GLOBAL ELECTROMAGNETIC INDUCTION

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Abstract. Methods of analysis of long period geomagnetic variations (periods over a few hours), the available electromagnetic response function estimates, and the effect of lateral inhomogeneity within the Earth are reviewed. Recent advances in the inversion of response function data to produce conductivity-depth profiles are mentioned, and aspects of the inverse problem specific to global (spherical Earth) induction are discussed.

There is a rapid rise in electrical conductivity between about 400 km and 800 km, but whether this is a gradual change or consists of one or several steps is not resolvable with the presently available data and naive inversion algorithm used here. At the greatest depths to which presently analysed variations penetrate (1000–1500 km), available data have some difficulty in resolving finer structure, but there are indications that the electrical structure of the continents becomes more laterally homogeneous as depth increases. Previously published inversions see lateral variations in electrical structure down to at least 500 km, and at shallower depths these variations are unambiguously resolved.

1. Introduction

Varying electric currents in the Earth's magnetosphere and ionosphere produce a magnetic variation field at the Earth's surface. This externally produced variation field (e) induces electric currents within the Earth, which in turn produce secondary magnetic field variations (i) measurable at the Earth's surface. The electromagnetic response i/e is dependent upon both the form (spherical harmonic composition) of the external variation field involved, and upon the electrical structure of the Earth. The response at different frequencies contains information about the electrical structure at different depths. Thus, if we have knowledge both of the form of e and of the response, we can deduce information about this structure.

The first question that must be approached is what is meant by 'global' electromagnetic induction. It is used to be believed that at comparatively short periods (a few hours) the structure of the deeper Earth and the spatial scale lengths associated with variations of these periods were such that the Earth would behave as a radially symmetric sphere, and thus a single 'global' response function could be defined. As data quality and analysis procedures have improved, it has become clear that such an assumption is not valid, certainly not for periods shorter than some days. Thus, I have chosen to define 'global induction' as that produced in the Earth by fields of global scale (primarily solar quiet (*Sq*) and storm time (*Dst*) fields), generally analysed using data from a globally distributed array of geomagnetic observatories. Such analyses provide information primarily about the electrical conductivity of the Earth's upper mantle at depths between 200 and 2000 km.

This is intended to be a critical review so that although many of the assumptions and flaws discussed are of small significance where first used, they are mentioned as

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they could be of greater significance if a similar analysis procedure were used inappropriately elsewhere. Needless to say, it is the better analyses which are criticized the most.

1.1. Basic Theory

The theory of electromagnetic induction in a (radially symmetric) sphere has been discussed in detail by many previous authors (e.g., Banks, 1969; Rokityanski, 1982), and only a very brief outline is given here.

The three orthogonal components of the magnetic variation fields at the Earth's surface (B_r, B_θ, B_φ) can be written in terms of i and e as (Rokityanski, 1982; Jady and Marshall, 1984)

$$\begin{aligned} B_r &= -\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[ne_n^m - (n+1)\,i_n^m]\,p_n^m(\cos\theta)\,e^{im\phi} \\ B_\theta &= -\sum_{n=1}^{\infty}\sum_{m=-n}^{n}(e_n^m + i_n^m)\frac{\mathrm{d}P_n^m(\cos\theta)}{\mathrm{d}\theta}\,e^{im\phi} \\ B_\phi &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n}(e_n^m + i_n^m)\frac{P_n^m(\cos\theta)}{\sin\theta}\,im\,e^{im\phi} \end{aligned} \tag{1}$$

where r, θ, and ϕ are radius, co-latitude and longitude respectively, and e_n^m and i_n^m are the parts of the external and internal variation fields corresponding to a spherical harmonic of degree n and order m. P_n^m is an associated Legendre function. B_r, B_θ, B_ϕ, e_n^m and i_n^m may be expressed as either the frequency components of the variation fields, or the values at a given instant of time.

If we have a sufficient geographic density of measurements of the three magnetic components on the surface of the Earth then we can conduct a spherical harmonic analysis of the variation fields and thus obtain the parts of the variation fields associated with each constituent spherical harmonic, e.g.,

$$B_{rn}^m = -[ne_n^m - (n+1)i_n^m]P_n^m(\cos\theta)e^{im}{}_\phi. \tag{2}$$

Scaling with respect to latitude and dividing the scaled vertical component by the scaled northward component we obtain the response (Banks, 1969)

$$W_n^m = \frac{(ne_n^m - (n+1)\,i_n^m)}{(e_n^m + i_n^m)} \tag{3}$$

which is simply related to the ratio of internal and external parts (Q_n^m)

$$Q_n^m = \frac{i_n^m}{e_n^m} = \frac{n - W_n^m}{(n+1) + W_n^m}. \tag{4}$$

A similar expression to Equation (3) can be obtained by taking the ratio of the vertical and azimuthal components.

The response functions thus calculated will in general be different for different spherical harmonics because of the different spatial wavelengths of the inducing fields, but for a radially symmetric Earth the derived Earth models are simply related (Weidelt, 1972).

At angular frequency ω for zonal harmonics the response function W_n is related to the admittance $C_n(\omega)$ (Weidelt, 1972), the impedance $I_n(\omega)$ and apparent resistivity $\varrho_a(\omega, n)$ via

$$C_n(\omega) = \frac{aW(\omega)}{n(n+1)} \tag{5}$$

$$I_n(\omega) = i\omega\mu_0 C_n(\omega) \tag{6}$$

$$\varrho_a(\omega, n) = \frac{|I_n(\omega)|^2}{\omega\mu_0} \tag{7}$$

where a is the radius of the Earth, and μ_0 the permeability of free space.

In practice we have only (noisy) estimates of the field at a limited number of geographic locations (geomagnetic observations). This means that we can fit only a limited number of spherical harmonics to the data, and that these harmonics are not orthogonal. However, at mid-latitudes the spatial form of the inducing fields at the periods of interest are such that these models of the fields are believed to be adequate (see below).

If the Earth is significantly laterally inhomogeneous then 'mode mixing' can occur; i.e., a P_n^m external field can produce a whole (spatial) spectrum of internal fields, which severely complicates the analysis. Response functions relating the various field components at the Earth's surface can still be derived, and still contain useful information about the Earth's deep structure, but care is required in deducing the Earth structure from such data. This problem is discussed in detail below.

2. The Available Response Function Data

Figure 1 (from Filloux, 1980) shows part of the geomagnetic spectrum. The periods of interest here are those labelled 'solar quiet' and longer. It can be seen that the spectrum rises more or less continually as period increases. At very long periods the externally produced part of the spectrum is swamped by the internally produced 'secular variation'. It is possible to derive estimates of lower mantle conductivity from analyses of secular variation by making severe assumptions about the form of magnetic variation fields at the core-mantle boundary (e.g., McDonald, 1957; Currie, 1968). Less model dependent results are obtained from the rapid secular acceleration impulse ('jerk') Of 1969–70. Such analyses are difficult because of the poor quality of much of the very long period geomagnetic observatory data, and other factors. Most such 'jerks' can in fact be shown to be of external origin, and are related to the solar sunspot cycle (see e.g., Srivastava and Abbas, 1984).

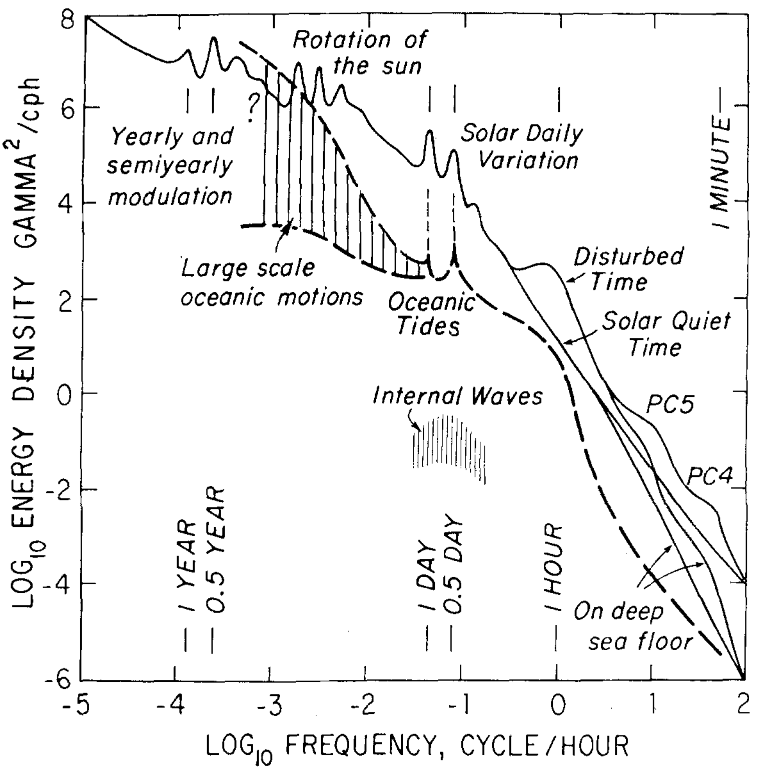

Fig. 1. Spectra of natural magnetic fluctuations on Earth at mid latitudes, for both quiet and disturbed times, on the surface and at the ocean floor. The level of signals related to oceanic motions is also shown (from Filloux, 1980). Of primary interest here is the externally produced part of the variation field (solid line) at periods over a few hours.

Statistical analyses of the 1969–70 jerk (Achache *et al.*, 1980; Malin and Hodder, 1982) indicate that it is of internal origin. However, there is still some controversy about this conclusion due to the jerk's coincidence with sunspot maximum and the surprisingly low lower mantle conductivities ($\approx 300\ \Omega^{-1}\ m^{-1}$) which seem required to explain the data. While Backus (1983) shows that such low conductivities are not required near the core-mantle interface, the work of Le Mouel and Courtillot (1984) suggests that the average conductivity of the lower mantle must still be lower than was previously believed, if the jerk is indeed of internal origin. These results will not be further considered in this review.

Figure 1 shows that between periods of a few hours and several years the geomagnetic spectrum consists of a steadily rising 'continuum', with a series of lines superimposed at 3, 6, 8, 12, and 24 hr; 9, 13.5, and 27 days; and 6 and 12 months. In fact there is also a spectral peak at 11 yr, but this is not clear in the figure. Various workers have found additional spectral peaks at periods from 2 to 60 yr. Many of these seem to be products of the analysis procedure used, or poor baseline control, and even if they are real no separation of internal and external parts has been achieved. Thus in 'global' induction studies we are restricted to periods from a few hours to 11 yr. Many response function estimates have been published, especially for the daily variations. The following discussion is not intended to give an exhaustive list of the available data, but rather to consider the merits of the various analysis techniques.

2.1. Analysis of Daily Variations

Much has been written about the analysis of S ('solar') and L ('lunar') daily variations, which are sometimes refered to as 'geomagnetic tides' (Winch, 1981 discusses 17 analyses prior to his own). S is primarily due to the daily modulation of the properties of the upper atmosphere caused by the rotation of the Earth with respect to the Sun. L is due to the tidal influence of the Moon upon the hydrosphere and atmosphere.

