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GLOBAL GEOMAGNETIC SOUNDING – METHODS AND RESULTS

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The methods of global geomagnetic sounding and the results obtained by these techniques are examined. The principles and limitations of modelling methods, exact inversion methods, and heuristic (Backus-Gilbert) methods are discussed. The evolution of our picture of the earth's overall radial electrical conductivity distribution with successive improvements in data and mathematical techniques is described. The prospects for improving this picture are also discussed.

1. Introduction

It is now about 80 years since the electrical conductivity of the earth was first studied. In 1883 Lamb mathematically formulated the problem of electromagnetic induction in a spherical earth by external magnetic field variations. Using this theory, Schuster (1889) deduced from magnetic variation data that the earth acted like a conducting body. Since then both the methods and results of geomagnetic deep sounding have improved considerably. This article will discuss them in the context of the earth as a whole and its overall radial conductivity distribution.

The central equation of geomagnetic deep sounding is the induction equation:

$$\nabla^2 A = i\omega\mu\sigma(r)\,A$$

Here A is the magnetic vector potential, ω the angular frequency, σ the conductivity and μ the permeability. There are excellent physical reasons (Tozer, 1959) for supposing μ to be very close to μ_0 in the deep earth, so that the conductivity is the only variable coefficient in the above equation. Only poloidal fields are involved in induction processes; this restricts A to the form: (Lahiri and Price, 1939):

$$A = \mathbf{r} \times \nabla\psi$$

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inside the earth, where ψ also satisfies the induction equation. ψ can be expanded as a sum over spherical harmonics:

$$\psi = \sum_{n=1}^{\infty} \sum_{m=-n}^{+n} f_n^m(r)\, P_n^m(\cos\theta)\, e^{im\varphi}$$

where $f_n^m(r)$ satisfies the radial induction equation:

$$\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} f_n^m = [n(n+1) + \mu_0\, i\,\omega\,\sigma(r)\, r^2]\, f_n^m$$

Outside the earth, the field can be expressed as the gradient of a potential W, which can also be expanded in spherical harmonics:

$$W = \sum_{n=1}^{\infty} \sum_{m=-n}^{+n} \left[e_n^m \left(\frac{r}{a}\right)^n + i_n^m \left(\frac{r}{a}\right)^{-n-1} \right] P_n^m(\cos\theta)\, e^{im\varphi}$$

Here a is the radius of the earth, e_n^m is the amplitude of the external inducing field in a particular spherical harmonic and i_n^m the amplitude of the corresponding internal induced field. The solutions and their radial derivatives above and below the surface of the earth must be fitted smoothly together. The resulting ratio of the induced amplitude i_n^m to the inducing field amplitude e_n^m is $S_n^m(\omega)$, the geomagnetic response of the earth in the spherical harmonic mode P_n^m. It is a function of frequency and depends of course on the conductivity distribution.

The central problem of geomagnetic deep sounding is to obtain the radial conductivity distribution from information about one of more of the $S_n^m(\omega)$. This is the subject of this paper. It will start by discussing the modelling method, which was the first to be applied, and then go on to the more sophisticated methods that have recently been developed.

2. Modelling methods

Modelling consists simply of guessing a conductivity distribution and comparing the calculated response of this model to the actual data. This carries no guarantee that the final answer will be unique, but has the advantage that the direct problem is fairly easy to solve. Before the advent of the electronic computer, even the modelling method was rather restricted in scope. Conductivity estimates had to be based on data which were easily obtained and on models which could be solved analytically. The useful data were therefore restricted to large and well defined magnetic variations, such as the quiet daily variation (S_q) and its harmonics, or magnetic storm variations (D_{st}). The S_q-variations have as their source two current loops in the day-lit hemisphere of the ionosphere, and therefore have a fundamental period of a day. The spatial distribution is such that the variation is dominated by the spherical harmonic P_n^{n-1} at the n-th multiple of the daily frequency, up to about $n = 4$. The storm time

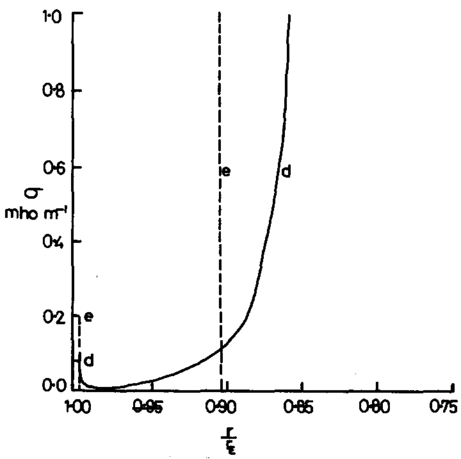

Fig. 1. Lahiri-Price (1939) models of upper-mantle conductivity.

variations are generated by an equatorial ring current, and are well described by the P_1^0 spherical harmonic alone. The periods seen in typical storms are of the order of days.

