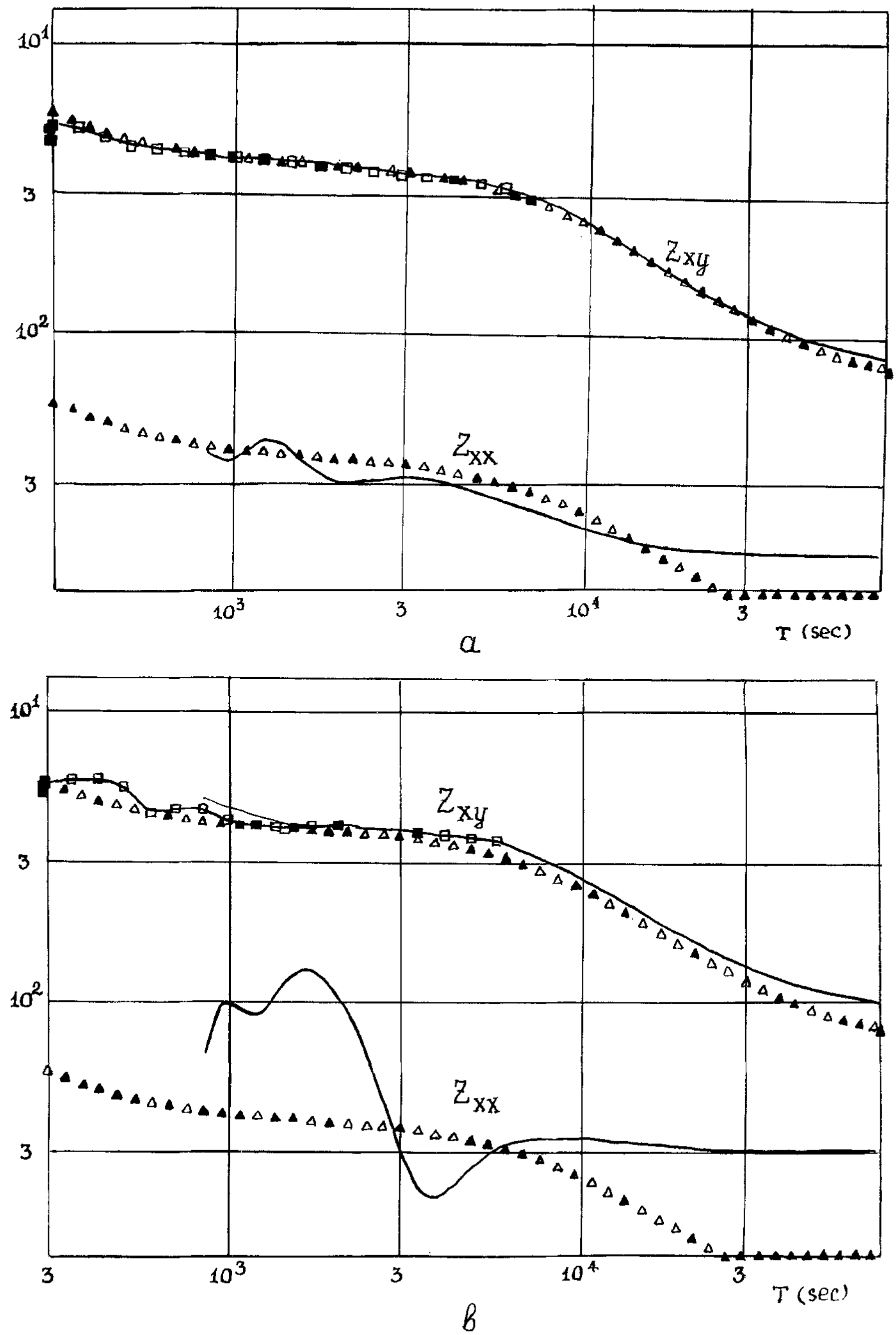

Fig. 8. Processing results of test D-3 by means of integral equation (IEM) and generalized harmonical (GHAM) methods (a) undisturbed model field variations; (b) model field variations disturbed by 15% noise; $LR = 1024$, (△ △ △ △) are the exact theoretical curves: (——) is for IEM; (□□□) is for CHAM.