The variations most commonly used in induction studies are called 'solar quiet' or Sq. These are regular variations which dominated the surface magnetic variation field at mid-latitude during times of low magnetic activity. The electric currents producing Sq flow mainly in the ionosphere, although there is also a contribution from the magnetosphere. The equivalent current system for Sq in the day hemisphere basically consists of two current vortices, one in the northern, and one in the southern, hemisphere, that in the 'summer' hemisphere being the larger. In the night hemisphere there is a similar equivalent current system of smaller amplitude. Obviously then. the Sq fields at the Earth's surface have a fairly complex spatial structure, and spherical harmonic models of degree up to at least four or five are required to describe them adequately.

In fact the ionosphere and magnetosphere, and the associated magnetic variations fields are 'never truly quiet but rather at different degrees of restlessness' (Campbell, 1977). Thus the Sq field on any day contains irregularities. It is usual to attempt to reduce the effect of the irregularities, and the consequent instability in the spherical harmonic analysis (SHA), by averaging the Sq field over time. Monthly means (i.e., the average of five International Quiet Days) have sometimes been used, but it seems that averaging over longer periods may really be necessary (see Parkinson, 1971). In their thorough analysis, Malin and Gupta (1977) used the 8 most 'quiet' days during the IGY. Matsushita and Maeda (1965) chose 15 quiet days from each of the three 'Lloyd seasons'. The longitudinal averaging scheme described below can also help to smooth out such irregularities in Sq.

The most used method of analysis of Sq data for use in induction studies seems to be first to separate the frequency components of the data at each observatory by using an expression of form

$$Sq = \sum_{p=1}^{4} \left[k_p \cos\left(\frac{2\pi}{24} pt\right) + l_p \sin\left(\frac{2\pi}{24} pt\right) \right] \tag{8}$$

where t is universal time (UT) (see e.g., Malin and Gupta, 1977; Rokityanski, 1982, who also produced contour maps of k_p and l_p).

Before applying Equation (8) to the data a baseline must be chosen. The daily mean value is often used (implicitly), but Malin and Gupta recommend the use of midnight values.

The data must then be subjected to a spherical harmonic analysis (SHA). Equatorial stations must generally be excluded because the equatorial electrojet produces fields of short spatial wavelengths unamenable to a limited SHA. For similar reasons auroral stations are sometimes excluded. One SHA scheme is that suggested by Schuster (1889) which assumes that the *Sq* field is a function of local time (LT) only, and not of UT. Under this assumption the data can be scaled to a common longitude. This introduces an error due to the partial UT dependence of the data, but separates the symmetric part of the variations, and greatly stabilizes the SHA. The vast majority of workers conduct an SHA based on least squares. An alternative is to interpolate the field between stations and integrate over a sphere. Fainberg (1983) suggests that the results from the latter technique may be superior. If this is the case it is presumably because of the additional *a priori* information provided by the form of interpolation chosen.

An alternative to the Schuster scheme is to conduct a full SHA of the globally distributed area, using either the Fourier coefficients or fields values at successive instants of UT. The results from such analyses can be unreliable. Berdichevski *et al.* (1976) demonstrated that the method is very sensitive to the number and location of observatories used, and to the spherical harmonic model chosen. With a large number of roughly evenly geographically distributed observatories, and with the use of pilot analyses to discover the optimum spatial spectrum, these problems can be greatly reduced (e.g., Malin and Gupta, 1977, who in addition also used appropriate weighting for the data from each observatory). However, the study of Hobbs *et al.* (1981) reaffirmed the extreme sensitivity of the spherical harmonic model obtained upon the distribution of observatories used.

Whatever method of analysis is chosen, some appropriate coordinate system must be selected prior to SHA. Geographic or geomagnetic coordinate systems can be used, but because of the restricted SHA possible, better results are apparently obtained if special coordinate systems related to the spatial form the *Sq* variations are used. Berdichevski *et al.* (1972) and Borisova (1973) used a coordinate system derived from the isolines along which the form and amplitude of the *Sq* variations varied least, which isoline a particular observatory belonged to being determined by visual inspection of data and by $B_r\!:\!B_\theta$ correlation.

A dip-latitude coordinate system was used by, for example, Matsushita and Maeda (1965). Here latitude is substituted by dip-latitude found from the expression

$$\tan \theta_d = \frac{1}{2} \tan I \tag{9}$$

where I is magnetic dip.

The major terms in the spherical harmonic description of Sq are P_{m+1}^m, where $m = 1, 2, 3, 4$ for frequency 1, 2, 3, 4, cpd: and during the equinoxes when the current systems can be expected to be almost symmetric about the equator the field is well represented by these terms alone. If we assume that the field is completely described by one spherical harmonic, then we can define an 'Sq-effective' coordinate system from the axes of the horizontal polarization ellipse, in which the Northward and Eastward components are in quadrature (Schmucker, 1974). The latitude in this coordinate system is given by

$$\tan^{-1}\left(\frac{X_0}{Y_0}\right) = \cos\theta_e - \frac{\sin\theta_e \tan\theta_e}{m} \tag{10}$$

where X_0 and Y_0 are the northward and eastward axes of the polarization ellipse in the old coordinate system. The coordinate system is thus defined for each Fourier component and for each observatory, and can vary with time. Schmucker found little variability in these coordinates from day to day, and that the responses estimated using the pairs of components (B_r and B_θ) and (B_r and B_ϕ) agreed well. The drawback to this technique is the initial assumption about the spherical harmonic composition. Parkinson (1980) presents data showing the bias in the response functions that can results from making such an assumption, but it is not clear whether the data he used was from the equinoxes when the approximation would be expected to be most valid. According to Matsushita and Maeda (1965) even during the equinoxes the average Sq current systems are asymmetric. Because of the small influence of source field effects on Sq response functions (see Section 4) the data from a single station can be used to calculate an average response which closely approximates the zero wave-number response. In his more recent work Schmucker (1984) utilised a technique based on this property to avoid making assumptions about the form of the inducing field.

We must now consider what is perhaps the most serious problem facing analysis of long period electromagnetic induction data, that of the sensitivity of such data to lateral variations of conductivity within the Earth. Indeed, Sq data has often been used to look for and delineate areas of 'anomalous' electrical structure (see the review by Parkinson, 1980).

The influence of such laterally inhomogeneous structure is discussed fully in Section 3. Here it is important only to note that the spatial scale lengths associated with daily variations (e.g., the skin depth) are generally small compared to the average spacing of the observatories used for the SHA. Thus the data from an individual observatory will tend to 'see' only the structure in its near vicinity, and not that in those of its neighbours. This, together with the rapid and ubiquitous variations in the Earth's near surface structure, means that SHA of degree up to four or five may not be sufficient to describe the surface variations fields. This is especially true for the vertical component which is the most sensitive to such lateral variations in conductivity.

Arrays of temporary observation stations surrounding observatories and operating for some months have been used to identify and analyse 'anomalous' fields related to multi-dimensionally within the Earth (e.g., Bennett and Lilley, 1973), but obviously such data is not generally available.

One way to stabilize the SHA is to smooth the data beforehand, in the hope that the effect on the response of laterally heterogeneous structure will be averaged out. This is achieved implicitly in using the Schuster scheme mentioned above, where the data is averaged longitudinally. One disadvantage of this is the more or less arbitrary amount of averaging which goes into the process (being dependent on the geographic distribution of available observatories).

Various authors have attempted to avoid the problem by rejecting those 'anomalous' stations which are strongly affected by such lateral inhomogeneities. This is sometimes refered to as the separation of 'normal' field. Parkinson (1974) rejected those stations whose residuals when fitted to an initial SHA was greater than some arbitrary value. Berdichevski *et al.*, 1976 rejected those stations which perturbed the smooth pattern of their isoline maps (mentioned above). Borisova (1973) constructed a synthetic *Sq* field and used a comparison of the synthetic and actual data to isolate 'anomalous' stations. Fainberg (1975) expanded the *Sq* variation field into its 'principle components' and thus found the 'normal' field, which he defines as that which has the same temporal behaviour worldwide. In his 1983 paper Fainberg compares the 'normal' field separated using this method with that produced by Borisova, and finds them to be in good agreement.

The 'principle components' method can reduce the noise in the data. Effectively what it does is to fit to the data a time variation of form derived from the data itself, rather than for example a sinusoid. Fourier analysis and SHA must then be carried out to obtain the response. As the Earth is laterally inhomogeneous the phase difference between the internal and external parts is different at different locations. Thus even if the external part has the same temporal behaviour worldwide, the temporal behaviour of *Sq* will vary from place to place. The principle components method filters out this variation in temporal structure arbitrarily, allowing only variation in the amplitude of the *Sq* field. This can be seen as an advantage or a disadvantage, depending on the context of the analysis.

Geographically inadequate data coverage and the consequent instability of the SHA perhaps make the sort of data conditioning described above difficult to avoid. However, in view of the known multi-dimensional nature of the Earth's near surface structure, data from all locations can be expected to be 'anomalous' in some sense. (Note also the discussion in Section 4 about 'global average' responses.) Therefore it would seem preferable to approach the problem using some analysis technique based in the first instance upon data from individual observatories, e.g., the scheme of Schmucker described above. Of the 120 stations included in his 1974 analysis Schmucker found that the mathematical conditions for one dimensionally (Weidelt, 1972) were fulfilled by only 23. These were all in mid-latitudes ($20° - 50°$) and over 800 km from the nearest ocean. Subsequently Schmucker (1982) used an SHA representation of the horizontal field, together with the local vertical fields to derive local *Sq* response functions, and also a spatial gradient technique to obtain comparable results. Apparently the results from all three analyses agreed well with the second proving to be the best, but there was an inconsistency between the phases of the response from these analysis and those of longer period. In his most recent work (Schmucker, 1984) the single source term restriction is avoided (see above), and this leads to more consistent results.

Data from analyses other than those of *Sq* can be used for induction studies e.g., Malin (1973) and Winch (1981) used two and a half and two years respectively of continuous data, rejecting only five disturbed data each month, to obtain results for *S* and *L*. The spatial structure of *S* is more complex than that of *Sq*. *L* is typically one tenth the power of *Sq* (Malin and Gupta, 1977). Therefore these variations do not seem so well suited to use in induction studies as *Sq*.

2.2. Magnetic Storm Analyses

The UT dependent part of the surface magnetic variation field associated with the recovery phase (the slow, near exponential recovery over several days of the fields to their pre-storm values) of magnetic storms (*Dst*) is due primarily to the magnetospheric 'ring current'. This effectively flows in a circle around the Earth in a plane close to that of the geomagnetic equator, at a height of several Earth radii. Because of its distance from the Earth, and its simple form, the magnetic field variations at the Earth's surface associated with a symmetric ring current can be represented by very simple spherical harmonic models; zonal (i.e., longitude independent) and of low degree. As the position of the ring current is controlled by the Earth's main geomagnetic field, the geomagnetic coordinated system is the correct one for analysis of these variations.