In 1919 Chapman fitted a uniform sphere model to S_q- data. For a best fitting model he obtained a uniform sphere of radius 0.96 earth radii and a conductivity of 0.036 mho/m. In 1930, after Price (1930, 1931) had developed the theory of electromagnetic induction in the earth to cope with aperiodic variations, he and Chapman fitted a uniform sphere model to storm time variations. These contain components with periods several times longer than S_q and therefore penetrate more deeply. They found a radius of 0.94 R_E and a conductivity of 0.44 mho/m, more than ten times Chapman's earlier result. This was the first indication that the earth's conductivity rises rapidly with depth.

In 1939, Lahiri and Price obtained analytic solutions for a model whose conductivity varied as an arbitrary power of the radius. They fitted a five-parameter model to both S_q- and D_{st}-variations. The result is shown in Fig.1. The conductivity varies as an arbitrary inverse power of radius below some arbitrary depth. Above that it is zero except for a thin surface conducting layer. This surface layer was not introduced by them as part of the initial model; they could not fit the data without it.

The two curves marked d and e in the figure are the extremes of the range of distributions that gave a reasonable fit to the data. Again, the major conlusion that was drawn from this was that the conductivity rises quite rapidly, apparently around 700 km depth. Between the surface and 700 km, the conductivity is very low. The surface layer seems to represent the oceans, although the limited resolution available in a five-parameter model prevents this interpretation from being conclusive. The integrated conductivity of this layer in model d, for example, is the same as that of a uniform ocean covering the entire earth to a depth of about 0.5 km. Although there is considerably more sea water on the actual earth than this, it is broken up by the land masses, and one might reasonably expect this to reduce the inductive effect to that found by Lahiri and Price.

This oceanic effect needs further comment. The oceans constitute a large deviation from the lateral homogeneity required by the earth models. Chapman and Whitehead pointed out in 1923 that they would have significant effects on variations as rapid as the S_q - variation. Schmucker (1970) has recently summarized the evidence that S_q is indeed affected by the oceans and local anomalies in the upper mantle. In a global analysis, one hopes that these effects will average out, but clearly this is not a very satisfactory way of doing things.

So far only the conductivity determinations made by studying induction by external fields have been discussed. None of these is able to say anything about the conductivity very deep in the mantle. Estimates of the very deep conductivity have been based on the secular variation of the main geomagnetic field. The secular variation is presumed to originate in the earth's core; what we see of it at the earth's surface is determined by the filtering properties of the mantle. The conducting mantle attenuates the rapid variations more than the slow ones, and the spectrum of the secular variation at the earth's surface can be expected to cut off above some maximum frequency. This frequency is observed to be about 0.25 cycles/year (Currie, 1968). A number of investigators (Runcorn, 1955; McDonald, 1957; Yukutake, 1959, 1965; Rochester, 1960; Smylie, 1965) have shown that a conductivity of 100 mho/m in the lower mantle (plus or minus half an order of magnitude) will account for this. This approach is, of course, based on the assumption that the observed cutoff frequency is not an intrinsic

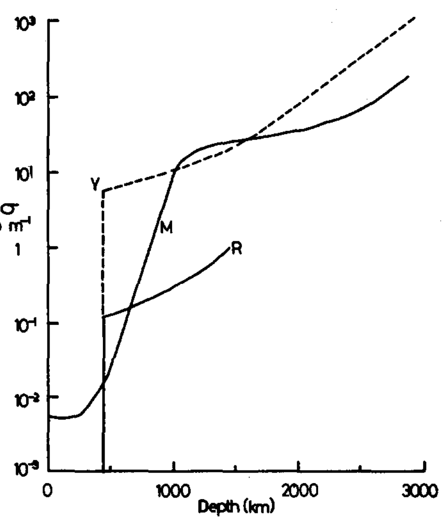

Fig.2. Estimates of mantle conductivity (R = Rikitake, 1950; M = Price-McDonald, 1957; Y = Yukutake, 1959).

property of the secular variation source. McDonald (1957) combined his secular variation result for the lower mantle with the Lahiri-Price estimates for the upper mantle to produce a well known composite model of the entire mantle. This is the curve marked M in Fig.2. Also shown are two other curves obtained about the same time. The curve marked R is due to Rikitake (1950, 1966) who used as data several different effects, such as S_q, D_{st}, magnetic bays and solar flare effects. Curve Y is due to Yukutake (1959) and is based on his study of the secular variation.

