ELECTROMAGNETIC INDUCTION IN THE WORLD OCEAN

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Abstract. The general state of global investigations of electromagnetic induction in the oceans and also some new results obtained in the past few years in the U.S.S.R. are considered.

Introduction

The problem of electromagnetic induction in the World Ocean is one of the principal problems in modern geomagnetism. This is so not only because it leads to the possibility of studying the electrical conductivity of the Earth's crust and upper mantle lying immediately below the ocean, but also because without a knowledge of the spatial characteristics of electromagnetic fields induced in the ocean and extending far towards the continents it is not possible to give detailed interpretations of electromagnetic soundings on the land.

Depending on the dimensions of studied fields and the methods of their analysis the problem of electromagnetic induction in the oceans can be divided into two aspects: global and local investigations. For global investigations large-scale variations of the field comparable in dimensions with the continents and the oceans are studied. The object of local investigations is the fine structure of the field lying close to isolated islands, coastlines and so on.

In the present paper, the general state of global investigations of electromagnetic induction in the oceans is considered and some new results obtained in the past few years in the U.S.S.R. are presented.

Short Historical Outline

The first attempt to take account of the effect of the World Ocean on the results of deep sounding of the Earth was made by Chapman and Whitehead (1922). The work considered the effect of a hypothetical ocean of constant depth covering the Earth, and it was concluded that induced fields are strongly distorted by the oceans. This view has also been supported by Lahiri and Price (1939) and Benkova (1941). It is evident that the effect of currents in the oceans has been over-estimated in these works because in them the effect of continents separating the oceans has not been taken into account.

The next steps in the study of this problem were the works of De-Wet, Price, Rikitake and Yokoyama, and Nagata. Using data on ocean depths and assuming continents to be non-conducting De-Wet (1949) by the method of relaxation computed current systems

Geophysical Surveys **4** (1980) 157-171. 0046-5763/80/0041 – 0157$02.25.

induced in the oceans by Sq-variations. The calculations showed that in the Pacific, Atlantic and Indian oceans current vortexes are formed. Price (1949) considered electromagnetic induction in a non-uniform thin layer and deduced boundary conditions connecting the scalar magnetic potential on both sides of the layer. He also proposed two iteration procedures for calculating fields in this model. The Price theory played an important role in subsequent calculations of electromagnetic induction; this theory has still not lost its significance. Using this theory, Rikitake (1950) considered as a model of the ocean a spherical sheet 5 km thick having a conductivity of 4 Sm. When this mode was excited by Dst variations he found that two current vortexes were formed in the north and south hemispheres. In their attempt to explain the anomaly of geomagnetic variations in Japan, Rikitake and Yokoyama (1955) considered a hemispherical model of an ocean excited by a uniform field. The calculations showed that the ocean almost completely shields fast variations and at the land/ocean boundary intensive fields are formed. This effect was named by Rikitake the 'coast' effect. On analyzing the results of calculations, Rikitake and Yokoyama (1955) noted that the results obtained by them needed improvement because a small number of harmonics was used in solving the problem and the effect of the conducting mantle was not taken into account. Nagata *et al.* (1955) verified the accuracy of calculations made by Rikitake and Yokoyama on a model of a hemispherical shell made of copper. The parameters of the model simulated a world ocean of depth 170 m excited by a field of Dst-variations. The results obtained by them were in good agreement with the data obtained by Rikitake and Yokoyama.

The next step in the study of induction in the World Ocean is related to the works of Ashour, Rikitake and Parkinson. Ashour (1950) investigated electromagnetic induction in an ocean having the form of a circular disk of radius r_0 with total conductivity S, and found that at a depth of $D = 4$ km the time τ during which the currents decaying freely decrease to 1/e of their initial value equals $4.2 \times 10^{-7}\ r_0 S$. In the case of the Pacific Ocean $r_0 = 4.10^6$ m, $S = 1.6 \times 10^4$ Sm and $\tau \sim 7$ to 8 hours. Ashour solved the problem of induction in a perfectly conducting thin hemispherical layer excited by a uniform external field (1965) and showed that induced currents concentrated close to the coastline. The magnetic field of the currents increases the horizontal component of the primary field in the ocean and reduces the radial component on the land side. These effects are more distinct in the neighbourhood of the coastline and weaken with increasing distance from the coast. The model proposed by Ashour describes the coast effect for fast variations better than the model advanced by Rikitake (1955), but this model cannot be used for slow variations of the type Sq and Dst in which the ocean cannot be replaced by a perfect conductor and in which it is necessary to take account of the effect of the conducting mantle. In 1961a, he investigated a model consisting of a perfectly conducting hemispherical layer inside which is a perfectly conducting core. The model was excited by a uniform external field. Rikitake concluded that the effect of the conducting core drastically reduces the ocean effect. This conclusion was criticized by Ashour (1965). A more exact estimate of the contribution of the conducting core in electromagnetic induction in the oceans was given by Rikitake in his paper (1961b). In this paper, Rikitake solved the problem of exciting a hemispherical ocean by spherical harmonics P_2^1 and P_3^2. The comparison of current functions calculated for the model with a core and without it shows that although the effect of the core is to decrease the ocean effect by 1.5–2 times, a significant contribution to the induced magnetic field from currents in the ocean remains.