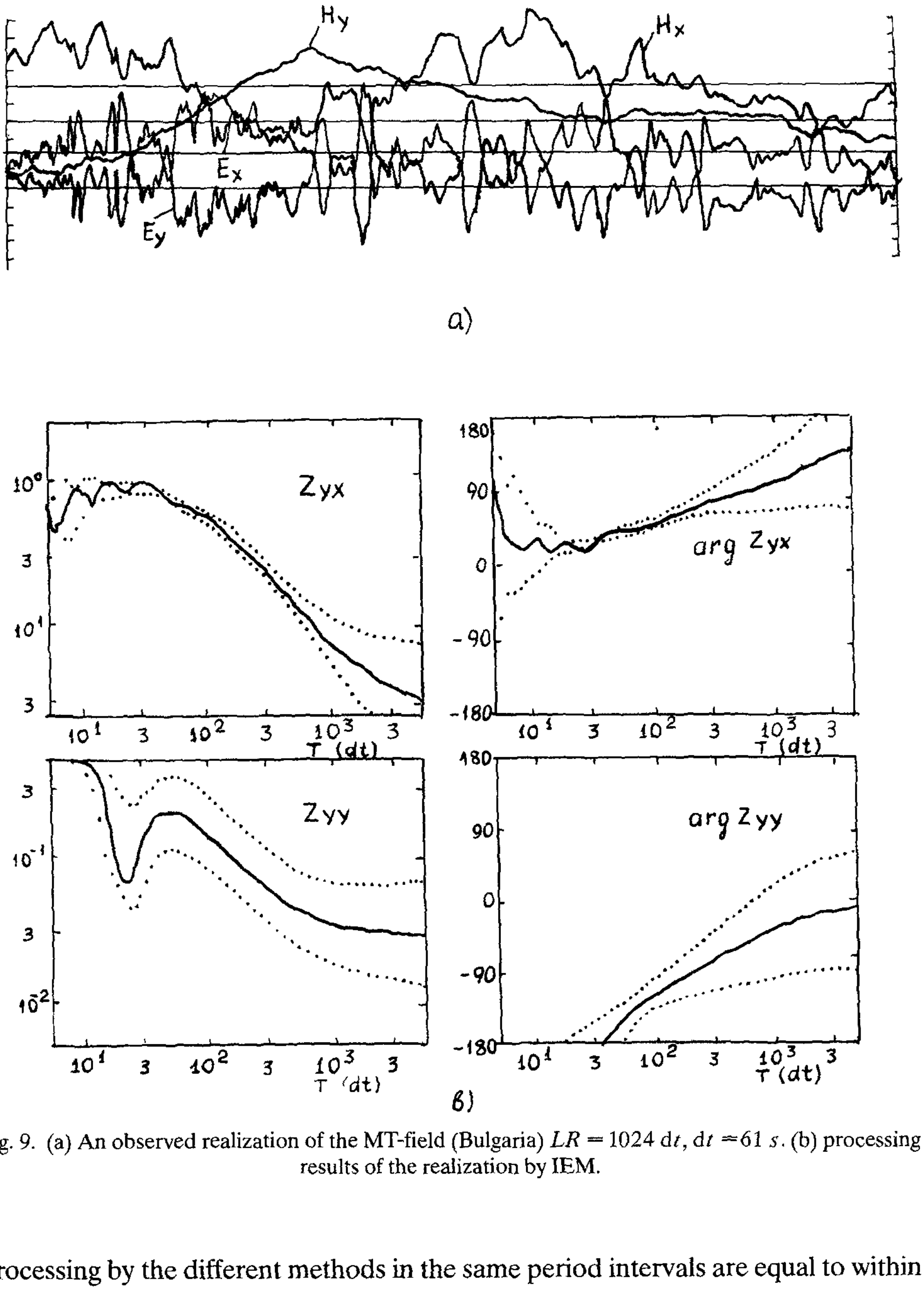

Fig. 9. (a) An observed realization of the MT-field (Bulgaria) $LR = 1024\, dt$, $dt = 61\, s$. (b) processing results of the realization by IEM.

processing by the different methods in the same period intervals are equal to within 10%. On the other hand the maximal derived period sequentially rises when one goes from the method of the generalized harmonic analysis to the integral equation method: $T_{\max 3} < T_{\max 2} < T_{\max 1}$.