This is a brief outline of the state of global geomagnetic sounding up to about a decade ago. After this time modelling methods improved enormously, mainly because of the application of electronic computers. There was no longer any need to restrict models to analytically solvable profiles, and computerized time-series analysis made available a larger range of data. An advance along these lines was made in 1963 by Eckhardt et al. In their paper they discussed the shorthcomings of the previously used short-period data, S_q in particular. They pointed out the discrepancies between various determinations of the S_q response and that the spatial structure of S_q was not as simple as it was assumed to be. Using time-series analysis, they went on to study geomagnetic variations of longer periods. They found a number of strong peaks in their spectra, in particular the semi-annual line and the harmonics of the 27-day solar period. The spatial distribution of these excitations was adequately described by the single spherical harmonic P_1^0. The horizontal field was in fact fitted to better than 5% at the five stations used in their analysis for the 9.8- and 13.5-day periods. This was consistent with the variations having as their source fluctuations in the equatorial ring current. Having concluded that these longer periods could be used for more reliable tests of conductivity models because of their simple spatial structure, they tested a number of models against their data. The data fitted the Price-McDonald model (Fig.2) quite well, significantly better than some of the alternatives such as the very sharply rising e-distribution of Lahiri and Price (Fig.1). They concluded that no revision of the Price-McDonald model was necessary to fit their data. They also found that even the 6-month period data were insensitive to depths greater than 1,000 km.

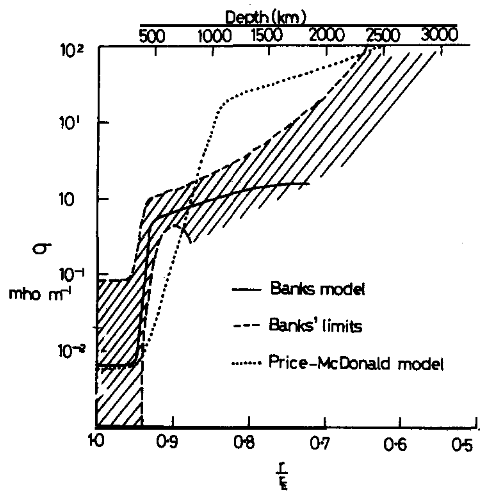

Fig.3. Mantle conductivity models of McDonald (1957) and Banks (1969) after Banks.

A paper by Banks (1969) carried the same approach considerably further. In estimating the response of the earth, Banks used not only the well defined peaks in the geomagnetic spectrum, but also the continuum between them. He also found that the single spherical harmonic P_1^0 described the variations quite well. He then obtained an estimate of the response of the earth in the P_1^0 mode over the entire range of frequencies from 0.01 to 0.25 cycles/day. Banks went on to fit a multilayered model to this response data, and produced a conductivity estimate that is shown in Fig.3. Also shown in the figure are upper and lower limits. He obtained these limits by varying each layer in the model individually until the computed response disagreed with the data. Because this does not allow for the cancellation of the effects of simultaneous variations of several layers, it is not a rigorous way of obtaining the true error limits, but it does roughly indicate the sensitivity of the model to changes. Below 1,000 km, the true error limits are probably much larger than he has indicated. The major difference between his curve and the Price-McDonald model (also shown in Fig. 3), which had been confirmed by Eckhardt et al., is that the steep rise takes place at 400 km depth in his model rather than 700 km. This difference is outside Banks' error limits. Whether this is a serious discrepancy depends on the spatial resolution of the estimates as well as the errors, and the modelling methods do not really tell us much about either of these. It is probably not serious; Banks (1972) has done a further analysis of his data and finds the depth of the rise to be much closer to the 700 km originally suggested by Lahiri and Price (1939).

3. Exact inverse methods

Modelling is often a laborious way of obtaining a solution. What is worse, it does not tell us if other dissimilar models also fit the data well. Direct procedures for calculating the conductivity from the data, if they can be found, assure us by their very existence that the solutions so obtained are unique. Such inverse methods, as they are called, have been developed recently along two lines, which I shall call exact and heuristic. It is the exact methods which define the conditions for uniqueness, and these will be described first.

