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NUMERICAL AND ANALOGUE MODELLING OF INDUCTION EFFECTS IN LATERALLY NON-UNIFORM CONDUCTORS

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Modelling of electromagnetic fields in laterally non-uniform geoelectric structures is briefly reviewed. Attention is paid to progress achieved specifically during the last three years in numerical and analogue techniques of solving two- and three-dimensional problems. Some new important results are also mentioned.

1. Introduction

Research into the induction effects of natural electromagnetic fields and investigation into the distribution of the electrical conductivity in the earth have recently made an important contribution to our knowledge of the earth's body. Methods of geomagnetic and electromagnetic sounding (MTS) over broad period bands have been extensively used in estimating the peculiarities of the conductivity distribution on global, regional, and local scales.

The first theoretical studies and estimates of the conductivity date back to 1939 (Lahiri and Price, 1939). Further basic papers, leading to the specific subject of this review, by Price (1950), Tikhonow (1950), Cagniard (1953) and several others can be referred to. The actual application of the electromagnetic methods to the earth's conductor raised new problems as to the interpretation of the results obtained by observing superficial fields on MTS sites or over large arrays of simultaneously operated magnetometers. The source effects on the one hand, and local perturbations of a system of induced currents by abrupt changes of the conductivity on the other hand, were supposed to explain the observed effects. The importance of lateral inhomogeneities was recognized and different approaches to the solution of the theoretical field distribution in these complex conductors have been worked out during recent years.

The development and the stage of treating the above problems at the time of the First Workshop in 1972 (Electromagnetic Induction in the Earth and Planets, 1973) was summed up by Jones and published subsequently (Jones, 1973a). Avoiding pure analytical methods, which were covered in session 2a of this Workshop, I shall confine myself to the results obtained by numerical and analogue methods during the last two years.

2. Two-dimensional models

Two-dimensionally inhomogeneous models approximate a broad family of important linear geological structures of long extent where two pronounced mutually perpendicular directions can be defined. A semi-infinite conductor occupies the region $z > 0$, and rather general changes of the electrical conductivity are confined to the zy-plane and the configuration of the conductivity is independent of the x-coordinate which, therefore, represents the axis of structure homogeneity.

The electromagnetic field is assumed to be oscillating with period $T = 2\pi/\omega$ long enough for displacement currents to be negligible (Price, 1950, 1967). Because all quantities are presumably independent of the x-coordinate, two independent field modes are decoupled from the general system of Maxwell's equations (the MKS system and a time factor $\exp(-i\omega t)$

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are used here). They are called the *E-polarization case* characterized by the equations:

$$\left.\begin{aligned} &\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma E_x \\ &\frac{\partial E_x}{\partial z} = \mathrm{i}\omega\mu H_y \\ &\frac{\partial E_x}{\partial y} = -\mathrm{i}\omega\mu H_z \end{aligned}\right| \qquad (1)$$

and the *H-polarization case* given by equations:

$$\left.\begin{aligned} &\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = \mathrm{i}\omega\mu H_x \\ &\frac{\partial H_x}{\partial z} = \sigma E_y \\ &\frac{\partial H_x}{\partial y} = -\sigma E_z \end{aligned}\right| \qquad (2)$$

It is seen from the systems 1 and 2 that E_x and H_x satisfy the equation:

$$\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -\mathrm{i}\omega\mu\sigma U$$

where U has to be replaced by either E_x or H_x for the relevant polarization. According to eqs. 1 and 2 the solutions for E_x and H_x are sufficient to obtain all other field components.

Eqs. 1 and 2 have to be solved with proper boundary conditions defined at internal interfaces where abrupt changes of conductivity take place, and at external boundaries limiting the extent of the model for $z \to \pm\infty$ and $y \to \pm\infty$. For detailed analysis of both kinds of boundary conditions and also for differences between the E- and H-polarization cases we refer to papers by Price (1950), Jones and Price (1970) and Schmucker (1971).