When studying the anomaly of a geomagnetic field in Japan, Rikitake (1964) considered induction in a spherical shell imitating the distribution of oceans and land. Rikitake inferred from his results that current vortexes are formed in the oceans and that the continents produce a significant effect on the structure of the geomagnetic field. At the same time he suggested that it is necessary to take account of the effect of the mantle and pointed out that the third degree harmonics are insufficient for a complete understanding of the variations of induced fields. An attempt was also made by Parkinson (1975) to calculate currents in the oceans. For these calculations he used the Price method after combining two iteration procedures . With the aim of increasing the convergence of the method Parkinson introduced an 'accelerating factor' that enabled him to perform calculations for T = 40 min. The conductivity of the oceans was given at the nodes of a $10 \times 10^{\circ}$ grid; the continents were assumed to be non-conducting. Parkinson succeeded in obtaining a solution indicative of the formation of current vortexes in the oceans, but a strong dependence of the covergence process of the solution upon the dimensions of the grid, and the instability of the solutions, do not permit us to consider these results to be reliable.

In the last decade the theory of electromagnetic induction in the oceans has been further developed by Rikitake, Ashour, Bullard and Parker, Hobbs, Hutson, Hewson-Brown, Kendall, Malin *et al.* The most advanced model calculated by Rikitake (1968) consists of a non-uniform spherical shell imitating the actual distribution of conductivity of the continents and the oceans together with a concentric perfectly conducting core. The model was excited by a uniform external field typical of the Dst-variations. The spatial spectrum of the field was presented as in earlier models by harmonics $n \leqslant 3$. From the solution to the problem sudden commencements (Sc) were theoretically calculated for geomagnetic observatories. Rikitake believes that for certain observatories the calculated and measured variations are in agreement with each other; at the same time at a number of observatories such an analogy is not noticed. This discrepancy according to Rikitake may be due to the small number of harmonics used in the analysis and to an imperfect model of the shell. The amplitudes of the current vortexes in the oceans are significantly suppressed by the effect of the perfectly conducting core. The maximum duration of τ for which calculations have been made is only 10 minutes; the author did not succeed in solving the problem for large duration impulses because of the limitations of the method.

An interesting model of induction was investigated by Ashour (1971). In this model, the Earth was taken as a perfectly conducting sphere together with a non-uniformly conducting hemisphere the conductance of which varies according to the following law:

$$\sigma = \sigma_0 (1 + \alpha) \cos\theta \, (1 + \alpha \cos^2 \theta)^{-1}$$

The inducing field is uniform and parallel to the axis of symmetry. This model differs from all earlier proposed models in that the conductivity of the ocean decreases gradually to zero at the boundary with the land, which is supposed to be non-conducting. Numerical calculations showed that the presence of a perfectly conducting core presumed by Ashour to be located at a depth of 800 km decreases the effect of induction in the oceans by up to 50%. With increasing frequency both the shielding action of the ocean and the coast effect increase. It should be noted that very useful boundary conditions were proposed by Ashour (1974) for a thin spherical shell.

Bullard and Parker (1970) proposed an important modification to Price's method. They presented Price's equation for the current function in the form of an integro-differential equation and carried out calculations as applied to an actual ocean. The conductivity of the continents, as in the models proposed by other authors, was taken as zero. The external field was given by a set of spherical harmonics. The iteration process advanced by Bullard and Parker converges for periods more than 7 hours and diverges for smaller periods. The calculations made by Bullard and Parker and also by Richards (1970) using this iteration scheme provide support to the formation of current vortexes in the oceans.

It should be emphasized that practical calculations of electromagnetic fields for a non-uniform thin shell model on the basis of Price's iteration algorithms run into serious difficulties. The processes are convergent either in the range of high frequencies (method 1) or in the range of low frequencies (method 2); these ranges do not overlap each other. The investigations by Hutson, Kendall, Malin and Hewson-Browne (1972, 1973a, b) became an important event in the development of the theory of electromagnetic induction in the oceans. They have obtained an integro-differential equation with respect to the current in the inhomogeneous layer which may be represented in the form of a generalized linear operator equation $(\lambda I - \hat{L}) f = g$. This equation may be solved by an iterative method. In the case of axisymmetrical models, $\hat{L}$ has been shown to be a compact operator. Consequently the iterative procedure converges. The authors did not give the proofs of convergence for the method in the general case but they noticed that, on physical grounds, this iterative procedure had to converge also. In subsequent papers Kendall (1978) and Hewson-Browne and Kendall (1978a, b) proposed a number of modifications to this method.