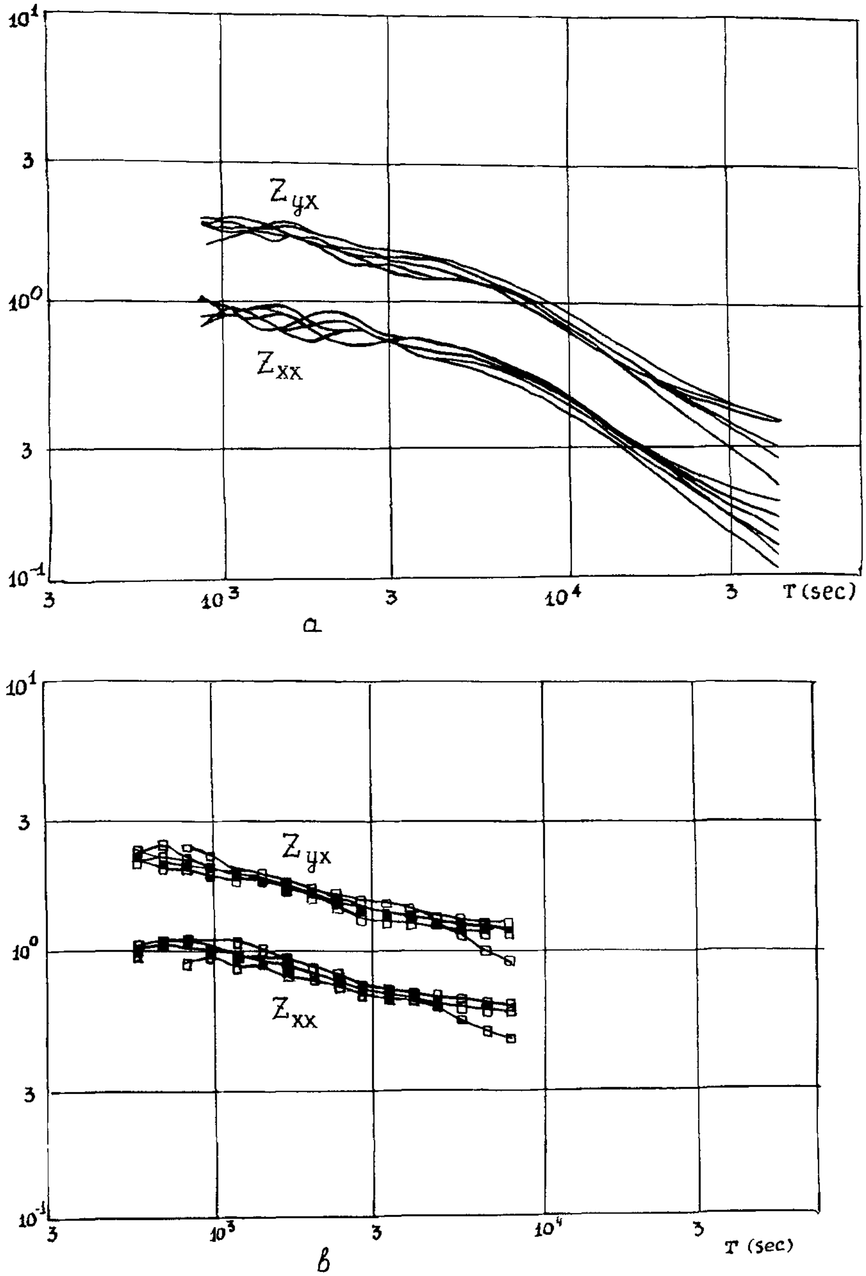

Fig. 10. Processing results of some observed realizations at one point in Bulgaria: (a) IEM; (b) GHAM.