In the pure numerical methods a two-dimensional structure of rather general conductivity distribution is first restricted to a finite region which is subsequently covered by a mesh of grid points, or by certain finite elements, or by conductive cells. The field distribution in the entire region is approximated by coupling the behaviour of partial fields in the elementary units. The boundary conditions are respected and the elementary fields are obtained by making use of certain basic principles of physics.

3. The finite-difference method

Starting with Neves (1957) the method was further developed by Price, Jones and Pascoe and has been extensively used in numerical modelling of induction problems. Full details of this technique were explained in Jones' review (Jones, 1973a), where also relevant references to original papers by Jones and Price (1969, 1970) and Jones and Pascoe (1971, 1972) can be found. In these papers a general computing program was developed for solving for the total field in two-dimensional models of a general nature. The application of the computational procedure was also extended to solving for just the perturbation component of the total field (Jones, 1972). This approach was further developed and the general computing program readapted for studying the frequency dependence of the geomagnetic perturbation fields due to lateral conductivities in the earth (Jones and Ainslie, 1973a). Models of a conducting half-space which consisted of two quarter-spaces of different conductivities (one-order difference) were analysed simultaneously with a model of a more conducting dike embedded in a half-space. Both the H- and E-polarization cases were considered for several different frequencies (1.0, 0.1, 0.033 and 0.01 Hz) and the perturbation field component profiles as a function of height above the surface of the conductive region were studied. The perturbation fields were shown to be strongly dependent on the frequency of an applied field, and significant differences were exhibited between the H- and E-polarization cases.

The method was further refined for solving specific problems where high conductivity contrasts must be assumed for representing adequately some specific conductive structures (oceanic–continental interface, island structures etc.). Two approaches were considered by Jones and Ainslie (1973b) to solve high conductivity contrast problems for the E-polarization case. In the first approach they increased by a factor of about four the number of points in the grid in the horizontal direction. Approximately the same result, however, was obtained by approaching the problem with a technique which the authors call the folding-in technique. This method approximates a solution successively from a grid with large grid dimensions to a more accurate solution for a grid with smaller horizontal grid dimensions.

A simple vertical contact of two different conductive regions, the shelf and the sloping shelf models were solved for conductivity contrasts of $4–6 \cdot 10^3$ between regions. Numerous results shown in the above paper prove the folding-in technique to be an efficient method of dealing with high conductivity contrasts. Specifically, it gives a substantial reduction in computing time compared with the large-grid approach and also appears to offer a better matching of the boundary conditions leading to a more accurate solution.

The finite-difference technique has also been developed since 1969 by our research group. Pure two-dimensional models covered with a mesh of grid points intentionally variable in both the horizontal and vertical directions were investigated for E- and H-polarization cases. In approximating the Laplacian operator we used Green's theorem. Contour graphs of relevant field components and apparent resistivities as functions of the period have resulted from our treatment. In the final step the MTS curves were derived at specific points of observation on the earth's surface. The curves were plotted as functions $\rho_z/\rho_1 = f(\lambda/h_1)$ where a normalizing resistivity ρ_1 was determined for such a short period that the field distribution was not affected by the inhomogeneity. The wave length λ was calculated from the given parameters according to the formula $\lambda = \sqrt{10 \rho T}$ (km), where h_1 (km) is the thickness of the superficial layer.

First, some models of block structures were studied (Nedoma and Praus, 1972; Praus et al., 1973a). The method was further extended by introducing approximations for inclined boundaries (Červ and Praus, 1972), and some models with dipping contacts were analysed (Praus et al., 1973b). These models were intended to account for realistic conductivity configurations in the marginal parts of sedimentary basins where relatively well conducting sediments were deposited on the slopes of a less conducting basement.

In Fig. 1 the apparent resistivity functions, normalized to ρ_1, are displayed for a simple infinite contact of two subregions for the E-polarization case. Different sloping angles are assumed. In Fig. 2 the apparent resistivity functions are shown along the profile across the margin of a sedimentary basin. The effects of the sloping contact are apparent from the graphs. A zone of depressed resistivities above the sloping contact is characteristic of this model.