In practice the storm time magnetic field is asymmetric (see e.g., Kingan *et al.*, 1980) due to asymmetry of the ring current (e.g., partial ring currents, Rokityanski, 1982); and to the superposition upon the ring current field of the intense disturbed daily variation S_D. This asymmetry can be reduced by averaging many storms with their commencements uniformly distributed throughout a day (e.g., Chapman and Whitehead, 1922; Chapman and Price, 1930; Benkova, 1953). Another approach is to take a single strong storm. The fitting of zonal harmonics only is equivalent to averaging with respect to longitude. Single storm analyses have been conducted by e.g., Rikitake and Sato (1957), Anderssen and Seneta (1969), Anderssen *et al.* (1970), Berdichevski *et al.* (1970), Lagutinskya *et al.* (1975), Fainberg *et al.* (1975a), Rotanova and Chimiddorge (1977), Devane (1977a, b) and Jady and Marshall (1984).

Most analyses suggest that the P_3 amplitude in the horizontal component is less than 10% of the amplitude of the P_1 part, with the higher degree terms being smaller still, although the results of Jady and Marshall indicate that harmonics of degree of up to 7 are statistically significant (although small), at some times. Jady *et al.* (1979) provide a method of comparison of different *Dst* analyses based on the properties of a distortion free filter.

Because of the time-asymmetric form of the *Dst* variations they are naturally suited to analysis in the time domain. The resulting time series of spherical harmonic terms can then be inverted using e.g., the method of Jady (1974), which avoids the problems associated with transformation into the frequency domain. Unfortunately, such inverse procedures are not so well developed as their frequency domain counterparts, and it is therefore desirable to obtain response function spectra. This can be achieved by transforming the results obtained by SHA at successive time intervals. Various attempts have been made to do this e.g., Takeuchi and Sato (1963), Fainberg (1983), Jady *et al.* (1983), but none seem completely successful. The alternative is to transform the data to the frequency domain prior to SHA (e.g., Devane, 1977a, b).

Various schemes for removing the daily variations, smoothing, and conducting the SHA have been used. Field values just prior to storm commencement, or some sort of average, perhaps over several days, can be used as a baseline. Both *Sq* and S_D have been used to remove the daily variations. Because of the day to day variability of the daily variations neither approach is completely satisfactory. High latitude and equatorial stations must generally be excluded from the SHA (see Jady and Marshall, 1984 for detailed discussion of these points). It is not clear whether doing the SHA before or after Fourier transformation is the best scheme. Rokityanski (1982) suggests that both schemes be used simultaneously and the one which provides the best result for the particular data set be selected. Fainberg *et al.* (1975b) used the method of 'principle components' to separate out a 'normal' field. Anomalous stations were rejected prior to SHA, which indicated that P_1 harmonics only were significant in the 'normal' fields. The results of Rotanova and Chimiddorge (1977), who used harmonics up to degree and order three, are widely scattered and sometimes physically unrealistic. Rokityanski (1982) states that this demonstrates the unsuitability of SHA of *Dst* fields using several harmonics, but the thorough analysis of Jady and Marshall (1984) seems to have been successful in this.

2.3. THE 27 DAY VARIATION AND ITS HARMONICS AND THE 'CONTINUUM'

At slightly longer periods than those so far discussed, the geomagnetic spectrum consists of a series of lines (at about 27, 13.5 and 9 days) superimposed upon a 'continuum' of power. These spectral lines are caused by the 27 day recurrence tendency of magnetic storms, and are thus primarily due to variation in the intensity of the ring current system. The rotation of the Sun (period about 27 days) modulates the polarization of the inter-planetary magnetic field and energy density of the solar wind, which in turn control the likelihood of occurrence and activity level of magnetic storms. Banks (1969) showed that the continuum corresponds to a worldwide geophysical phenomena, has the same source as the spectral lines, and is valid for use in induction studies. As these variations are associated with the ring current system, the geomagnetic coordinate system is appropriate. Because of the time averaging involved in the study of variations of such long period, transient perturbations from symmetry tend to average out, and the external part of the variation field at the Earth's surface can be represented extremely well by low degree, zonal spherical harmonics. Currie (1974) suggested that power might leak to the higher frequencies from the P_2^0 annual variations causing a P_1^0 model to be inadequate, but later studies seem to have found no evidence for this. As the period increases the penetration into the Earth, and thus the volume of the Earth affecting the measured response, increases. This will tend to average out the effect of near surface lateral inhomogeneities. Furthermore if, as has been suggested by many authors (e.g., Vanyan *et al.*, 1977), the Earth becomes increasingly radially symmetric with depth, then it is these long period variations (penetration of the order of 1000 km) which may 'see' a radically symmetric structure.

The demonstrable unreliability of many published data at these periods is probably due partly to poor data (with very long time series baseline control is obviously important), partly to the influence of the multidimensionality effects discussed below, and partly to the use of comparatively unsophisticated analysis techniques. Because of their considered importance to 'global' induction studies, and the small number of available analyses, we will discuss the available data in some detail.

The early analyses of the such data (e.g., Rikitake, 1951) are probably unreliable due to the small amount of data used. Banks (1972) tabulates and analyses the data which were then available. Eckhardt *et al.* (1963) used data from five observatories to show that the spectral peaks at about 27, 13.5, and 9 days were well fitted by a P_1^0 spherical harmonic, and proceeded to estimate the response using the data from their 'best' mid-latitude station. Note also the revised response estimates by Eckhardt *et al.* quoted by Rokityanski (1982, p. 117).

Banks (1969) analysed the data from two stations to obtain response estimates at the line frequencies, and four stations for the continuum. With so few stations he obviously could not do an SHA, but had to assume a P_1^0 harmonic only. In estimating the continuum response Banks used a 'reference field' technique (similar to the used in single station magnetotellurics). Instead of simply taking the ratio

$$W_1 = -\frac{\langle B_r \rangle}{\langle B_\theta \rangle} \cot\theta \tag{11}$$

where the brackets may indicate averaging over frequency or different sections of data, we can use

$$W_1 = -\frac{\langle B_r B_\theta^* \rangle}{\langle B_\theta B_\theta^* \rangle} \cot\theta \tag{12}$$

(B_θ^* is the complex conjugate of B_θ). This has the advantage that uncorrelated noise in the radial component can be expected to average to zero. The disadvantage is that any noise in the tangential component downward biases the response function amplitude (phase is unbiased). Considering the very much greater noise level in B_r than in B_θ the removal of most of the noise at the expense of a small bias error is a reasonable approach. Banks quotes his B_θ component signal to noise ratio as greater than 6:1 in amplitude, suggesting at most a 3% underestimation of W_1.

However, comparison of Bank's results with those of later studies suggest that his response functions may be underestimated by a significantly greater amount than this (Fainberg, 1983). It should be noted that Bank's response function phases are 'non-physical' at the higher frequencies (i.e., they are inconsistent with P_1^0 induction in a radially symmetric Earth). While in a statistical sense the phases are not significantly 'non-physical', this suggests that the differences between Bank's estimates and those of other workers are not due to bias error. This difference could be due simply to noise in one or other data set, or to the violation of one or more of the assumptions made (e.g., the spherical harmonic representation), but the most probable explanation is the different locations of observations used and the consequent different effects from lateral inhomogeneity within the Earth, notably the oceans. The difference between Bank's continuum and line estimates supports this hypothesis, as the line estimates used only two of the four observations.

Dzodencukova (1975) calculated the response at 27, 13.5, and 9 days using the data from 12 'low noise' observatories. There is an inconsistency in Dzodencukova's results as published. Both Rokityanski (1982) and Fainberg (1983) suggest that the quoted magnetic ratio rather than the apparent resistivity is correct. Anderssen *et al.* (1979) used a technique similar to that of Banks to analyse two years of hourly mean data from 17 observatories. They found that the magnitude of the response at stations Thule–Qanaq and San Juan were 'anomalous' but used the remaining data to estimate a 'global' response in the period range 5 to 35 days. The phase of the response was found to be almost independent of period itself at periods over 15 days.

As the quantity of data and the analysis techniques used have improved, it has become easier to see inconsistences and problems in the data, and the effect of lateral inhomogeneity. Pecova *et al.* (1977) analysed three year sections of daily mean data from about 30 European observatories, and used the ratio of B_r and B_θ spectral amplitudes and the assumption of a P_1^0 harmonic only to estimate the response for the 27 day variations and its harmonic at each station. They found that W_1 was generally less over the northwestern part of Europe than in the South East. Fitting the fields to first degree harmonics of order 0 to 1 and plotting contours showed that the B_θ field was predominantly zonal for all three periods, while the B_r field showed a pronounced dependence on longitude. Results from different three year sections of data gave slightly different results but 'anomalies' were found to persist in the western part of the region. Assuming that the external inducing field was purely P_1^0, but that first order terms were required to describe the internal field they calculated a modified first degree response function. This was found to be essentially longitude independent, the remaining observatory 'anomalies' being more or less random.

In their 1980 paper (Pecova *et al.*, 1980) the analysis was extended to include about 50 observatories, still mostly in Europe but some globally distributed. Three year sections of data from different parts of the sunspot cycle were analysed to estimate the response 27, 13, and 9 days. Fitting to a P_1^0 harmonic gave similar results to their previous analysis, which they suggest means that the deep structure beneath Europe is little different from that of the whole Earth. A first and second degree and order

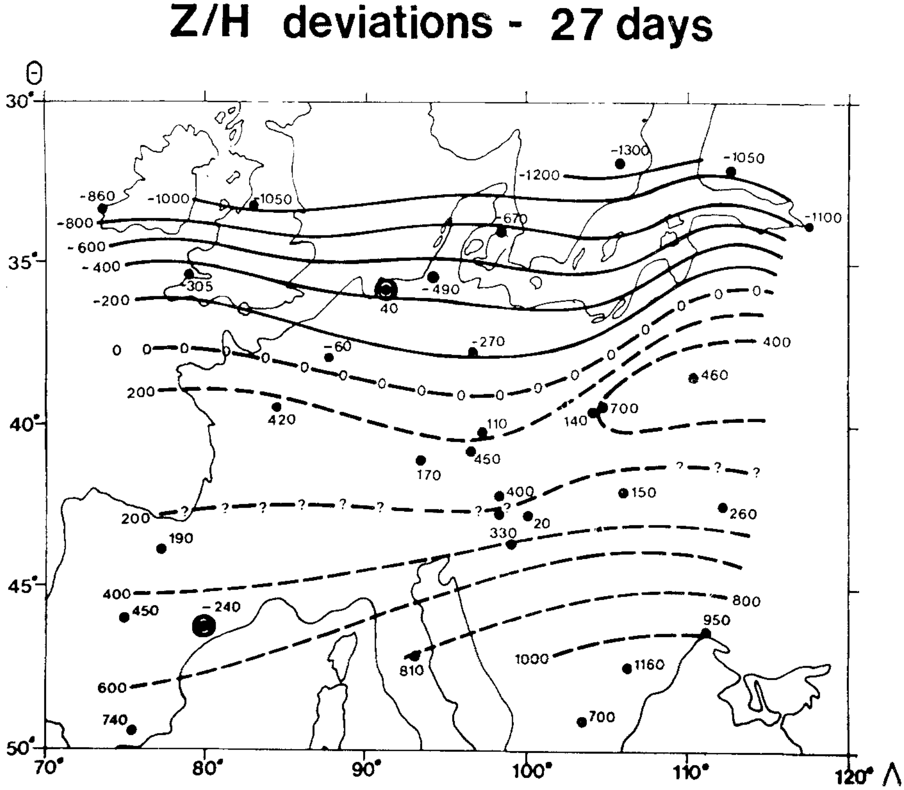

Fig. 2. Contour map of residuals between locally estimated Z/H ratio and a global W_1 response (multiplyed by 10^4) for the 27 day harmonic (from Pecova *et al.*, 1980).

spherical harmonic analysis demonstrated good agreement with the 1977 results; B_θ was predominantly zonal and B_r a function of both latitude and longitude, but they now found that a second degree representation was required to describe the B_r field structure over Europe. They then produced a contour map of residuals between the observed data and the 'global' response at the 27 day period (Figure 2). It can be seen that there are negative deviations to the North, and positive ones to the South. They suggest that the pattern may be due to some deep E–W striking structure below central Europe. They compare their data with that of Fainberg *et al.* (1975a) and find a similar structure.