The first significant result for exact inversion theory was obtained several decades ago in connection with a purely mathematical problem. In the equation:

$$\frac{d^2y}{dx^2} = [\lambda - V(x)]\, y$$

which, like the induction equation, is of Sturm-Liouville type, the eigenvalues for any particular set of homogeneous boundary conditions can be calculated if the function $V(x)$ is known. This procedure can be reversed. In 1945 Borg showed that knowledge of the two eigenvalue sets of the equation corresponding to two sets of homogeneous boundary conditions was sufficient to determine $V(x)$ uniquely. Borg's proof of this fact did not, however, give an actual method for doing this. This result is relevant to the inverse induction problem because the induction equation can be put in the above form by appropriate substitutions (Weidelt, 1970) and the response function, if known over all frequencies in a particular spherical harmonic, can be shown to determine the required eigenvalue sets (Bailey, 1970). This uniqueness property of the geomagnetic sounding problem was stated explicitly by Tichonov in 1965, although again an actual inversion procedure was not available.

Because Schroedinger's equation for scattering from a central potential can be reduced to the above form, where $V(x)$ is the potential, there was strong motivation in quantum mechanics to find a procedure for calculating the potential function from scattering data. This was done in 1951 by Gel'fand and Levitan. Their method involves reducing the problem to that of solving a linear integral equation, which, even if it needs to be solved numerically, can be solved exactly.

It took two decades for this technique to diffuse from quantum mechanics to geophysics. In 1970 Weidelt applied it to the geomagnetic sounding problem. At the same time Bailey (1970) developed independently another exact inversion method which reduces the problem to that of solving a first order non-linear integro-differential equation. Both methods necessarily invoke the same mathematical properties of the geomagnetic response function, but use them to reduce the problem to different simpler problems. Both methods arrive at the same result: that if the electromagnetic response of the earth is known at all frequencies, in any spherical harmonic mode, then only one radial conductivity distribution is possible. The latter method will be described first, since it makes the dependence of the conductivity distribution on the frequency dependence of the geomagnetic response clear in physical terms.

The response functions $S_n^m(f)$ (where f is the frequency) of the earth in a particular spherical harmonic P_n^m is defined as the ratio of internal generated field amplitude $i_n^m(f)$ to the external generating field amplitude $e_n^m(f)$. That is:

$$i_n^m(f) = S_n^m(f)\, e_n^m(f)$$

Fourier transformation of this equation back to the time domain yields a convolution, namely:

$$I_n^m(t) = \int_{-\infty}^{\infty} K_n^m(\tau)\, E_n^m(t-\tau)\, \mathrm{d}\tau$$

$K_n^m(\tau)$, the impulse response, is the Fourier transform of $S_n^m(f)$; I_n^m and E_n^m are the Fourier transforms of i_n^m and e_n^m, respectively. Clearly $K_n^m(\tau)$ must be zero for negative τ; the internally generated field cannot depend on future values of the external source field. This requirement of causality can be translated into a set of integral constraints on $S_n^m(\omega)$, namely:

$$\mathrm{Re}\, S_n^m(\omega) = \frac{2}{\pi}\int_0^{\infty} \frac{u\, \mathrm{Im}\, S_n^m(u)\, \mathrm{d}u}{\omega^2 - u^2} + S_n^m(\infty)$$

$$\mathrm{Im}\, S_n^m(\omega) = \frac{-2\omega}{\pi}\int_0^{\infty} \frac{\mathrm{Re}\, S_n^m(u) - S_n^m(\infty)\, \mathrm{d}u}{\omega^2 - u^2}$$

These are the well known Kramers-Kronig dispersion relations, and the geomagnetic response function must satisfy them.

The geomagnetic response function is normally defined for the earth as a whole. It can also be defined for any concentric subsphere of the earth. Eckhardt (1963) made this the basis of a method of directly calculating the response function of the earth for any conductivity distribution. The response function $S_n^m(r,\omega)$ of the subsphere of radius r obeys the equation:

$$\frac{\partial S_n^m(\omega,r)}{\partial r} = \frac{i\omega\mu_0 r\sigma(r)(n+1)}{n(2n+1)}\left[S_n^m - \frac{n}{n+1}\right]^2 - \frac{2n+1}{r} S_n^m$$

S_n^m is zero at the centre of the earth. Therefore, given the conductivity $\sigma(r)$, this equation can be integrated from the centre to the surface of the earth to determine the surface response function. Conversely, if S_n^m is known at the surface, it can be calculated at any depth by integrating this equation downwards rather than upwards. The result we obtain depends of course on what conductivity distribution we choose. If we do this, it turns out that only one conductivity distribution will preserve the causality, as defined by the Kramers-Kronig relations, of S_n^m at all radii; this is the correct conductivity distribution.