During 1970-1976 Hobbs published a series of papers (1970, 1975, 1976) in which he analysed in detail the available methods for solving the problem of electromagnetic induction in the oceans, and successively improved Price's iteration methods. The researches of Hobbs culminated in the development of a method processing good convergence and identified by the author as analytic continuation. This method reduces to a certain transformation of power expansions into new series converging at all frequencies. Hobbs proposed a method for determining optimum parameters of the transformation and demonstrated on model examples the suitability of this method for practical calculations.

The works of Berdichevsky and Zhdanov devoted to the analysis of anomalies of variable geomagnetic fields were published in the U.S.S.R. (1973, 1974). A method for taking account of the effect of surface irregularities upon the results of global sounding is advanced in these works. According to this paper the impedance Z_n of a spherical model of the Earth consisting of a non-uniform thin shell separated from the conducting mantle by an insulator can be determined from the measured impedance Z_m as $Z_n = K_n Z_m$, where K_n is a correction factor that accounts for the effect of the irregular thin shell and is given by $K_n = (1 - \psi_n / R_0 h_\theta)^{-1}$. Here, R_0 is radius of the Earth, ψ_n is a current function, which is the solution of the system of equations of the form

$$i\omega\mu_0 r_0^2 h_r = \frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!} \sum_{k=0}^{\infty} \sum_{n=-k}^{k} \psi_k^n a_k^n$$

$$a_k^n = \int_0^{\pi} \int_0^{2\pi} \left\{ P_k^{|n|}(\cos\theta) \left[-\frac{k(k+1)}{S} - \frac{in}{\sin^2\theta} \frac{\partial}{\partial\phi}\left(\frac{1}{S}\right) + n \operatorname{ctg}\theta \frac{\partial}{\partial\theta}\left(\frac{1}{S}\right) \right] - \right.$$

$$\left. - P_k^{n+1}(\cos\theta) \frac{\partial}{\partial\theta} \left(\frac{1}{S}\right) \right\} e^{-i(n-m)\phi} P_l^{|m|}(\cos\theta) \sin\theta \, d\theta d\phi$$

and the complex coefficients h_r and h_θ are the expansion coefficients of H_r and H_θ which are the components of the magnetic field expressible as the spherical harmonic series

$$H_r = \sum_n \sum_m h_r \, e^{-im\phi} P_n^m (\cos\theta),$$

$$H_\theta = \sum_n \sum_m h_\theta \, e^{-im\phi} dP_n^m (\cos\theta) / d\theta.$$

The papers considered in this review are devoted mainly to theoretical analyses of induction in the oceans. They reveal that conductivity gradients in the Earth's upper layer can exert a pronounced effect on the structure of geomagnetic field variations. This effect must weaken with the decrease in frequency but it remains appreciable even for a 24-hour period. The presence of a conducting mantle reduces the effect of surface inhomogeneities but does not suppress it completely. At low frequencies the magnetic effect of anomalous currents may be observed at distances of hundreds and even thousands of kilometers from the anomalies.

In certain papers, attempts have been made to investigate experimentally the effect of the ocean on geomagnetic variations. For example, Parkinson, in his papers published in 1962 and 1964, analysed bay disturbances in Australia and showed that Parkinson vectors were directed perpendicular to the coast in the direction of the deep ocean. Parkinson carried out physical modelling and concluded that the observed picture may be due to sharp conductivity variations in the Earth's crust and upper mantle close to the coastline. Similar anomalies were observed by Schmucker and Cox in California, Lilley and others in Australia and in other regions. It should be noted that these anomalies have been determined mainly from data on bay disturbances and have been interpreted as local phenomena for the regions in which they were observed. In recent years, a number of papers containing elements of analysis of the coast effect have been published. Since in these papers only the local anomalies close to sharp gradients of conductivity are investigated separately from global effects no discussion of these papers will be given.

Ocean Effect for Dst-variation Fields

Out of all geomagnetic variations observed on the Earth's surface Dst-variations have the most simple spatial structure. The primary field of Dst-variations in the first approximation may be regarded as uniform. Therefore, it is quite natural to expect that the global part of the anomalous field, which is related to induction in the world ocean, will be easily isolated in individual Dst-variations. A detailed analysis of Dst-variations was performed by Fainberg, Dubrovsky and Lagutinskaya (1975). The main information about analysed storms is presented in Table I.