Examination of Figure 2 shows that there are sometimes marked differences in the residuals at stations which are very close to each other. This gives some idea of the noise level in the data, but on the whole the pattern of their contourrs is quite convincing. The almost longitude independent character of the contours shown in Figure 2 suggests the possibility of an inadequate SHA of the (predominantly zonal) spherical harmonics. Pecova *et al.* (1980) used only first and second degree harmonics. Various studies of magnetic storms, and longer period variations, suggest that the P_3 harmonic may have an amplitude of up to 10% of that of P_1, although

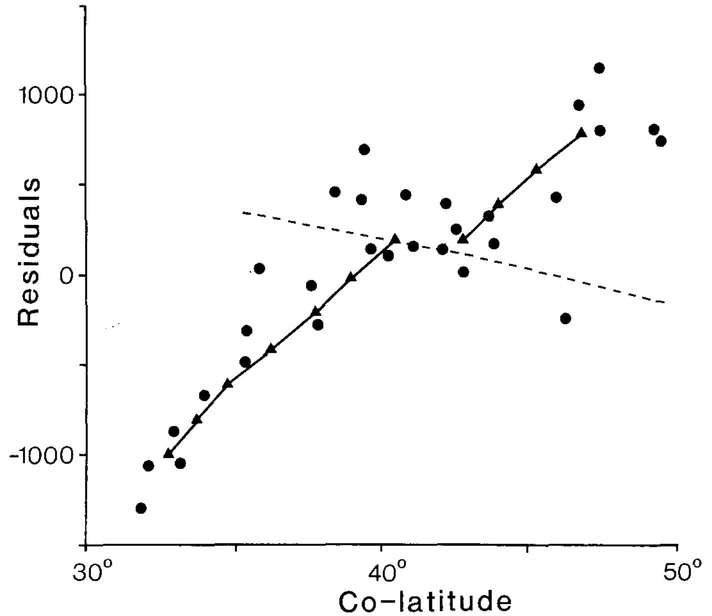

Fig. 3. The same data as Figure 2, plotted as a function of (geographic) latitude. Also shown are a smoothed version of the variation across central Europe (from Pecova *et al.*'s contours-solid line), and (dotted line) the expected behaviour of the response function there if it is assumed that there is no P_3 power present, but in fact P_3/P_1 amplitude is 10% (see Section 3).

this may well be overestimated because of noise in the data. The associated Legendre functions of the first kind (Schmidt quasi-normalised) for P_2^0 and P_3^0 harmonics are

$$\begin{aligned} P_1(\cos(\theta)) &= \cos(\theta) \\ P_3(\cos(\theta)) &= 1/2(5\cos^3(\theta) - 3\cos(\theta)). \end{aligned} \tag{13}$$

Using these equations, and Equations (1) and (3), we can calculate the error which would be present in the estimation of W_1 if it were assumed that there was no P_3 power present, but in fact $e_3 : e_1$ was about 10% (amplitude). i_3 can be estimated from forward modelling using the procedure of Banks (1969). Figure 3 shows the latitudinal behaviour we would expect under these conditions. It can be seen that the curve slopes the wrong way to explain Pecova's results. If we allow that e_1 and e_3 are not in phase (probably physically unrealistic) then the curve can be made to slope the other way, but the amplitude is still much too small to explain the empirical results. Thus Pecova *et al.*'s results can not be explained by an inadequate spherical harmonic representation of the external field. These results are further discussed in Section 3.

Taking advantage of the recent theoretical advances of Banks (1975, 1981), the response at 17 globally distributed observatories was calculated by Roberts (1984). The analysis procedure was based on the use of 'complex demodulation and careful data selection. Because of the form of Equations (1), the B_r field is much more sensitive to lateral inhomogeneities within the Earth than is the B_θ field, and it is also much more noisy. Thus a P_1^0, P_3^0 SHA is probably sufficient to describe the B_θ field, but not B_r. Taking advantage of this fact, an SHA of B_θ was conducted in order to give a global 'noise free' B_θ field. This was then used with B_r single station data to produce a W_1 response at each of the 17 observatories. The use of a global B_θ field has the additional advantage that while we may not remove the bias errors completely, the bias error at each frequency is the same at all observatories, meaning that results are directly comparable. The disadvantages of this analysis are the small number of observatories used in the SHA (a simultaneous data set of 14 yr was required), and the assumption of lack of bias from P_3 power in the B_r field induced by the external P_3 field.

An extensive data set is at present under preparation by Schultz and Larsen (1983) and should provide a valuable addition to the available data.

2.4. THE RESPONSE AT 6 MONTHS 12 MONTHS AND 11 YEARS

The existence of a spectra line at a period of six months is clearly established, although it is often difficult to separate from noise. Its source mechanism is not known for certain, it could be due to special interactions between the solar wind and the magnetospheric boundary (see Campbell, 1980 for references). Eckhardt *et al.* (1963) and Banks (1969) estimated the response from single stations, assuming a P_1 harmonic. Currie (1966) demonstrated the worldwide nature of the semi-annual variation, but found no clear latitudinal trend in either the horizontal or vertical component data, suggesting that the hitherto assumed description by a P_1^0 spherical harmonic may be inadequate. Courtillot and Le Mouël (1976) also suggest that a simple P_1^0 model is inadequate.

The annual variation in geomagnetic activity seems to be produced by a quite different mechanism from the semi-annual variation, and has a different spatial structure (Currie, 1966). It has been suggested that the origin of the annual line is an ionospheric dynamo mechanism (e.g., Courtillot and Le Mouël, 1976). However, Malin and Isikara (1976) suggest that its origin in non-polar regions may be primarily an annual variation in the latitude of the ring current due to the tilt of the Earth's axis to the ecliptic and the consequent asymmetric distortion of the magnetosphere, with a lesser contribution from the annual modulation of ionospheric properties (see also Currie, 1976; Campbell, 1980; Mukherjee and Rajaram, 1982). The works of Currie (1966) and Banks (1969) suggested that the annual variation was predominantly P_2^0. Thorough SHA of the annual variation by Malin and Isikara confirmed this conclusion but also found significant P_1^0 and P_2^1 terms. Courtillot and Le Mouël also consider a P_2^0 model inadequate. In the opinion of Rokityanski (1982) even the best analyses of the annual variations can determine the apparent resistivity with an accuracy probably as low as an order of magnitude. In his extensive analysis of S and L Winch (1981) found that his estimates of the response at six and twelve months were 'non-physical', the internal part being greater than the external part. He attributes this to drift in baselines. Obviously some observatories are more susceptible to such effects than others (e.g., those with large annual variations in temperature), so selection of data to isolate 'good' stations could improve matters.

Baseline control is obviously also a severe problem in attempting to analyse the 11 yr variations of the geomagnetic field. It appears that these are of external origin and are primarily due to a modulation of ring current activity associated with the sun-spot cycle. There is also significant sun-spot cycle modulation of S (Malin *et al.*, 1975). In addition to the problems associated with the analysis of shorter period variations there is the complication of severe contamination of the data by the internally produced secular variation. This must be removed by subtraction of a linear trend or polynomial fit, or by band-pass filtering.

Yukutake (1965) indicated the P_1 character of the 11 yr variations, and estimated the response. His analysis was based on band-pass filtering, a procedure criticised by several later authors, and despite careful data selection his analysis produced 'non-physical' results. (Note the error in Yukutakes calculations pointed out by Rokityanski, 1982; p. 121). Most authors prefer to fit sinusoids in the time domain, rather than Fourier transform, to avoid introducing distortion. Courtillot and LeMouël (1976) used parabolic trend removal and maximum entropy spectra to obtain response estimates at 11, 5.5, and 3.7 yr. That at 11 yr was found to be 'non-physical' and the error estimates associated with the response at 5.5 and 3.7 yr are very large.

Alldrege (1976) removed a linear trend and then fitted sinusoids to the data. He eventually fitted a P_1 harmonic only, using data from only the 'best' one third of his observatories. He was unable to differentiate the phase of the response from zero. Fainberg (1983) recalculated the response on the basis of Alldrege's data.

Isikara (1977) removed a polynomial trend, the degree of which being determined by reference to the 'best fit' of the residuals to solar activity indices. Under the assumption of a P_1 harmonic only he found his averaged results to be 'non-physical', and therefore rejected observatories where $W_1 < 1$ and reaveraged.

Harwood (1977) and Harwood and Malin (1977) fitted polynomials to remove the secular variation and fitted sinusoids determined from solar activity variation. After discarding some data, 81 stations were subjected to an SHA. The P_1^0 term was found to be dominant but with significant higher degree and order terms, the P_2^0 part apparently being of internal origin. Comparison of Harwood and Malin's results with those of Alldrege, Courtillot and Le Mouël, and Isikara suggests that the results of the earlier analyses may be biased by the lack of an adequate SHA.

Ducruix *et al.* (1980) used a method similar to that of Courtillot and Le Mouël (1976), and found that the use of an 'adequate' secular variation model gave 'physical' response estimates at 11 yr assuming a P_1^0 harmonic only.

2.5. A SUMMARY OF SECTION 2

Different analyses of Sq give significantly different results. This is due in part to the complexity and variability of the external inducing current system, and partly to the great influence of lateral variations in the Earth's electrical structure. Analyses producing 'global' estimates of the response are forced to average the data from different locations in a more or less arbitrary fashion. Rejection of anomalous stations (i.e., those strongly affected by lateral heterogeneity) can stabilise the analysis. Because of the unquestioned influence of lateral heterogeneity on the daily variations it seems clear that 'global' analyses can never be more than a first approximation to reality. Using for example the techniques of Schmucker it is possible to obtain geographically localised results.

Analysis of longer period variations is in some ways easier because of the greater simplicity of the external inducing field. Problems similar to those encountered in *Sq* analyses (related to e.g., transient distortion of the external current system) are found for single storm *Dst* analyses. In addition, time domain *Dst* studies have the problem of the lack of sophisticated time-domain inversion procedures. Undistorted transformation into the frequency domain can be problematical.