It is easy to see how an inverse method may be built on this. Given the response function at some radius (initially the surface) we determine the local conductivity which will preserve the causality if we integrate Eckhardt's equation downwards a very small distance dr. This conductivity can be shown to be:

$$\sigma(r) = \frac{1}{8}\frac{n(2n+1)^2}{r(n+1)} \left| \mathrm{Re}\left[\int_0^{\infty}\left\{S_n^m(\omega,r) - \frac{n}{n+1}\right\}^2 \mathrm{d}\omega\right]^{-1}\right|$$

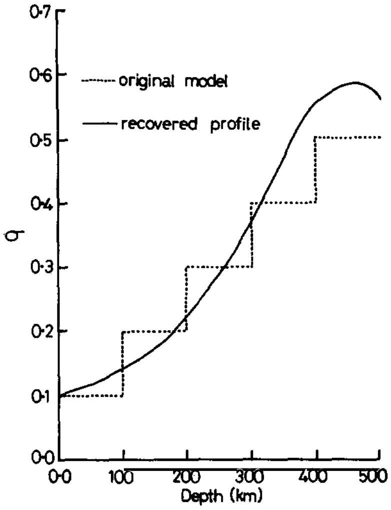

Fig. 4. Smoothing of staircase model by exact inversion method.

We calculate the response function at the new radius $r - dr$ and begin the process again. Repeated iteration recovers the conductivity profile down to any depth we please.

The Gel'fand-Levitan method as applied by Weidelt does not work in this way. It is necessarily based, however, on the same mathematical properties of the response function that correspond to the physical principle of causality. As the details are complicated, only the mechanics of the process will be described. Since the Gel'fand-Levitan method was originally applied to Schroedinger's equation, Weidelt begins by substituting new variables in the induction equation to arrive at Schroedinger's equation. From the response function, he derives a new function R. He also defines a function A from which the conductivity can easily be derived. The two are related by the equation:

$$A(x,y) = R(x+y) + \int_{-y}^{x} A(x,t)R(t+y)\,\mathrm{d}t$$

This is easily solved for A and thus the conductivity. The reduction of the problem to this equation is the core of the Gel'fand-Levitan method.

As both Weidelt and Bailey have found, these exact methods do not perform as well as modelling methods when given realistically noisy and truncated data. The modelling method allows the modeller to use outside knowledge in setting up his models, and, more important, to treat suspicious looking data with a pinch of salt. Exact methods attempt an exact reconstruction of the conductivity from the given data; if this data are in error, no such conductivity profile may exist.

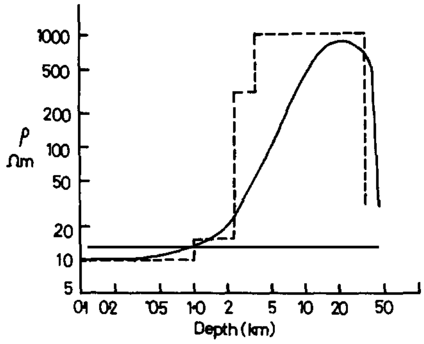

Fig. 5. Comparison of modelling (Vozoff and Ellis, 1966; dashed line) and exact inversion method (Weidelt, 1970; solid line).

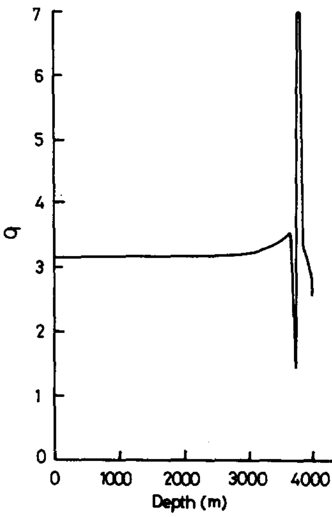

Fig. 6. Effect of finite depth penetration: exact inversion method applied to uniform sphere data.

Fig. 4 and 5 show one of the characteristics of the exact methods, their tendency to give the smoothest conductivity profile consistent with the data. In Fig.4, the dotted line shows a staircase profile from which artificial data were computed. The solid curve is the result generated by my method using these data. The maximum data frequency used corresponds to a skin depth of about 100 km, so the inversion resolves no structure finer than this. Fig.5 shows the same effect obtained by Weidelt's method. Here, real magnetotelluric data were used; the dotted line was obtained by modelling (Vozoff and Ellis, 1966) and the solid line by Weidelt. As we shall see later this is essentially a limitation of the data, not of the method. The lack of high-frequency information obscures the fine structure no matter how the data are treated; modelling methods merely put the fine structure into the model a priori.