TABLE I

Dst No.	Date of commencement	Duration of storm in hours	Number of observatories
I	8. 06. 1958	140	87
II	4. 12. 1958	100	79
III	28. 06. 1958	108	69

Individual Dst-variations were determined by subtracting averages for the five international quiet day Sq-variations from the field of magnetic storms. Since at high latitudes it is difficult to isolate Dst-variations, the high latitude observatories were excluded from consideration. The procedure of isolating Dst and of estimating the error of harmonic analysis is available in the works of Lagutinskaya *et al.* (1973, 1974). According to recommendations given in these works the analysis was restricted to the first three odd harmonics.

The storm field was subjected to spherical harmonic analysis (SHA). The calculations showed that the harmonics determined for $1 \leqslant n \leqslant 5$ are stable enough. This means that the spatial dimensions of the studied anomalies are 8–13 thousand kilometers, i.e. they embrace areas comparable with some continents and oceans. Zonal $n = 3{,}5$ and first sectorial harmonics constitute 15–20% of the first zonal harmonic; the remaining harmonics do not exceed 10–15%. The values of apparent resistance calculated from the zonal and first sectorial harmonics agree well with the results of global soundings of the Earth (Berdichevsky *et al.*, 1976) and therefore the field described by these harmonics can be taken as normal. The anomalous internal field can be found at observatories by synthesizing harmonics. Figure 1 shows the major axes of the polarization ellipses calculated from the

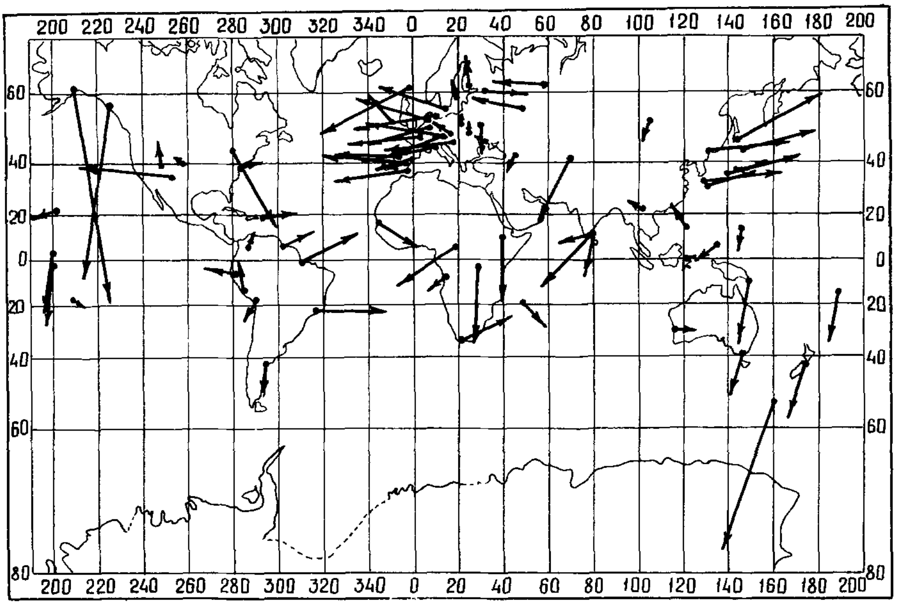

Fig. 1. Polarisation vectors of anomalous induced magnetic field. Dst–I, third harmonic.

third time-dependent harmonic of the anomalous field. From Figure 1 it is evident that a large majority of vectors are directed perpendicular to the coastline. The centres towards which the field vectors are directed lie in the Pacific, Indian and north part of the Atlantic oceans. Similar maps have been drawn for all periods for the three storms. The orientation of the vectors in the direction of the oceans is distincly shown on all the maps.

By using the coefficients of SHA we shall construct normalized equivalent current systems responsible for the anomalous magnetic fields. Figures 2(a) and (b) present maps of current systems corresponding to the third harmonic of storm No. I. The maps are drawn for $\alpha = \pi/6$ and $\alpha = 2\pi/3$. When calculating the coefficients of SHA the amplitude of the external field was normalized. From the figures it may be seen that a system of current vortexes is induced in the oceans in the course of the storms: two vortexes each in the Atlantic and Pacific oceans and one in the Indian ocean. From storm to storm these vortexes are somewhat displaced, but their centres remain within the limits of the oceans. The phases within the limits of each vortex vary very slowly; a rapid change of phase of 120–180° takes place on passing from one vortex to another.