Estimation of the response from the continuum part of the spectrum, and the 27 day line and its harmonics, is perhaps easier due to the time averaging of distortions and to the additional spatial averaging related to the longer period. The influence of lateral inhomogeneity within the Earth is still seen, but it may be possible to remove it by the spatial averaging effect of the SHA (see next section). As for the shorter period variations, it seems desirable to examine geographically localised response estimates.

Estimation of the responses at very long periods (6 months to 11 yr) is extremely difficult because of baseline drift and the possible complex form of the inducing field. While such data are undoubtedly imprecise, they are potentially very useful because of the great depth to which the variations penetrate, but conclusions about the structure of the Earth based on such data must obviously be treated with caution.

Figure 4 shows some apparent resistivities and associated phases. Because of their great number only a few of the many daily variation results are plotted. The scatter of the results is considerable. This is due both to noise in the data and to the lateral inhomogeneity of the Earth.

3. The Influence of Lateral Conductivity Variations

With regard to studies of very long period data, the problem of lateral conductivity inhomogeneities within the Earth, and their influence upon the response function, falls into two fairly distinct areas. The first relates to the influence of the thin, but comparatively highly conducting oceans, the second to the structure of the solid Earth itself.

It has been speculated for some time (e.g., Chapman and Whitehead, 1922) that induction in the oceans by magnetic variations of external origin might have a significant effect on *Sq* data. It was thought that the response at longer periods than *Sq* would be much less affected (see e.g., Banks, 1972), but it is now clear that the oceans can strongly affect both daily and longer period variations at observatories near coasts. Schmucker (1974) found that his *Sq* response data was consistent with simple 'one-dimensional' induction only at sites more than 800 km from the nearest

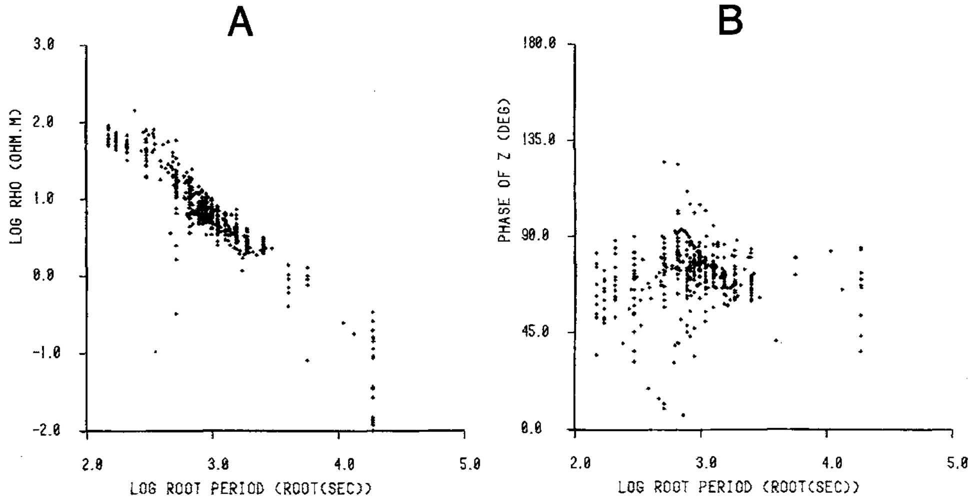

Fig. 4. (A) Apparent resistivity and (B) phase of impedance from a large number of published analyses. Only some of the available *Sq* data is plotted.

deep ocean. Roberts (1984) found that response estimates from the 'continuum' part of the spectrum were strongly distorted at oceanic observatories. Modelling studies such as those of Hobbs and Dawes (1979) indicate strong distortion of the *Sq* field near coasts. Larsen (1975) identified an 'island effect' in data from Hawaii at periods up to several days, and proceeded to model the effect and thus remove it. Fainberg (1980) shows the influence of the presence of the oceans on *Dst* data. In his 1983 paper he discusses mathematical modelling of the electric currents induced in the world's oceans using the techniques of Fainberg *et al.* (1983) and Fainberg and Singer (1980), where the oceans are assumed to be a thin sheet of variable conductivity isolated from the underlying medium (Price, 1949). These studies indicate that the results from SHA of daily variations are strongly distorted but (surprisingly) those from longer periods are not. This apparent contradiction with empirical results may be due to the smoothing effect of the SHA, simply to the distribution of observatories used, to the exclusion of vertical current flow in the model, or to the effect of gross multidimensionality within the Earth.

Figure 5 shows the 27 day response of Pecova *et al.* (1980) at European observatories (see Figure 2) plotted against approximate distance from the nearest ocean. Also shown are two error bars, the larger from the greatest difference in the response at two neighbouring (very close) stations, and the smaller from the average of several such calculations. They therefore give some idea of the uncertainty in the estimated responses due to instrumental noise, very near surface effects etc. It can be seen that the responses at more than a couple of hundred kilometres from the nearest coast are essentially the same, while those near the southern coasts of Europe are generally larger, and those near the northern coasts smaller. The pattern shown

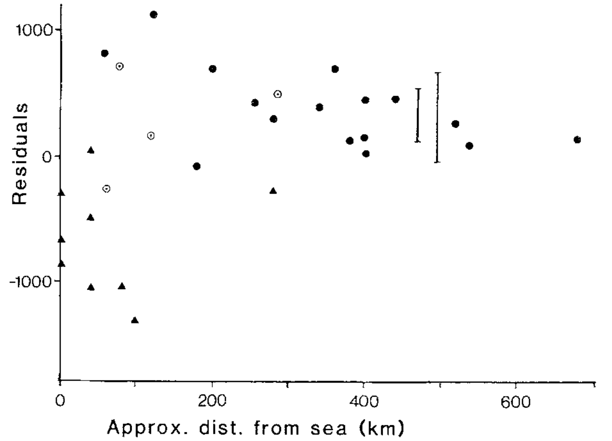

Fig. 5. The same data as in Figure 2, plotted against approximiate distance from the sea, triangles represents stations north of the zero line on Figure 2, and circles stations to the south. Open circles are the four stations on the Iberian peninsular.

in Figure 5 may, of course, not be due to the presence of the oceans as we might expect a coincident change in solid Earth structure.

To gain some insight into the effect upon the response of ocean-land boundaries, we can turn to the results of two-dimensional (2D) modelling. Because of the sphericity and complex three dimensionality of the Earth and the spatial form of the inducing fields at the long periods of interest here, the induction at coasts will in general be neither '*H*-polarisation' or '*E*-polarisation' but will contain elements of both. Because of these effects any conclusions we reach will be at best rough approximations, but should indicate with some sort of validity the spatial scale lengths involved.

First we will consider the *H*-polarisation case. This may seem strange as in the flat Earth 2D case there is no vertical magnetic field in this mode of induction, but the scale length deduced will still have relevance to our case. As has been pointed out by many authors the oceans are very thin compared to the skin depths at periods of one day and longer, and will act essentially as a thin sheet conductor. Such (2D) structures have been examined by e.g., Ranganayaki and Madden (1980) who show that the decay length of disturbance of the near surface telluric fields in the *H*-polarisation case is approximately frequency independent and is given by the formula

$$\delta = (\sigma_1 h_1 h_2/\sigma_2)^{1/2}$$

where σ_1 and h_2 are the conductivity and thickness of a conductive surface thin sheet, and σ_2 and h_2 those of an underlying (thin sheet) resistor. The amplitude of the distortion decreases with increasing period. Using this formula Ranganayaki and Madden estimate this decay length to be perhaps 1000 km on the oceanic side of the boundary. This was challenged by Drury (1981) who suggested a somewhat smaller value. On the land side of the boundary, where the surface layer is much less conducting, the decay distance is much smaller (see Ranganayaki and Madden,

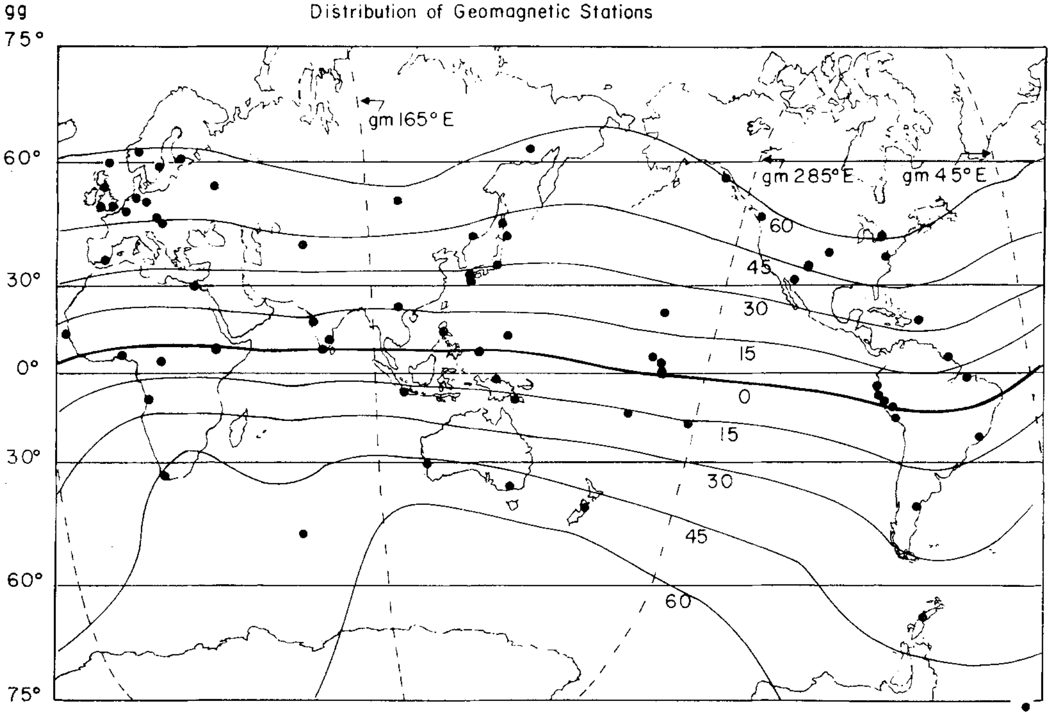

Fig. 6. Distribution of 69 IGY geomagnetic stations used in the *Sq* study of Matsushita and Meada (1965). The broken lines show the boundaries of the three longitudinal zones used (see Section 3).

Figure 1, also Dawson *et al.*, 1982). Its value is highly dependent upon the model parameters chosen, but is probably a few hundred kilometres. Another approach is to take a 'realistic' Earth model, including an ocean, and use one of the 'full' 2D programs. This was done using Brewitt–Taylor and Weaver's (1976) program. In the *E*-polarisation at 20 days period a few plausible Earth models suggested decay lengths of the magnetic transfer function of 100–300 km, but again this is strongly dependent on the model parameters chosen.