Another limitation inherent in real data is that the lowest frequency used may not penetrate beyond a certain depth. What does the exact method do in this case? Fig.6 shows Bailey's method applied to band-limited data corresponding to a uniform sphere model. Beyond a certain depth, the errors in the local response function have grown to be larger than the information content, and the solution explodes.

Although the exact methods have practical drawbacks, their very existence assures us that a perfectly known response function can in principle uniquely determine the conductivity, and indicates that an imperfectly known one will give us at least an approximate solution. It should be pointed out here that the Gel'fand-Levitan method has also been applied to the secular variation problem by Johnson and Smylie (1970). They show that the transfer function of the mantle for secular variations as a function of frequency can be used to determine the conductivity profile in the mantle. Since our knowledge of the secular variation source spectrum is very limited, there are unfortunately no data at present to which this can be applied.

4. Heuristic inverse theory

The exact inversion methods are not suited to quantitatively describing the errors and the smoothing that result from real data. The heuristic method that will be described here was specifically designed to do this and therefore produces very useful results. Other heuristic methods of varying degrees of rigor and usefulness exist (e.g., Moskvicev, 1965; Miecznik, 1966; Wu, 1968; Nabetani and Rankin, 1969; Schmucker, 1969) but have not been applied to the global problem. The method was developed by Backus and Gilbert (1967, 1968, 1970) for solving in a practical way a wide range of geophysical inverse problems. Parker (1970) applied it to the geomagnetic sounding problem, and his paper is the basis of this outline of the method.

In principle the method is simple. The problem is linearized; that is, an initial conductivity profile must be chosen by informed guesswork, and this profile is adjusted to fit the data exactly. If this guess is widely inaccurate, the adjustment procedure, which is based on linear perturbation theory, may not converge. If it is a reasonable guess, it will converge. Let us formulate this mathematically. Let the data consist of N measured observable quantities, denoted by g_i. It does not matter what these quantities are as long as they are functions of the profile. As well as measuring these quantities we can compute what they ought to be for the initial model profile. We can also calculate how small changes in our model profile will affect these observables. To first order, these must be of the form:

$$\delta g_i = \int_a^b K_i(r)\, \delta\sigma(r)\, dr$$

where $a-b$ is the interval over which the conductivity exists. The kernels K_i can be computed numerically. By taking the difference between the observed values of the g_i and the values calculated for our model, we can also obtain the required δg_i.

How do we obtain the required perturbation $\delta\sigma(r)$ of our model to fit the data exactly? To answer this, consider a linear combination of the δg_i, such as:

$$L = \sum_{i=1}^{N} a_i \delta g_i$$

where the coefficients a_i are as yet undetermined. Substitute the expressions for the δg_i involving the kernels K_i and the perturbation $\delta\sigma(r)$. This gives:

$$L = \int_a^b \left[\sum_{i=1}^{N} a_i K_i(r) \right] \delta\sigma(r)\, dr$$

If we could choose the coefficients a_i so that the quantity in square brackets was a Dirac delta function centered on some radius of interest r_0, then L would be simply $\delta\sigma(r_0)$, the quantity we require. In practice, we cannot obtain a delta function, but, by choosing the coefficients a_i carefully, we can usually get a function that is sharply peaked at r_0. L is then an estimate of $\delta\sigma(r_0)$ with a resolution equal to the width of this function. Once we have found the coefficients a_i for each height of interest, we can compute $\delta\sigma(r)$ simply as:

$$\delta\sigma(r) = \sum_{i=1}^{N} a_i\,(r)\,\delta\,g_i$$

At the same time we have automatically computed the resolution of all our estimates.

If the formula above involves strong cancellation between terms, the resulting errors in $\delta\sigma(r)$ (which can be calculated using simple error theory) may be much larger than the original experimental errors in the δg_i. In this case, broadening the resolution by adjusting the a_i reduces the errors in $\delta\sigma(r)$. This situation is analogous to the Heisenberg uncertainty principle; accuracy and spatial resolution cannot simultaneously be optimized. This is where the Backus-Gilbert method displays its power; not only does it tell us both the resolution and the accuracy of our estimate, but it allows us to vary the compromise between them to suit our needs. There are a number of numerical methods for finding the coefficients a_i that simultaneously attempt to minimize the smoothing and the errors. This is primarily a computational problem and I will not go into it here. They are described in the papers of Backus and Gilbert, and others (see Wiggins, 1972).