A question arises as to whether the anomalies of internal fields are produced by irregularities of the primary field. The calculations of external current systems and their comparison with the internal systems show that vortexes of anomalous field cannot be produced only by the anomalous external field. Firstly, the induced anomalous fields have more intensity than internal fields; secondly, the configuration and location of current systems of internal fields are maintained from storm to storm at all frequencies

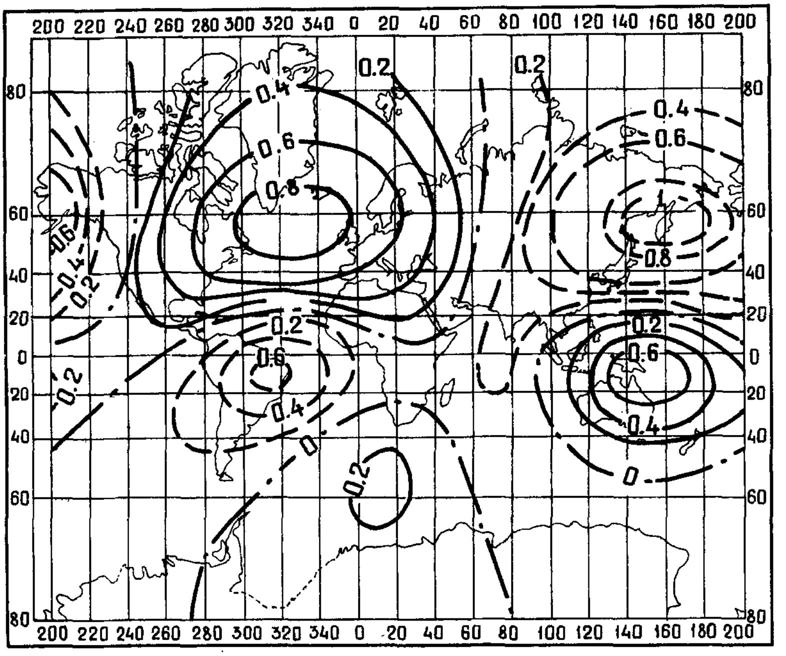

Fig. 2(a). Current streamline systems of anomalous induced magnetic field. Dst–I, third harmonic. $\alpha = \pi/6$.

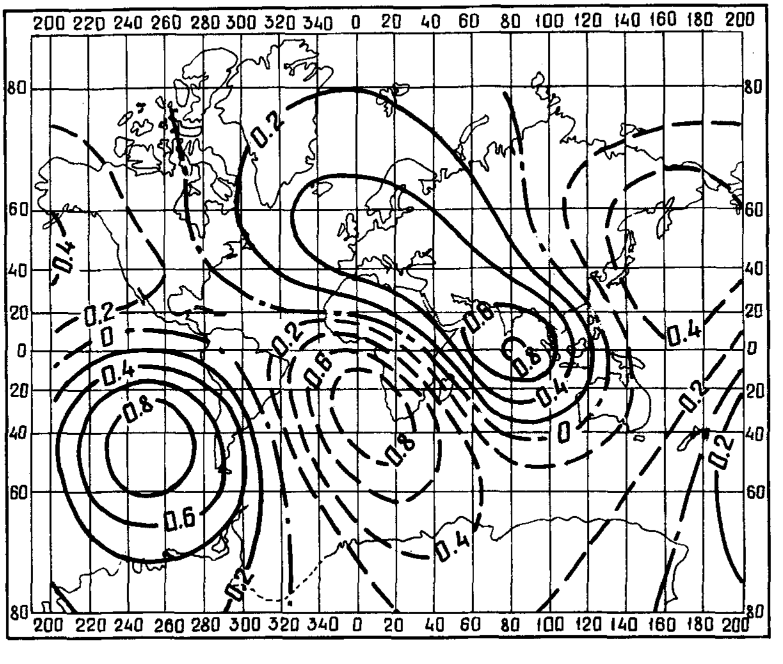

Fig. 2(b). Current streamline systems of anomalous induced magnetic field. Dst–I, third harmonic. $\alpha = 2\pi/3$.

despite significant variations in the structure of external fields.

The analysis enables a conclusion to be drawn that during world magnetic storms a global system of current vortexes is induced in the Earth, the centres of which are confined to the oceans, i.e. the ocean effect occurs. The coast effects which appear in a number of regions in the form of abrupt changes in geomagnetic variations close to the coastline apparently are indicative of the appearance of this global phenomenon.

Interpretation of the Ocean Effect

For exhaustive interpretation of the ocean effect it is necessary to solve the problems of induction in the Earth model consisting of a conducting core ('C' zone of the upper mantle), a non-uniform intermediate conducting layer (asthenosphere) and a non-uniform spherical shell. The solution of the problem in this formulation involves significant difficulties. For this reason, it is advisable at the first stage to restrict ourselves to the analysis of a simplified model, i.e. of induction in a non-uniform spherical shell by a uniform external field. The solution of such a problem for the first three spherical harmonics was given by Rikitake (1964).

When constructing the model Rikitake took the conductivity of sea water to be σ = 4S/m, the depths of the oceans were taken from bathymetric maps; for continents σ was taken as 10^{-3} S/m for an average thickness $h = 10^4$ m. The values of conductance were given at the nodes of a $15 \times 15°$ grid. Although the model proposed by Rikitake takes into account the actual distribution of conductivity only in general terms, nevertheless it is quite suitable for a qualitative study of the structure of an anomalous field.