Figure 6 from Matsushita and Maeda (1965) shows a proportion of available geomagnetic observatories and indicates the significance of the problem. For various reasons many of the observatories are close to the sea, and are therefore prone to oceanic distortion effects. Matsushita and Maeda, and other workers (for a compilation see Rokityanski 1982) have attempted to examine the gross variations of upper mantle structure by analysing *Sq* data in three longitudinal zones (marked on Figure 6). Such results indicate that in the American zone the mantle has a higher conductivity than in the other two zones considered, but this conclusion must be treated with caution. Figure 6 shows that of the 18 stations Matsushita and Maeda had in the American zone 14 are near a coastline. Another point is that many of these observatories are in subduction zone regions (note the concentration on the west

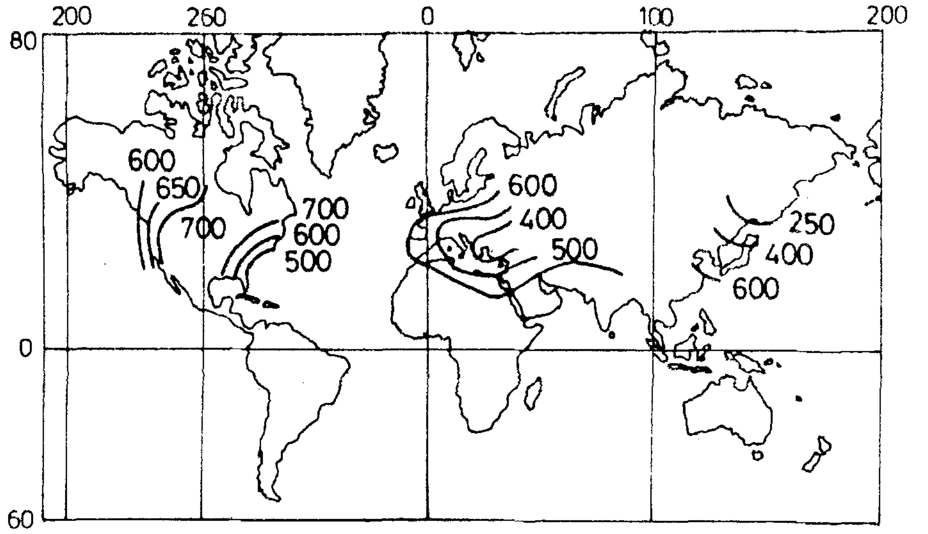

Fig. 7. Map of apparent depths (km) of a perfect substitute conductor from Berdichevski *et al.* (1976). (The figure is reproduced from Schmucker (1979).

coast of S. America). It is known that anomalously high mantle conductivities are often found in such regions (see e.g., the discussion in Roberts, 1983). Because of the comparatively short spatial wavelengths involved the west coast observatories will to a large extent 'see' only the near west coast structure. Thus, even if the results of Matsushita and Maeda give a fair indication of the average conductivity below their observatory sites, there is little reason to believe that this is a valid figure for the whole of the Americas.

It has been suggested for some time that the oceans themselves are not sufficient to account for some observed coast effects (e.g., Lilley and Parker, 1976), which implies a coincident change in solid Earth structure. Both electromagnetic results (see Law, 1984) and results from other fields indicate that the oceanic and continental lithospheres are indeed generally quite different. Figure 7 shows the depth to a perfect substitute conductor as deduced using the spatial gradient method and daily variation data by Berdichevski *et al* (1976). They attribute the pattern to variations in solid Earth structure, despite the proximity of the oceans.

It is well known that the electrical conductivity of the crust, and that of the upper mantle down to say 400 km, can be extremely laterally inhomogeneous even within continental areas (see e.g., Roberts, 1983). If, as seems certain, convection cells exist within the upper mantle, then we have every reason to expect associated lateral variations in conductivity of at least an order of magnitude, probably extending to at least six or seven hundred kilometres in depth. If there is whole mantle convection, or a distinct set of convection cells within the lower mantle, then we might expect significant electrical heterogeneity right down to the core. Comparison of the results of Larsen (1975) from Hawaii with those of other areas (see Figure 8) seems clearly

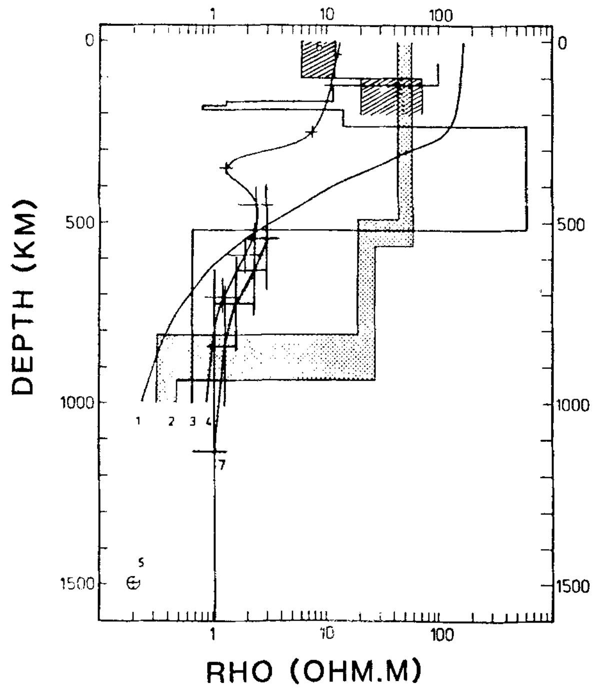

Fig. 8. Resistivity-depth profiles from (1) Banks; (2) Schmucker (1974); (3) Tucson model (Larsen, 1977); (4) Hawaii model (Larsen, 1975); (5) Isikara (1977); (6) Poehls and Von Herzen (1976); (7) Hobbs (1984); (modified from Haak, 1980).

to show the presence of a mantle 'plume', which may well originate from thermal instability at the core-mantle boundary (see e.g., Loper and Stacey, 1984). Such plumes imply lateral variations in conductivity at all depths within the mantle, although these variations may be restricted to quite small geographic areas.

If lateral inhomogeneity extends to great depth, then obviously the thin sheet approximation breaks down. The appropriate disturbance decay length in this case is not that given by Equation (14), but rather some effective skin depth, and this will therefore increase with period. The small lateral scale of many variations of structure within the crust and upper mantle *may* mean that their effect is averaged out for long period variations (see also Vanyan, 1981). If lateral variation in conductivity at great depths is insignificant or is restricted to small geographic areas, then a 'global' response function and associated 'global' conductivity depth profile might be appropriate. If there is significant large scale heterogeneity then response estimates and inversions for different geographic areas are required. The problem becomes intractable only if the Earth is laterally inhomogeneous on a scale less than the effective skin depth, but not on such a small scale that spatial averaging is effective, i.e., if we cannot treat the inverse problem as 'one-dimensional'. Which case is appropriate can only be determined by reference to actual data.

4. Inversion

In considering the 'global' induction problem we are primarily interested in the one-dimensional (1D) inverse problem. While the Earth is not 'one-dimensional', in the case of a flat Earth such algorithms can be used if the effective scale length of lateral conductivity variations within the Earth is greater than the spatial scale lengths associated with the electromagnetic fields (skin depth). If this criteria is significantly violated then, because of the paucity of 'global' induction data, it is probably not meaningful to attempt rigorous mathematical inversion; we are restricted to drawing more qualitative conclusions from our data.

It was shown by Weidelt (1972) that, for a spherical Earth and a given spherical harmonic inducing field, the derived (radially symmetric) conductivity-depth profile can be readily transformed into that for a flat Earth and uniform inducing field. This allows us to analyse our 'global' data using the 1D algorithms developed for local magnetotelluric studies. There have been substantial theoretical advances in this area over the last few years. Thus, from the parameter models of e.g., Lahiri and Price (1939) and the forward modelling procedures of e.g., Banks (1969), we now find at our disposal a bewildering number of sophisticated automatic 1D inversion programs. The problem now is not finding a model to fit the data (if one exists), but finding a model which is 'best' in terms of some objective criteria, is physically realistic, and produces some meaningful estimates of resolution. By physically realistic we mean here that the model must conform to our *a priori* restrictions on what we believe the world must be like, and perhaps must be consistent with other forms of data. 1D inversion techniques are reviewed by Parker (1983). There are still problems and ambiguities associated with such algorithms, but with high quality data these seem not too significant.

The general 1D inverse problem will not be further discussed here, we will restrict ourselves to those aspects specific to the spherical Earth problem.

The first thing to be considered is the relationship between the response relating to different spherical harmonics. If the penetration into the Earth (skin depth) is small compared to the spatial wavelength of the inducing fields and the curvature of the Earth, then the data sets can be simply combined without adjustment. Whether this condition is fulfilled depends upon the period and the conductivity of the Earth at the relevant depths. Because of the highly variable nature of the Earth's near surface structure it is a little difficult to generalise with regard to the daily variations, but for all the models tried by the present author the difference in the apparent resistivity associated with a P_1 field and that associated with the expected degree of the inducing field is small (under 10%). This justifies the technique used by Schmucker (1984) in his *Sq* analyses (Section 2.1). For the annual variation, whose primary constituent is P_2^0, the effect is more significant. Various models based on those in the literature indicate that the apparent resistivities associated with the P_1^0 and P_2^0 modes of induction differ by perhaps 25%, the actual value being of course dependent on the real Earth structure. Some studies suggest that there may be significant modes other than P_1^0 in the 6 month and 11 yr variations. Because of the very great penetration of the 11 yr variation compared to the radius of the Earth, the spherical harmonic composition is obviously of great significance.

Thus to be 'correct' we can invert the very long period responses relating to the different spatial harmonics by using e.g., the approximate technique of Schmucker, using the appropriate spatial scale length, or we can use an inversion technique based upon a forward scheme that allows the inclusion of information on the spherical harmonic content.

These effects are generally small, and compared to other effects (e.g., imprecision in the data, the effect of lateral inhomogeneities) are probably insignificant, meaning that the inversion of $C(\omega)$ or $\varrho_A(\omega)$ ignoring the harmonic degree may well be a valid first approximation. However, as response function quality improves the effect may become of more significance.

As has been discussed in the previous section, one of the major problems in global induction studies is the possibility of different mantle structures in different geographic areas. To draw any firm conclusions about lateral conductivity variations within the mantle by using response data from different regions we obviously require resolution calculations. Such calculations are difficult because of the linerarisation of the inverse problem required. There are numerous examples in the literature of resolutions obtained from inversion algorithms which were later shown to be over-optimistic.

A related problem is that the error estimates associated with 'global' response estimates are usually model dependent e.g., they are derived from the fit of the data to an SHA under the assumption of a radially symmetric Earth. Inadequacy of the spherical harmonic model, especially if we reject 'anomalous' stations, or the presence of significant multidimensionality, may mean that the derived response function standard errors and ultimately the conductivity-depth resolutions may not truly describe reality. In other words we must be careful about over-interpreting our data on the basis of statistical criteria, as the statistics used often do not tell the whole story.