At present, structures with different stratification on both sides of the model are being analysed. In Fig. 3 the apparent resistivities are plotted for differ-

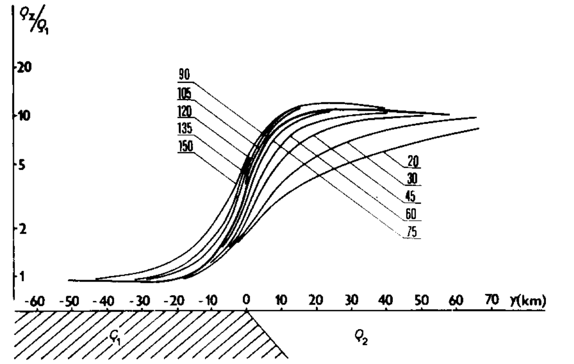

Fig. 1. E-polarization case.
Apparent resistivity curves, normalized with respect to ρ_1 along the sloping contact of two blocks with electrical resistivities $\rho_1 = 10^2$ and $\rho_2 = 10^3$ Ωm. The curves are labelled according to the sloping angles (90°: vertical contact, $\alpha < 90°$: underthrusting of more conductive material). Period $T = 10$ sec.

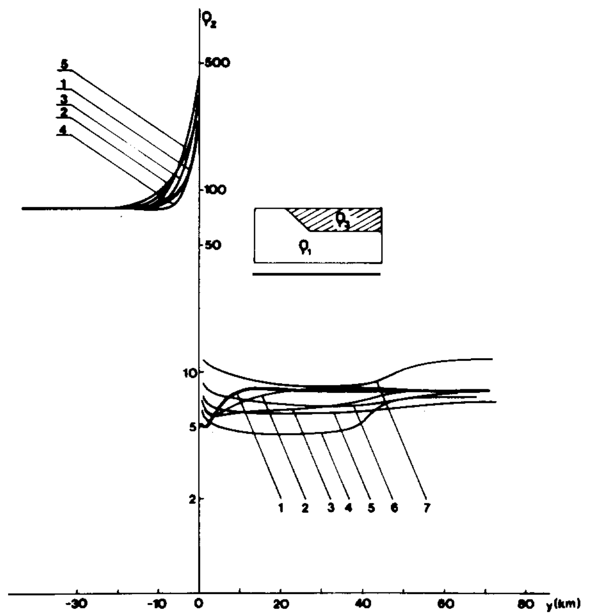

Fig. 2. H-polarization case.
Resistivities, proportional to apparent resistivities across an inclined margin (45°) of a basin. Parameters of the model are $\rho_1 = 10^3$, $\rho_3 = 10^2$ Ωm, $h_1 = 40$ km. The curves are labelled according to the source field periods (sec): *1* = 1; *2* = 5; *3* = 10; *4* = 25; *5* = 50; *6* = 100; *7* = 250 sec.