Parker has applied the Backus-Gilbert method to the geomagnetic induction problem. As observables he used the modulus of the geomagnetic response of the earth in the P_i^0 mode at 35 frequencies between 0.01 and 0.2 cycles/day; as data he used Banks' results (1969). He found that the Backus-Gilbert method would not generate any model that exactly fitted Banks' data. He did find it possible to fit it within one standard deviation. He concluded that Banks' data contains unphysical features, but that the true response curve lies within one standard deviation of Banks' data. Parker's conductivity profile down to a depth of about 1,500 km is shown in Fig. 7 with 20% errors and the corresponding resolution widths, with Bank's model for comparison. Below this depth errors are so large as to make any estimate meaningless. The two curves disagree significantly near the surface. Parker's value of 0.1 mho/m there is more than a standard deviation greater than Banks' value of 0.005 mho/m. In Fig. 8 Parker's curve is compared with Bailey's exact inversion of the same data. These results resemble each other more than either resemble Banks' curve, at least above the depth (about 1,000 km) where the exact method is overwhelmed by accumulated errors.

Very recently Parker (1972) has gone on to publish a paper which shows a very realistic approach to geophysics, called "Inverse Theory with Grossly Inadequate Data". The inversion problem of geomagnetic sounding, as we have seen, requires a very extensive, in fact infinite, range of data to yield an exact conductivity profile. What, then, is the use of making a single measurement such as the 11-year response of the earth? Clearly even this one measurement must tell us something about the earth, even if it is only in the form of a constraint on a single parameter, such as the mean conductivity. To this problem Parker has applied the methods of Backus and Gilbert and shown how one calculates such constraints. He illustrates this with an example drawn from the geomagnetic inversion problem: he shows that the complex response of the earth in the P_i^0 mode at the single period of 100 days is enough to show that the conductivity in the earth must somewhere exceed 0.84 mho/m. In itself, this result is not new, intended as it was as an example. The main purpose of the paper is to show exactly how very small amounts can be used rigorously to full advantage.

5. Results

We have concentrated so far on the methods, and the results have been quoted without much comment. In finishing, I will try to summarize the information obtained so far about the actual conductivity distribution. All recently obtained profiles display three common features: a region of low conductivity in the top several hundred kilometers, followed by a sharp rise, and a very poorly defined region below 1,000 km depth where the conductivity may level off. Within this framework, there is still significant disagreement.

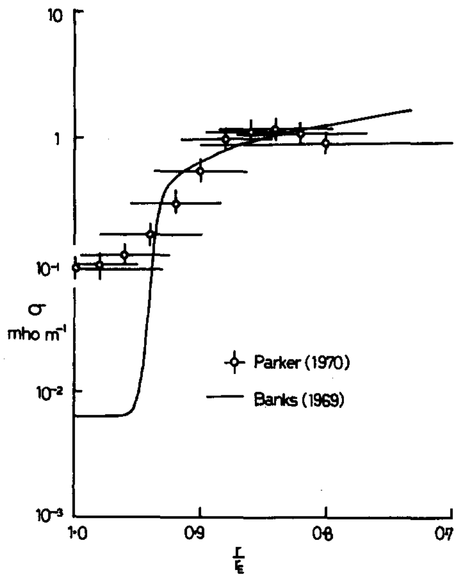

Fig.7. Upper-mantle conductivity models of Banks (1969) and Parker (1970).

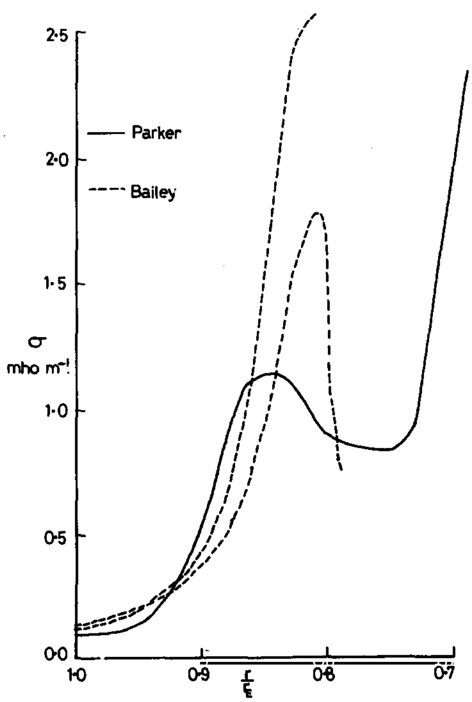

Fig.8. Comparison of exact (Bailey, 1970) and heuristic (Parker, 1970) methods using Banks (1969) data.