Another point is that, because of the non-linear mapping between the response function and the conductivity-depth profile, if the Earth's electrical structure is different at different geographic locations there is no guarantee that a 'global average' response function will produce a valid 'global average' conductivity depth profile. The best way to avoid this potential problem would be to invert data from the different locations independently. Unfortunately not much such 'regional' data is available at very long periods, and even when it is the inversions are often unstable due to the comparatively large noise levels in such data. Another approach might be the averaging of response function data from stations where we might expect the underlying Earth structure to be similar, and the inversion of this average (This leads us back to the 'anomalous' station rejection used by many authors).

The inversion procedure can be stabilised to some extent, and the effect of near surface structure allowed for, by the combination of local and global response data

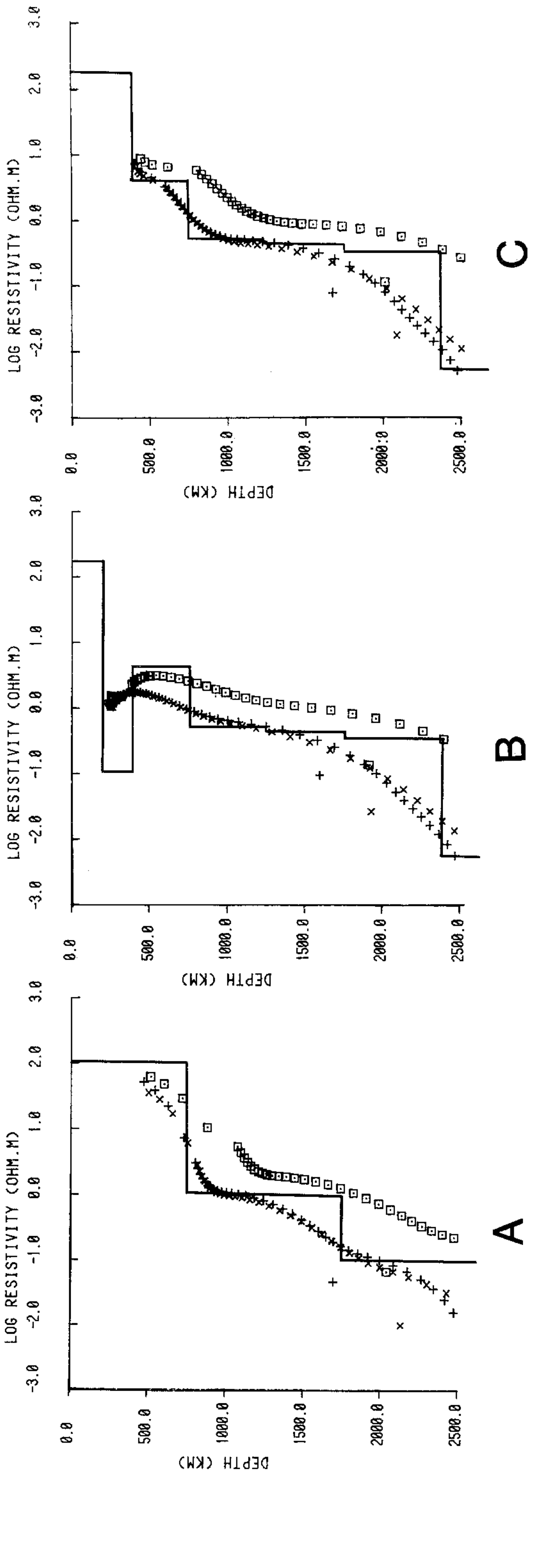

Fig. 9. Schmucker inversions of synthetic response data produced by forward modelling of the model shown (solid line) using the method of Banks (1969). (+) direct Schmucker inversion; (×) transformed to a spherical Earth (Weidelt, 1972); squares are the modified Schmucker inverse of Pecova *et al.* (1980).

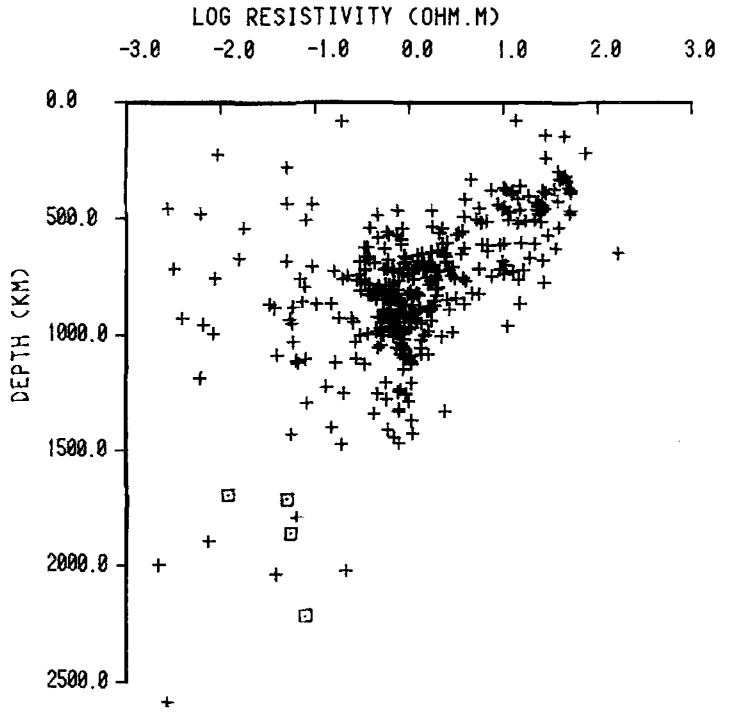

Fig. 10. Unmodified Schmucker inversions of the suitable data (i.e., that with both amplitude and phase). The squares represent the response at one year period.

(Kovtun *et al.*, 1982). If the two data sets are genuinely compatible this is of course valid, but obviously the reliability of conclusions drawn from such analyses about the very deep structure are still totally dependent upon the realiability of the long period data.

Most inversion routines work on the whole spectrum of response data. We do not have a consistent data set, and rather than invert some sort of smoothed average it seems preferable initially to invert the individual response estimates using Schmucker's (1974) approximate inverse. Rokityanski (1982) suggests that this scheme works well if the conductivity is a monotonically increasing function of depth, which is what we suspect we have (more or less) here. To test the effectiveness of the inversion procedure the response functions associated with the models shown in Figure 9 were calculated using the (spherical Earth) algorithms of Banks (1969), and the appropriate spherical harmonics. Also shown are the Schmucker inverse both with and without transformation to a spherical Earth via Weidelt's transformations. Schmucker inversion assumes a flat Earth and a clearly defined spatial wavelength of the inducing field, and in the spherical Earth case there is some question as to which is the most valid approximation of this wavelength for a given spherical harmonic. This wavelength is usually described by n (Equation (5)), but Pecova *et al.* (1980) consider a value of $n + 1/2$ to be more appropriate. Modified inversions based on this spatial wavelength are also shown in Figure 9.

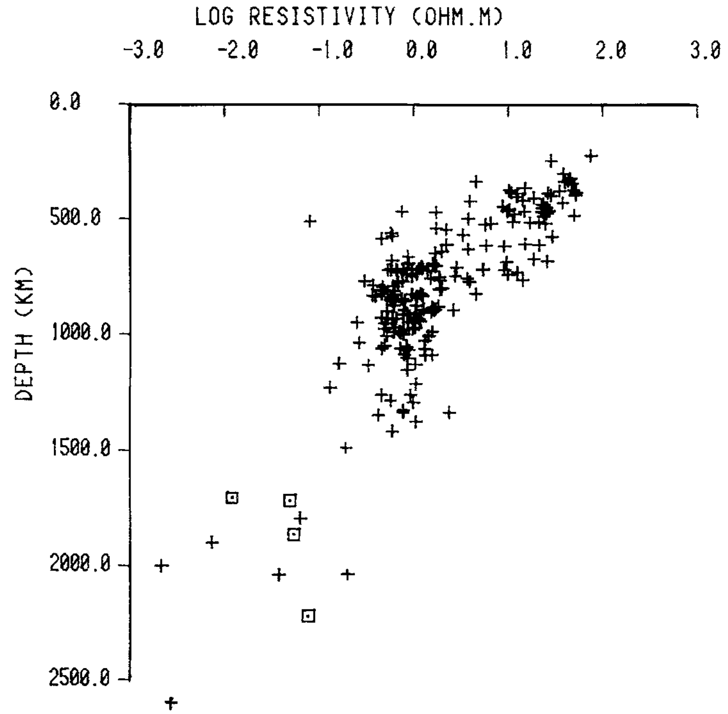

Fig. 11. As Figure 10, but with some of the unreliable data excluded. The squares represent the response at one year period.

We can see that the accuracy of the inversion is dependent upon what the structure actually is. We generally get a reasonable approximation to the original model, but (unsurprisingly) the inversion is unable to resolve detail. Note the effect of the harmonic degree on the inversion of data of one year period (single points lying to the left of the main curves).

To summarise our discussion of inversion: Many sophisticated mathematical techniques are now available. While even for the one-dimensional case none of these are fully general, we can certainly obtain meaningful conductivity-depth profiles with comparative ease. Definitive estimation of the resolution associated with these profiles is much more difficult. While rigorous statistical techniques are undoubtedly required, the statistics used rarely define all the potential imprecision in the problem and must therefore be interpreted with care, bearing in mind the physics of the problem. Simple model studies suggest the Schmucker inversion is a reasonable first approximation to reality.

5. Conductivity-Depth Estimates

Figure 10 shows Schmucker inversions of the data shown in Figure 4, and allows us to examine the data in more detail. Both amplitude and phase are required for Schmucker inversion. Phase is not available for some data. It can be estimated from the gradient of amplitudes (Weidelt, 1972) but as this means inter-relating the various

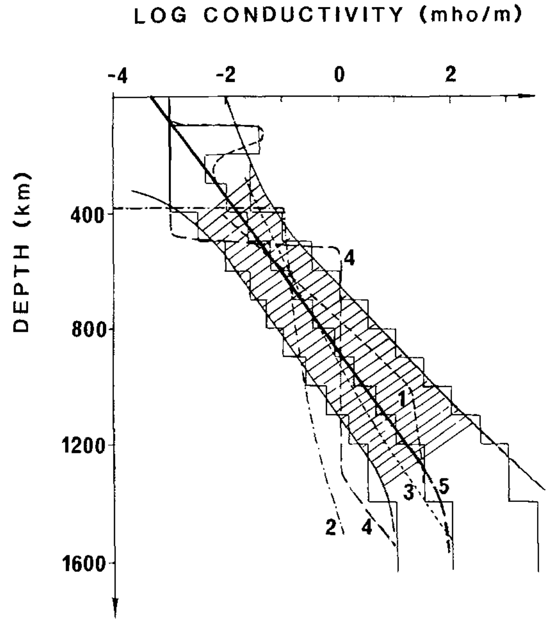

Fig. 12. Conductivity-depth profiles; (1) model of the Price-McDonald (Lahiri and Price, 1939; McDonald, 1957; Rikitake, 1973); (2) Rikitake (1966); (3) Berdichevski *et al.* (1976); (4) Dmitriev *et al.* (1977); (5) interpretation of global GDS data by an exponent (after Roktiyanski, 1982). The hatched area is Rokityanski's 95% confidence zone, derived from the models shown as stepped lines.

data this has not been done here. Phaseless data can of course be included in more sophisticated inversion procedures. The scatter of the data in Figure 10 is much clearer than in Figure 4. A large amount of this is due to the unreliable phase estimates of some analyses. Rejecting some data, e.g., that with demonstrably unreliable phase, and that which relates to oceanic stations, we obtain Figure 11.