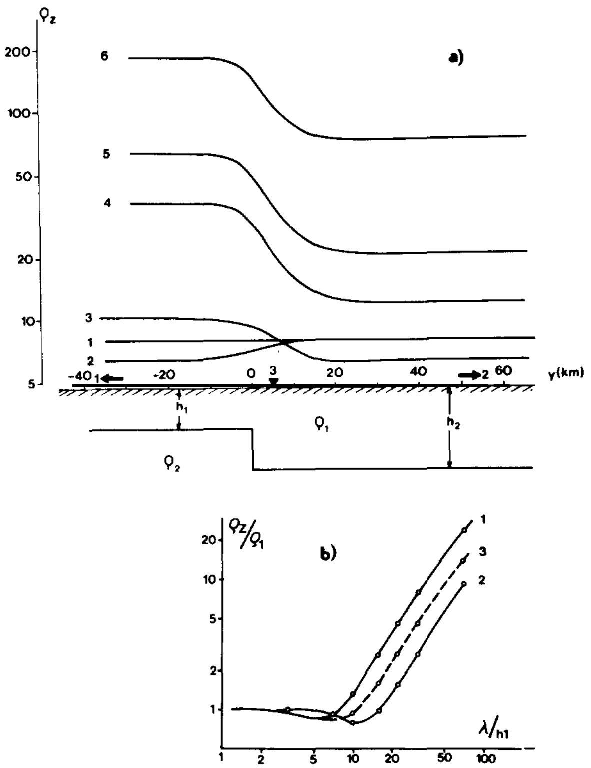

Fig. 3. H-polarization case.
a. The resistivities proportional to apparent resistivities across a less conductive step in the basement. $\rho_1 = 10$, $\rho_2 = 10^3$ Ωm, $h_1 = 10$ km, $h_2 = 20$ km. Periods (sec): $1 = 1$; $2 = 25$; $3 = 100$; $4 = 500$; $5 = 1{,}000$; $6 = 5{,}000$ sec.
b. The MTS sounding curves: *1* for $y < -40$ km. *2* for $y > 40$ km. *3* for $y = 5$ km.

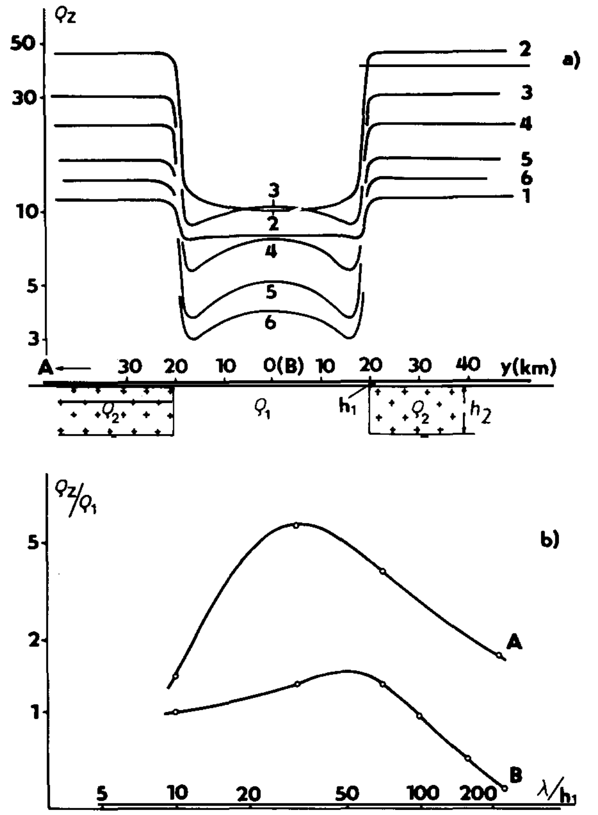

Fig. 4. H-polarization case.
a. Resistivities proportional to apparent resistivities across two non-conductive blocks placed in a homogeneous medium. Parameters of the model are $\rho_1 = 10$, $\rho_2 = 10^3$ Ωm, $h_1 = 1$ km. Curves are labelled according to source field periods (sec): $1 = 1$; $2 = 10$; $3 = 50$; $4 = 100$; $5 = 250$; $6 = 500$ sec.
b. The sounding curves: A for $|y| > 40$ km, B for $y = 0$.

ent periods along the profile across a less-conducting step in the basement (Červ and Praus, 1974). In Fig. 3b the corresponding MTS curves are shown for three particular points of observation. In Figs. 4a, b the apparent resistivity functions are plotted along the profile across two less-conducting blocks placed in a homogeneous medium. Strong edge effects are apparent in Fig. 4a near the block boundaries. Two MTS curves far from the vertical boundaries (A) and in the central part of the model (B) are displayed in Fig. 4b. A fictitious more conductive layer is suggested by the form of the MTS curve at site B although the medium is presumably homogeneous.

In treating the anomalous induction effects of island-type structures considerable attention was paid by Jones' group to extend the finite-difference technique to treating three-dimensional models. In these problems the field components remain coupled and there is no possibility of separating any specific direction of polarization. There are altogether twelve field quantities to be determined (six field components and the corresponding phases), and this amount of information becomes a problem to deal with.