An example of a normalized function, corresponding to an anomalous field, for a period of T = 24 hours and $\alpha = \pi/6$ is presented in Figure 3. It is evident that the external field excites current vortexes in the oceans, apart from this they coincide in their location, configuration and intensity with the vortexes isolated in Dst-variations. A good agreement of experimental data with the results of calculations provides support for the conclusion that the ocean effect is produced by induction in the non-uniformly conducting Earth.

It is obvious that induction in the oceans is only a part of the ocean effect. Actually, as was shown by Ashour (1971), the effect of the conducting core reduces induction effects by about 50%. Therefore, for a qualitative explanation of the ocean effect one has to assume that the irregularities of conductivity in the upper mantle make definite contributions to the anomalous fields. This conclusion is in agreement with the data collected by Parkinson for Australia. Two proposals can be made about the nature of non-homogeneities in the upper mantle: (a) the asthenosphere underlying the oceans is much more developed compared to the asthenosphere underlying the continents (Vanyan *et al.*, 1977); (b) the areas of increased conductivity in the upper mantle are confined to the transition zones from the continents to the oceans. It is quite possible that both these factors act at the same time. The available data are insufficient for separating fields caused by the water shell, currents in the transition zones and the non-homogeneity of the asthenosphere beneath the oceans. To solve such a problem it is necessary to take

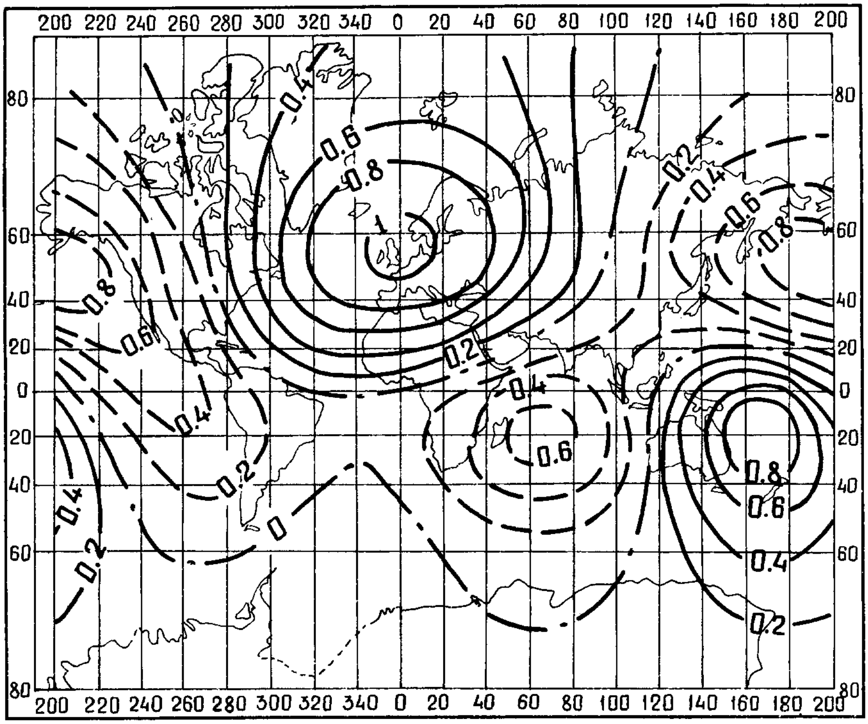

Fig. 3. Current streamline systems of anomalous induced magnetic field calculated for non-uniform thin shell. $\alpha = \pi/6$.

measurements along lengthy profiles intersecting continents, transition zones and the oceans. It is important to point out that the ocean effect opens up the fundamental possibility of studying conductivity anomalies in the Earth's crust and upper mantle in the areas of oceans and the zones of their contact with the continents.

Total Conductivity of Sedimentary Cover and Water Shell of the Earth

An important step in the solution of the problem of electromagnetic induction in the oceans is the construction of maps for the total conductivity of the non-uniform near-surface layer.

Sharp gradients of conductivity bring about significant distortions in the results of electromagnetic soundings, complicate their interpretation and involve difficulties in the construction of current systems responsible for the excitation of geomagnetic variations. The nature of the distortions depends upon the intensity and dimensions of the irregularities, the depths of their occurrence, the period and the structure of the variations.

We are familiar with the schemes of total conductivity compiled by Rikitake (1964) and Bullard and Parker (1970). These schemes take account of the actual distribution of S only in general terms, and are not suitable for the study of fine structure of induced fields.