Bearing in mind the model calculations shown in Figure 9, we can attempt to interpret Figure 11 and draw some tentative conclusions. Some of the scatter in Figure 11 is undoubtedly due to noise, but we can use the envelope of the distribution to define rough (conservative) bounds to the possible conductivity at a given depth. The large range of conductivities indicated at depths between 400 and 900 km may be related to the influence of lateral inhomogeneities within the Earth (cf. the 'electrical asthenosphere'), or could be merely a product of the large number of analyses at the relevant periods. Interestingly the structure seems to become more consistent at slightly greater depths, although this could be just a manifestation of the smaller number of analyses at these periods. The very deep estimates come from the one year and eleven year variations and are therefore comparatively unreliable and should be accorded little weight. The relationship between the Schmucker inversions of the 11 yr and 12 month data (squares in Figure 11) is not consistent with the pattern shown in Figure 9. Also, we would not expect the one year and eleven year variations to 'see' to the same depth. Therefore it seems clear that, as we

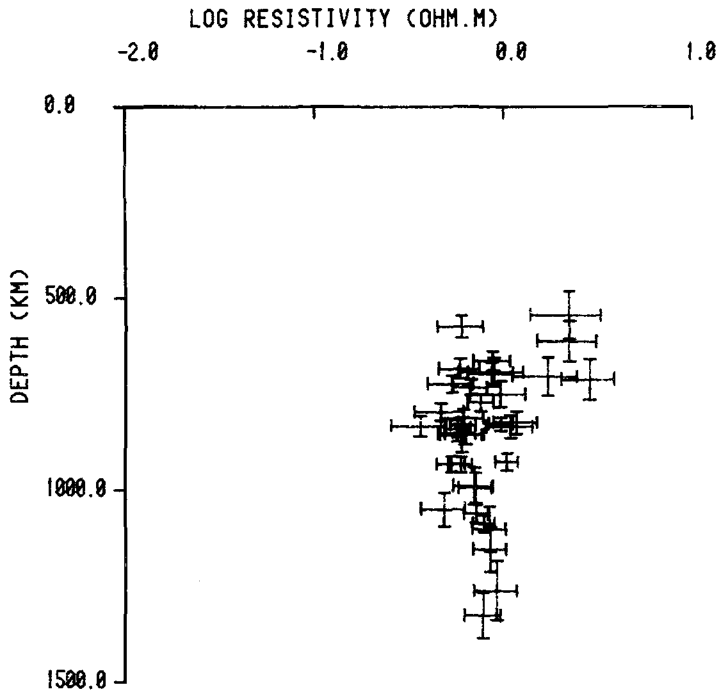

Fig. 13. Unmodified Schmucker inversion of data from Fürstenfeldbruck, Niemegk, Wingst and Wein-Koblenz with 95% confidence limits). Data from Roberts (1984).

suspected, both or either of the one and eleven year response estimates are unreliable.

It is tempting to identify the proposed step increase in conductivity at about 400 km (see e.g., Banks, 1969), but the apparent sharp step in Figure 11 is probably due only to the restricted period range of the data. There certainly seems to be a rapid increase in conductivity between 400 and 800 km, but whether this is a smooth change or consists of one or more steps cannot be resolved on Figure 11. Whether or not the Earth is radially symmetric below (say) 800 km, Figure 11 suggests that we probably cannot resolve lateral inhomogeneities at these depths using the presently available data.

All the above conclusions are tentative. The use of more sophisticated inversion procedures, especially if coupled to the selection of the 'most reliable' data set, could produce different results, and would increase the resolution dramatically. Rather than do this here we turn to some of the previously published results (Figures 8 and 12). Note that the abscissa on Figure 12 is conductivity not resistivity. With the exclusion of the higher frequency data Figures 8 and 12 are derived from largely the same data sets. It is therefore unsurprising to see the same general pattern and spread of results. While most of the results on Figure 8 are global averages, the results from Hawaii and Tucson show much more clearly than Figure 11 the possible location dependent nature of the Earth's structure. As in Figure 11, the structure appears to

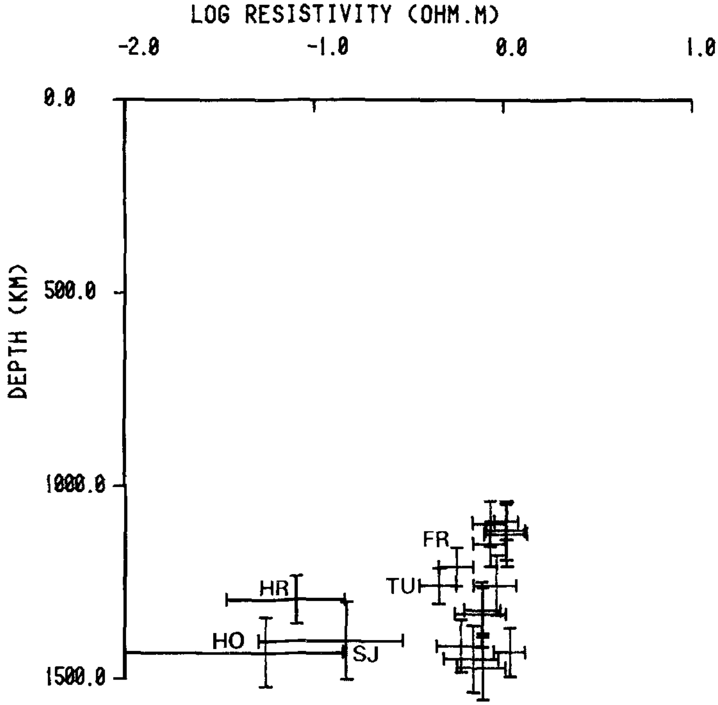

Fig. 14. Unmodified Schmucker inversion of response data at a period of about 70 days from 17 globally distributed observatories. Data from Roberts (1984). 95% confidence limits.

become more uniform with depth, and if the error bars (e.g., of Larsen) are to be believed then the conductivity at these depths is reasonably well resolved.

To examine further the possibility of laterally inhomogeneous structure at depth the data of Roberts (1984) have been used. Figure 13 shows Schmucker inversions of data from four neighbouring European observatories, with the associated error estimates. As the stations are close we would expect them to 'see' the same deeper structure, and this is clearly the case. This gives us some confidence in the data. There are apparently great differences in structure at shallower depths, and these differences seem to be resolvable. Figures 14 and 15 show Schmucker inversions of the response from 17 observatories scattered around the world at periods of about 70 days and 18 days respectively. Most of the data in Figure 14 are fairly consistent. The three 'anomalous' values are from Hermanus (HR), Honolulu (HO) and San Juan (SJ), and may be strongly affected by the presence of the oceans even at these long periods. Alternatively, the 'anomalous' response at the island stations may indicate a difference in the deep structure beneath continents and oceans. It is clear that such simple inversion of data from other locations cannot resolve differences in structure at these depths. One possible Exception to this is the data from Tucson (TU) and Fredericksberg (FR) (the only two American stations in the analysis). It seems that the conductivity of the upper mantle below these two stations (and possibly for

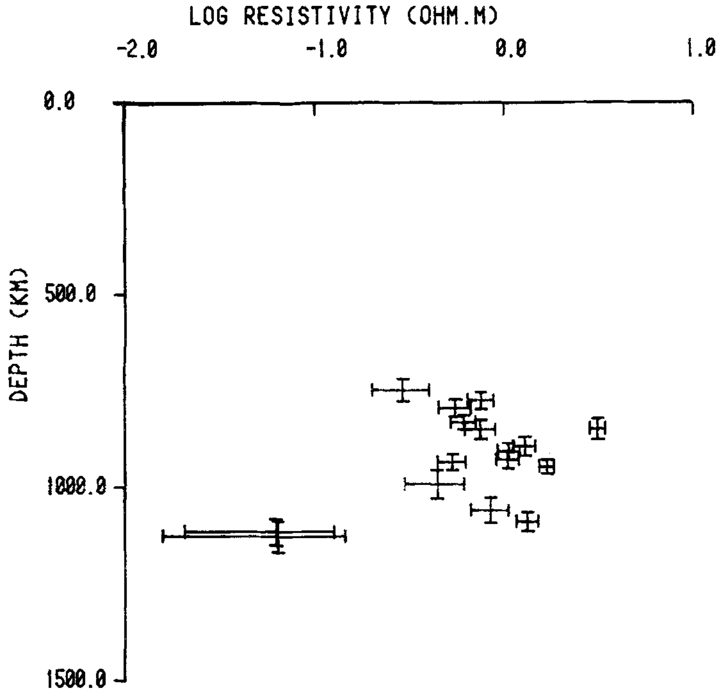

Fig. 15. Unmodified Schmucker inversion of response data at a period of about 18 days from 17 globally distributed observatories. Data from Roberts (1984).

the whole U.S.A.?) may be (resolvably) higher than that for the other locations (cf., the results of Matsushita and Maeda (1965), Section 3).

The results shown in Figure 15 suggest that the response functions are significantly different at this period (18 days). Whether these differences are due to deep or nearer surface structure, or to contaminated data, is difficult to say with certainty, but it seems likely that all three effects have some influence. The two most 'anomalous' stations shown are now Coimbra and Toledo in Spain. The phase of the response from Hermanus has become 'non-physical. so it cannot be Schmucker inverted. The response from Honolulu and San Juan are fairly consistent with most others at these periods.

To summarize: The available data at the longest periods probably cannot resolve lateral variations in continental conductivity, except for very tentative differences in the results from the U.S.A. and elsewhere. The responses from oceanic locations are significantly different from that of the continental areas, but it could be that even at these periods the response is significantly distorted by the presence of the oceans. At slightly shorter periods the responses from different locations are resolvably different. While this is probably due to both the influence of deep and nearer surface structure, it seems that lateral variation in structure is resolvable down to several hundred kilometres.

6. Conclusion

Most workers in this field, agree that there is a rapid rise in conductivity between 400 and 800 km depth, but the resolution claimed by many workers are probably over-optimistic, at least at the greater depths. The situation is likely to improve in the next few years when a large amount of new response data should become available. In order to interpret the significance of a given conductivity basic assumptions have to be made about the nature of the chemical composition of the mantle at depth. Also the effect of pressure, oxygen fugacity, amount and linkage of partial melt in the mantle are of such significance that any simple interpretation of conductivity in terms of temperature, as made by many early workers, is inadequate (see Haak (1980) and Shankland (1981) for a discussion of these points). Despite the problems involved in deducing temperature from electrical conductivity, the assumptions required do not seem so severe as those required for the interpretation of other forms of data (e.g., heat flow). It seems probable, then, that the most reliable information about the present-day thermal state of the mantle can be obtained from electromagnetic measurements.

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