Preliminary results of treating three-dimensional models were published by Jones and Pascoe (1972) and the method was extended for more general models in Lines (1972). Some of the results for single- and two-island structures were published in two papers (Lines and Jones, 1973a, b). The computational results reported in the above papers were subject to discussion concerning the interpretation of results (Nyland, 1973) and basic principles of the approximation method used (Rankin, 1973). In their replies the authors have shown that the procedure is a valid approximation to the geophysical situation and is adequate for their investigations. (Lines and Jones, 1973c; Jones, 1973b).

Referring to the basic paper by Price (1950) and using the method of Peltier and Hermance (1971) the authors Hibbs and Jones (1973) applied the finite-difference technique to solve for the electromagnetic fields induced in the earth by a non-uniform current source. They calculated the apparent resistivities from the Cagniard formula for models in which the earth's conductor has lateral and horizontal discontinuities and the source field is of a Gaussian electrojet. For this type of source they found the apparent resistivities at longer periods (greater than about 10^4 sec) to be considerably different from those for a uniform source.

At this point I should mention a discussion which has appeared quite recently. The authors Williamson et al. (1974) suggested an approximation for the Laplacian operator different from that used in the general programs developed by Jones and Pascoe (1971, 1972). Williamson et al. also showed that significant differences between their results and the Jones–Pascoe results may appear when calculating the electromagnetic response if meshes with strongly irregular grid spacings are considered. In their numerical example they intentionally defined a grid very unwisely to get enhanced errors. However, this was not the way the J–P program was used. In the works published by Jones and his co-authors gradual changes in grid spacings are usually defined, which makes the differences between the two methods small, between 3–5% increasing to about 10% near the interface for apparent resistivity and less for the field values themselves, as shown in the reply by Jones and Thomson (1974).

Jones and Thomson explain that the different finite-difference form of the Laplacian is related to the manner in which the first derivatives are approximated. Jones and Pascoe assumed a central difference form for the first derivatives from the outset, whereas the method of Williamson et al. implies the approximation of the first derivatives at a particular point by a weighted average of the derivatives in the adjacent regions. This modification introduces a greater degree of accuracy into the J–P finite-differences scheme and simple changes to the relevant subroutines of the original program are given.

From the discussion it is clear that the grid must be carefully chosen in any case and as nearly uniform as possible to get an approximation within a reasonable number of iterations.

4. The transmission surface analogy

The use of the transmission line and transmission surface analogies for solving Maxwell's equations in a two-dimensional form arises from the similarity of Maxwell's equations governing the electromagnetic field components and the transmission line equations for current and voltage on the transmission line. The analogy emphasizes the role of the impedance as the important physical parameter coupling E and H fields and shows that the cross-coupled first-order Maxwell equations are more basic than the associated uncoupled Helmholtz equation for separate components.

This analogy has been extensively used for studying perturbations of the electromagnetic field by lateral inhomogeneities. For basic theory we refer to papers by Madden and Thompson (1965), Madden and Swift Jr. (1969), and Swift Jr. (1971). For a transmission surface the equations read:

$$\left.\begin{aligned} &\frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = -YV \\ &\frac{\partial V}{\partial z} = -ZI_z \\ &\frac{\partial V}{\partial y} = -ZI_y \end{aligned}\right| \qquad (3)$$

where V and I are the voltage and current along the line and Z and Y are the distributed impedance and admittance parameters per unit length, respectively. The basic analogy is between equations 3 and one of the systems 1 or 2. In each of the polarizations one field component, linearly polarized along the axis of the structure homogeneity, is cross-coupled with the two components of the other field. Thus, by associating V with H_x (or E_x) and the components of I with E (or H) we can express the distributed circuit parameters of the equivalent transmission surface in terms of the relevant parameters. In defining the proper associations we must respect the principles of energy conservation.

The associations can be summed up for both polarizations as given in Table I.

In introducing the external boundary conditions for the H-polarization case we assume a constant voltage associated with H_x = constant at $z = 0$. For the E-polarization case a constant vertical current is associated with the H_y component which is constant at the top of an air layer. The conditions at the boundaries which limit the horizontal extent on both sides of a model are obtained by solving a one-dimensional transmission line problem to get the voltage boundary values.