More detailed and exact maps of S have been compiled in the U.S.S.R. by a collection of authors (1978). They include conductivity maps for the U.S.S.R., North and South America, the S map for Western Europe, and the map for the global distribution of S of the oceans and continents. The maps of total conductivity of the Earth's sedimentary cover are compiled from the data of electrical prospecting: MTS, MTP, etc., in different modifications, of sounding methods using constant current and in certain regions – from the data of direct measurements in wells. For many regions electrical prospecting data were very few; well logging for sedimentary cover also was not available. In these conditions a decisive role is played by seismic data on the thickness of sedimentary cover and on general geological and geophysical information on lithologofacies and tectonic pecularities of certain regions.

Many difficulties arise in the construction of maps for regions in which variations of S are very abrupt and which are difficult to represent in the selected scale of maps. For this reason for the majority of mountainous regions S is taken to be less than 10 S.

The S maps for water shell were composed from the data on ocean depths averaged over a $5 \times 5°$ grid. The dependence of specific electrical conductivity of sea water upon temperature and salinity was taken into account in the determination of S. The conductivity of sea sediments varies over wide limits depending upon a number of factors and may significantly differ from the conductivity of the offshore strip. This difference is most significant for friable bottom deposits having high porosity, water content and mineralization. Very scanty information is available on the resistance of sea sediments measured in naturally occurring conditions. Electrical prospecting in water areas was started recently, only some MTS and VES are available. By taking into account all the available data on electrical conductivity of friable sea deposits, ρ for sediments is taken equal to 1.5 Ωm for the calculations of S. The Arctic and the Antarctic are the least studied. For this reason in these regions the isolines are drawn tentatively. A definitive map of the values of S was drawn on the basis of averages over $5 \times 5°$ squares. The map of S for water shell and the Earth's sedimentary cover is shown in Figure 4. A general consideration of the map shows that the continents and the oceans differ significantly in their conductivity. The most abrupt gradients of S are noticed close to the transition from a continent to an ocean. It is only in these regions that the coast effect will occur. The coast and edge effects can be noticed not only at the boundaries of the continents, but in any zones of large gradients, for example, the open ocean close to abyssal depths, inside the continents etc. At the same time, the regions of conductance of 1000–3000 S are widespread at the land. This conductance is equivalent to a sea depth of 300–1000 m. The areas with S more than 10000 S (South Caspian depression) are equivalent to a sea depth of 3000 m. In certain regions continents intersect wide zones with S more than 1000 S and make channels for the flow of global surface currents. Together with local effects the global currents complicate the picture of the field and present problems in interpreting experimental data.

It may be pointed out that these maps are unsuitable for geological conclusions because geological data were employed in making these maps for the continents. For this reason, at this stage the given maps should first of all be considered as a model for

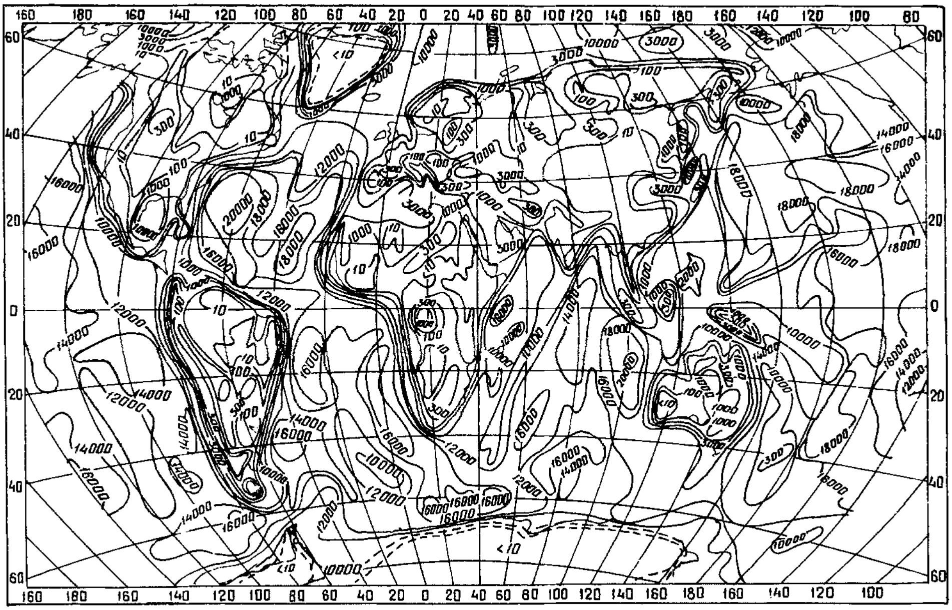

Fig. 4. Map of total conductivity of water shell and the Earth's sedimentary cover.

the study of the Earth's electromagnetic fields. In future, with the accumulation of experimental data on electrical conductivity more detailed maps may be compiled suitable for geological and tectonic analyses. It is conceivable that such maps for the whole Earth or a large part of it can be composed even now with the combined efforts of many countries.