For the near surface conductivity, both Parker (1970) and Bailey (1970) using Banks' (1969) data find values of 0.1 mho/m or greater. On the other hand, Banks as well as Lahiri and Price (1939) and some magnetotelluric studies (e.g., Caner, 1971) find values less than 0.05 mho/m. Parker states that this difference is greater than the errors. The discrepancy between Parker's and Banks' results obtained from the same data can probably be put down to the inherent ambiguities of the modelling method that Banks used, but this does not explain the other low values obtained. Data errors seem to be the most likely explanation of this. Banks' high-frequency response (periods from 5 to 10 days) is significantly different from Chapman and Price's storm response for the same periods. Banks himself stated that his high-frequency results have suspicious features: in particular, his results for the phase of the response at high frequencies are physically impossible. Although Parker used only Banks' amplitudes and not the phases, the inconsistencies in Banks' data should make us wary of accepting Parker's result without confirmation. Parker's rejection of the lower values of the conductivity is based on error bounds derived from Banks' estimate of errors in the response; Banks had no means of estimating these accurately.

The deeper conductivity, in particular the rise from 300 to 700 km depth, obtained by Parker depends more on Banks' low-frequency data, and is probably more reliable. The calculated resolution of Parker's profile is everywhere worse than 0.05 R_E or 300 km. Because of this, we cannot say how steep the rise is or precisely where it is located. This is annoying, because it has been suggested that all or part of this rise may in fact be a discontinuity associated with a phase change in the mantle (Akimoto and Fujisawa, 1965). Parker's result does not rule this out, as Fig.9 shows. Here a discontinuity at 700 km depth has been smoothed with a resolving kernel of spread 0.05 R_E; the result fits Parker's model quite well.

The region below 1,000 km is very imperfectly probed. Banks' data do seem to indicate the existence of a plateau here at about 1 mho/m. Parker has shown, however, that this levelling off is just at the limit of resolution. Clearly the curve cannot remain at this value down to much greater depths, as it has to rise by a factor of 100 towards the bottom

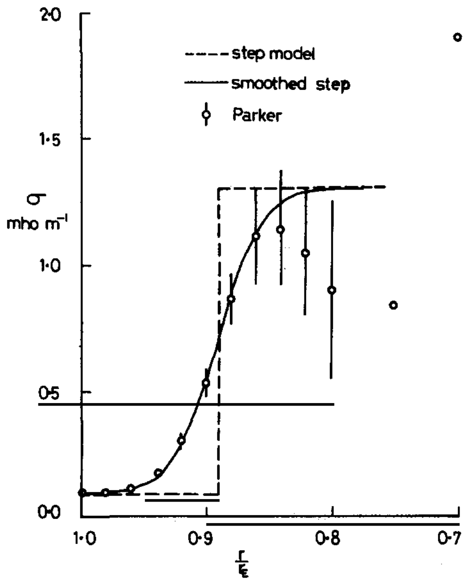

Fig.9. Parker (1970) profile compared with smoothed step model.

of the mantle to explain the secular variation results.

How much can our knowledge be improved? Attempts to extend the lower frequency limit of the data have not been particularly successful. Eckhardt et al. (1963), for example, could not extract the 11-year sunspot variation from the secular variation. On the other hand, to use data much above 0.2 cycles/day invites errors due to shallow deviations from spherical symmetry, which we are not interested in here. A radical extension of the useful data bandwidth seems to be ruled out. The other possibility is to improve the accuracy of the present data. Parker has looked at the improvements that this would produce. He found that if Banks' data had been obtained with no errors at all, the resolution could still not be narrowed much below 300 km. As a consequence, geomagnetic deep sounding on a global basis appears inadequate to prove or disprove the existence of discontinuities in the upper mantle. On the other hand, penetration with this resolution to the bottom of the mantle would be possible in principle.

The assumption of no errors at all is exceedingly unrealistic. Parker repeated the same calculation assuming 2% errors in Banks' data. He found that this would delineate the conductivity profile down to about 2,000 km with the same accuracy and resolution as are now obtained down to 1,000 km. This would tell us whether a plateau exists below 1,000 km. Whether or not the required data can be obtained with this accuracy remains to be seen. Banks used a limited amount of observatory data in deriving his response function. It seems likely that an exhaustive study along the same lines could significantly reduce the errors. Finally, it appears that studies of the top few hundred kilometers are best left to local magnetotelluric and short-period geomagnetic methods. These high-resolution methods can deal with the local conductivity variations that global studies neglect.

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