After a specific model has been covered with a mesh of electrically homogenous cells, Kirchhoff's law of electrical current continuity can be written at each node and a set of equations is obtained. This set of equations is solved in the next step by a suitable numerical method (matrix inversion, iteration or Gaussian elimination).

The above approach of analogue modelling was used in the interpretation of electromagnetic surveys and MTS results. For instance, some fault zones appearing as thin conductive dikes were investigated by Vozoff (1972). In studying a buried graben in the western Canadian basin Peeples and Rankin (1972) applied the analogue technique to modelling the theoretical responses of both the polarization cases.

More recently a group of authors, Ku et al. (1973), has made use of the network analogue method in studying the topographic effects on electromagnetic fields. They give the analogue solution for rather complicated models where both topographic and buried lateral inhomogeneities are involved. In several figures the authors showed that various topographies may give rise to electromagnetic field distortions and they may account for some anomalous effects. Their results prove the analogue technique to be an important and efficient numerical method.

TABLE I

Associations for network analogy solution

E-polarization	H-polarization
$E_x \Leftrightarrow V$	$H_x \Leftrightarrow V$
$H_y \Leftrightarrow I$	$E_y \Leftrightarrow -I_z$
$H_z \Leftrightarrow -I_y$	$E_z \Leftrightarrow I_y$
$Z \Leftrightarrow -i\omega\mu$	$Z \Leftrightarrow \sigma$
$Y \Leftrightarrow \sigma$	$Y \Leftrightarrow -i\omega\mu$

5. The finite-element method

The introduction and application of the finite-element method to geophysics is quite new. Coggon (1971a, b) and Ryu (1971) have applied the technique to modelling lateral inhomogeneities buried in a conductive half-space.

The method is based on the physical principle stating that any electromagnetic system behaves in such a way as to minimize its total energy. In the finite-element approximation a two-dimensionally inhomogeneous structure is represented by a mesh of finite elements inside each of which the total electromagnetic energy functional is minimized. A variety of element types (triangular and quadrilateral) may be introduced for solving models with specific geometry.

Following Coggon (1971a) the corresponding electromagnetic energy functionals to be minimized in each eth element can be written as:

$$U = \int_{\text{vol}} (2\omega^2 \mu_e)^{-1} |(\nabla \times \boldsymbol{E}_e)^2 + k_e^2 \boldsymbol{E}^2| \mathrm{d}v \tag{4}$$

and

$$U = \int_{\text{vol}} \mathrm{i}[2\omega(\sigma_e - \mathrm{i}\omega\epsilon_e)]^{-1} |(\nabla \times \boldsymbol{H}_e)^2 + k_e^2 \boldsymbol{H}^2| \mathrm{d}v \tag{5}$$

where index e specifies the following quantities in each eth element: $\boldsymbol{E}_e$ and $\boldsymbol{H}_e$ are the electric- and magnetic-field vectors respectively, μ_e is the magnetic permeability, $k_e = |\mu_e \epsilon_e \omega^2 + \mathrm{i}\omega\sigma_e\mu_e|^{\frac{1}{2}}$, σ_e is the electrical conductivity, and ϵ_e is the dielectric permittivity. The two-dimensional character of the problem makes it possible to replace the field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ in eqs. 4 and 5 by their scalar counterparts from eqs. 1 and 2.

The external boundary conditions are introduced in a similar way as with the other numerical methods. Minimization of the functionals 4 and 5 leads to a set of linear algebraic equations which are solved by usual procedures (matrix inversion, Gaussian elimination).

For actual results obtained by this approach we refer to the paper by Reddy and Rankin (1973). They used triangular elements for studying the effect of a two-dimensional sloping contact on the surface electromagnetic field in both the E- and H-polarization cases. Their graphs show that there are certain features common with our results reported previously (Praus et al., 1973b; figs. 1 and 2).