Method of Calculating Electromagnetic Fields Induced in Non-uniform Thin Shells

As mentioned above, Hutson, Kendall and Malin (1972, 1973) and Hobbs and Brignall (1976) proposed two new methods for rearranging iteration series converging in the range of low frequencies into series converging for all frequencies. Their approach, in principle, solves the problem of electromagnetic induction in non-uniform shells. Here we discuss another iteration procedure proposed by Zinger and Fainberg (1978) which is convenient for practical realization.

We shall consider a spherical model of the Earth consisting of an inhomogeneous thin shell of radius α and integral conductivity $S = S(\theta, \phi)$, in which is present a perfectly conducting core of radius r_0 ($r_0 < \alpha$). The shell is separated from the sources and the core by an insulator. The model is excited by the poloidal mode of an external magnetic field of frequency ω. We shall write the total conductivity S in the form $S = S_0 + \delta S$ where $S = 1/R$ and

$$R_o = (\sup R + \inf R)/2 \tag{1}$$

Then from Price's equation

$$\mathrm{div}_\perp (R \text{ grad } \psi) = i\omega\mu_o H_r \tag{2}$$

we can change over to the integro-differential equation

$$\psi = \psi_o(\theta,\phi) + \oint_\Sigma Q(\cos\gamma)\, \mathrm{div}_\perp [(\delta R/R_o)\, \nabla_\perp \psi]\, \mathrm{d}\Sigma \tag{3}$$

with the kernel

$$Q(\cos\gamma) = \sum_n \frac{2n+1}{n(n+1)} \, \frac{1}{1+iD_n} \, P_n(\cos\gamma) \tag{4}$$

in which

$$\eta = \omega\mu_o S_o r/R_o, \qquad \xi = r_o/\alpha \qquad D_n = \eta\, \frac{1-\xi^{2n+1}}{2n+1} \quad . \tag{5}$$

This equation can be solved by an iteration method. Let us consider the convergence of the iteration process. Considering functions $U(\theta,\phi)$, $\psi(\theta,\phi)$ as vectors in L_2-space we shall replace the integral in the right-hand of (3) by an operator type $\hat{L}\bar{U}$, where

$$\hat{L} = -\sum_n (1/(1+i\mathrm{D}_n))\hat{P}_{nm} \; .$$

Here P_{nm} is a projection-type operator on S_{nm}, and U is determined from the relationship

$$\Delta_\perp U = \mathrm{div}_\perp \left(\frac{\delta R}{R_o}\ \nabla_\perp \psi \right).$$

Let us introduce the norm

$$||\bar{U}||^2 = \oint_\Sigma |\nabla_\perp U|^2\ \mathrm{d}\Sigma$$

It is evident that

$$||\hat{L}\bar{U}|| \leqslant \max_n \left| \frac{1}{1+iD_n} \right|\ ||\bar{U}|| = ||\bar{U}|| \; . \tag{6}$$

From the definition of U it follows that

$$||\bar{U}||^2 = -\oint_{\Sigma}\tilde{U}\Delta_\perp U \mathrm{d}\Sigma = \left|\oint_{\Sigma}\frac{\delta R}{R_o}\nabla_\perp \tilde{U}\nabla_\perp \psi \mathrm{d}\Sigma\right| \leqslant \sup\left|\frac{\delta R}{R_o}\right| ||\bar{U}||\cdot||\bar{\psi}|| . \quad (7)$$

Here $\sim$ represents the compex conjugate. Thus,

$$||\bar{U}|| \leqslant \sup \left|\delta R/R_o\right| \cdot ||\bar{\psi}||.$$

By comparing (7) with (6) we see that

$$||\oint_{\Sigma} Q(\cos\gamma)\,\mathrm{div}_\perp(\delta R/R_o \nabla_\perp \psi)\,\mathrm{d}\Sigma|| \leqslant \sup|\delta R/R_o|\cdot||\bar{\psi}||$$

for any function $\psi(\theta,\phi)$. By taking into consideration (2), from the latter inequality it follows that the iteration process converges for all initial approximations $\psi(\theta,\phi)$. Thus, this method can be succesfully employed to calculate electromagnetic induction in the oceans.

Conclusions

The review of the state of the problem of electromagnetic induction in the oceans shows that in recent years significant progress has been made in this direction. General laws concerning induced fields have been established, theoretically and experimentally it has been shown that external fields induce current vortexes in the Earth, confined to the oceans. Inhomogeneities in the conductivity of the upper mantle play a definite role in induction. Maps for total conductivity of sedimentary cover and the Earth's water shell have been compiled, and methods for solving direct and inverse problems of electromagnetic induction in non-uniform thin layers have evolved. It is hoped that in the coming years the function of the oceans in the formation of geomagnetic field variations will be fully determined.

Acknowledgements

I would like to thank Professors A.A. Ashour, M.N. Berdichevsky, V.I. Dmitriev, P.C. Kendall and B.A. Hobbs for helpful discussions.

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