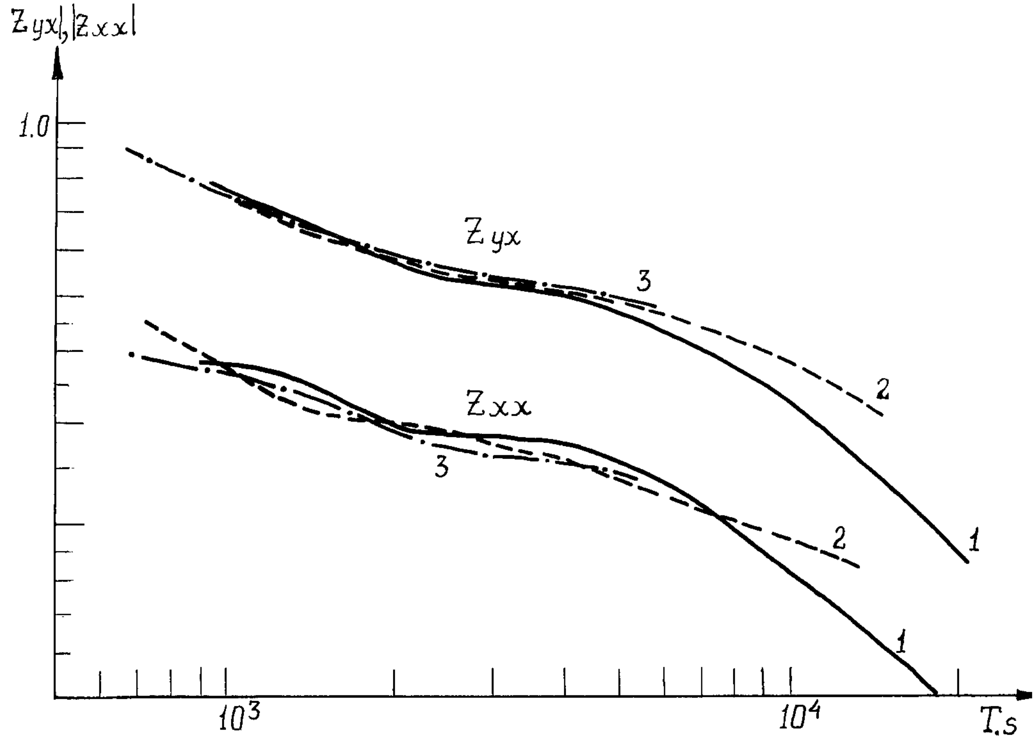

Fig. 11. Comparison of MT-processing results by means of integral equation (1), generalized mathematical filtration (2), and generalized harmonic analysis (3) methods.

Conclusion

(1) One of the main ways of improving accuracy of MT-data processing is by decreasing errors bound up with time variations in the field exitation model. One way of decreasing errors is by changing from ensemble field realization averaging of the transfer functions (that is necessary in statistical approaches of processing) to transfer functions determined from an isolated realization satisfying a system of two-component linear relationships of the MT-field, corresponding to uniform plane waves. The stage of selection of such realizations comes before transfer function determination.

(2) On the basis of the deterministic approach to MT-data processing involving apriori information on the transfer functions, it is possible to extract rather efficiently meaningful information from isolated realizations of MT-variations. In particular the integral equation and generalized filtration methods allow the transfer functions to be found for periods comparable with the realization duration.

(3) The estimation of errors in the determined transfer functions from isolated realization may be based on repeated exitations of assumed data by artificial noise, which models the real disturbances of the data. Comparing the MT-field com-

ponent variations to each other by given transfer functions along given geoelectric profiles allows us to judge in an unbiased way the processing programs used from different viewpoints (errors, frequency band, resolving power, etc.).

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