Numerical results obtained with quadrilateral elements consisting of four triangles for complex geological models are reported by Ward et al. (1973). The apparent resistivity response from a salt ridge and from island structures can be found in the above paper. Also the VLF responses from both conductive and magnetic semi-infinite dikes covered with a conductive overburden are given in the same paper. The effects of tilt angle and ellipticity on VLF responses from both the conductive and magnetic dikes were studied as well.

To conclude, the above results have proved the finite element approach to be an efficient method of modelling two-dimensional problems with a high degree of accuracy. On the other hand, increased requirements on computer capacity and related problems seem to be setting certain limits to extensive use of this method. Obviously, for the same reason no attempts have been reported as yet to solve three-dimensional models.

6. Analogue-model studies

Numerous results obtained by analogue modelling on laboratory scale models have proved this method to be a useful tool for investigating the problems related to perturbations of the field by lateral conductive inhomogeneities which do not yield readily to mathematical solution. The method is not restricted to simple two-dimensional problems, but rather complex two- and three-dimensional structures are tractable by the analogue-modelling technique. The theory, analogue-model configurations, measuring equipment and many results were summed up in Dosso's review (Dosso, 1973) for the First Workshop in 1972 (Electromagnetic Induction in the Earth and Planets, 1973).

Respecting the principles of electromagnetic similitude, which define the conditions for invariance of results under a change of scale, analogue-modelling configurations were developed by different research groups. The Dosso configuration was described in detail in his review (Dosso, 1973). It is of interest to supplement here some details on the equipment and results obtained by Soviet scientists.

There are two main groups carrying on laboratory scale analogue modelling. One group working in the Department of Earth Physics of Leningrad State University has been dealing with this method since 1959. A detailed description of the equipment was given by Brjunelli et al. (1969). Their analogue model consists of an overhead oscillating field source. It has the form of a quadrilateral solenoid with six windings of 3×2 m^2 dimensions each. It provides a source field uniform over 2×2 m^2 area of a large tank (4.2×4.2 m^2) containing concentrated salt solution (NaCl) with resistivity 0.055 Ωm. The field components E_x, H_y, H_z (or E_y, H_x, H_z) are detected by one electric and two magnetic dipoles which are placed together into a measuring unit. The modelling frequencies are between 0.4 and 50 kHz. From the measurements the behaviour of impedances and of related MTS curves were studied simultaneously with the H_z component anomalies for different inhomogeneous structures.

The research group analysed, for example, the distortions of MTS curves over a non-conductive step (Dobrovolskaya and Kovtun, 1970) and their results were consistent with those obtained analytically by Dmitriev (1969) and Dmitriev et al. (1973). Analyses of superficial field behaviour over a long triangular rise and depression of a non-conductive basement were performed by Kovtun et al. (1970). Modelling was also done for a three-dimensional non-conducting truncated cone embedded in a good conductor. With the same model the shielding effects of a thin insulating layer placed above the cone were investigated as well (Dobrovolskaya et al., 1970). The results are presented in the form of impedance profiles across the structure and some MTS curves are displayed.

The other research group dealing with analogue laboratory scale models is working at the Lvov Branch of the Institute of Geophysics, Academy of Sciences, Ukrainian SSR. The basic configuration of modelling equipment is essentially similar to that of the Leningrad group. It consists of an electrolytic tank ($4 \times 3.75 \times 0.65$ m^3) with saturated salt solution (NaCl). The applied source field is produced by a system of six wires (1.95×4.4 m^2) encompassing the whole electrolytic tank so that the primary uniform magnetic field inside the antenna-solenoid is used. More details were described by Bondarenko et al. (1971). An automated version of the above analogue configuration has been reported recently by Bondarenko et al. (1973).

This equipment was exploited for modelling electromagnetic fields over complex inhomogeneous structures. Recently, Kulik et al. (1974) have published the results of their studies of the frequency responses for conductors of different widths and for some models of the conductor with a variable cross-section.

7. Conclusion

The results mentioned in the preceeding parts of the review show that numerical and analogue methods of modelling have now become standard tools for interpreting induction and MTS results. I have not mentioned the very many papers making use of these approaches for the interpretation of data on regional or local induction studies. My attention was concentrated to papers where the above methods were further developed.

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