EFFECTS OF CRUSTAL CONDUCTIVITY HETEROGENEITIES ON THE ELECTROMAGNETIC FIELD

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Abstract. Examples of observed and computed effects of crustal conductivity heterogeneities are described. The physical processes responsible for these effects are discussed with special reference to their characteristic dimensions. Simple criteria allowing the recognition of the physical phenomena which govern the circulation of Earth currents are given.

These criteria are taken into account in a study of the electromagnetic fields relating to three typical effects of crustal conductivity heterogeneities. This study suggest that we should begin by interpreting data in frequency ranges for which the static distortion approximation of currents hold. The rough but correct model thus obtained can then be used as starting model in the interpretation of the whole data set, involving numerical modelling.

I. Introduction

The final objective of electromagnetic investigations, in both applied and fundamental geophysics, is to obtain original and accurate information about the area studied and to estimate the likelihood of the quantitative models produced. The achievement of this objective obviously requires highly reliable data, such as those obtained with the latest electromagnetic data acquisition and processing methods. It also requires interpretation methods which take due account of the effects of conductivity heterogeneities on the electromagnetic field.

Many solutions have already been proposed to eliminate the effects of these heterogeneities from the data acquired by magnetotelluric (MT) or geomagnetic soundings. One family of solutions can be applied when a situation can reasonably be described as two-dimensional (2-D). In magnetotellurics, this family of solutions involves rotation of the tensor and the search for the direction of the symmetrical axes of the best fitting 2-D structure through the minimisation of certain functions of the elements of the MT tensor (e.g. Swift, 1967; Jupp and Vozoff, 1976). In geomagnetic sounding, the anomalous electric currents are assumed to be induced in a 2-D structure defined as the extrapolation to infinity of the best fitting 2-D structure in the area where the anomalous field is observed (e.g. Fisher, 1984). However, when three dimensional (3-D) effects are present, these methods designed for 2-D situations lead to inadequate quantitative interpretations (e.g. Park et al., 1983). Some examples of 3-D effects corresponding to real and synthetical situations are given in section II of this review.
When faced with such inadequate interpretations the question again arises of how to interpret electromagnetic observations in such a way that it should be possible to discuss, on physical bases, the possibility that 3-D effects exist in the data and to provide for their elimination. The characteristics dimension which scales the area represented by a station then becomes a key parameter. Basically, this scaling length of the induction problem depends on the physical process which governs the behaviour of the electromagnetic field. For instance, the characteristic dimension corresponding to a galvanic coupling between a heterogeneous and a conductive layer through a resistive screen does not depend on the frequency of that field when the frequency is low enough (Berdichevski and Dimitriev, 1976). On the contrary, the intensity of the inductive phenomena, i.e. the self and mutual induction processes related to the existence of the anomalous flow of telluric currents, decreases with the frequency (LeMouel and Menvielle, 1982) and the associated characteristic dimensions are obviously frequency-dependent (e.g. Park, 1985). The first step in the data interpretation is therefore to identify the process(es) which govern(s) the electromagnetic field under observation. In Sections III and IV, we review the basic theoretical concepts necessary for this identification.

The aim of an electromagnetic investigation is to produce the most likely model of conductivity and to estimate its likelihood. We suggest starting interpretation by studying the electromagnetic field in simple situations, and using the rough but correct model thus obtained as a zero order model for any more refined interpretation. From this point of view, special attention should be paid to cases in which the distortion of currents by conductivity heterogeneities does not modify their temporal law (e.g. LeMouel and Menvielle, 1982; Wannamaker et al., 1984). This will make it possible to eliminate the effects of local heterogeneities on the MT sounding curve (e.g. Larsen, 1975; Andrieux and Wightman, 1984; Counil et al., 1984) and derive accurate conductivity variations with depth. More generally electromagnetic soundings can easily be interpreted in terms of the deflection of telluric currents by heterogeneities of conductivity at frequencies low enough for inductive phenomena to disappear (e.g. Edwards et al., 1971; Porath and Dziewonski, 1971; Rossignol, 1972; Babour et al., 1976; Galdeano et al., 1979; Babour and Mosnier, 1980; Woods and Lilley, 1980; Menvielle and Tarits, 1986). In Section V, we give typical examples of topographic effects and discuss methods of eliminating these effects using the theoretical results reviewed in the Sections III and IV.

II. Observed and Computed Effects of Crustal Conductivity Heterogeneities

An effect of crustal conductivity heterogeneities (hereafter called crustal effect) can be defined as the effect on the electromagnetic field, observed at an observation point P of the conductivity heterogeneities in the neighbourhood of that observation point, due either to the presence of heterogeneous bodies in the uppermost kilometers of the crust, or to that of mountains or hills. According to the most
generally accepted interpretation of this definition, there is a crustal effect whenever either an anomalous magnetic field or a departure from the one-dimensional MT tensor is observed. Consequently, very few situations are free of crustal effects, which can be described as the results of the changes in telluric currents caused by lateral heterogeneities of conductivity.

The purpose of the geomagnetic sounding is to show the anomalous variations in the geomagnetic field, and to try to interpret them in terms of lateral contrasts of conductivity. To a certain extent, geomagnetic sounding methods are therefore based upon the existence of effects related to the presence of conductivity heterogeneities.

The magnetotelluric sounding method, on the contrary, was developed by Cagniard (1953) for situations free of crustal effects, so that their presence may cause difficulties in the MT data interpretation. In layered situations, the transfer function (e.g. apparent resistivity ($\rho$) and phase ($\Phi$)) is the same whenever observed in any two perpendicular directions. This property disappears in the presence of a crustal effect. Figure 1 (Andrieux, personal communication) gives some typical examples of observed crustal effects. The apparent resistivity curves and phase curves were computed after expressing the tensor on the orthogonal basis in which its diagonal terms are minimal. In most cases, three frequency domains can be distinguished according to the behavior of the $\rho$- and $\Phi$-curves: the $\Phi$-curves may be identical, like the $\rho$-curves (domains (a) in Figures 1a and 1b) or the $\rho$-curves may be parallel (domains (b)), or, again, a significant divergence may be observed between the two $\rho$-curves and the two $\Phi$-curves (domains (c)). The situation corresponding to parallel $\rho$-curves and identical $\Phi$-curves has been extensively described by Andrieux and Wightman (1984); it corresponds to the static effect we shall discuss in section V. Figure 1c shows that this effect can be observed at some stations for frequencies ranging from 1000 Hz to 0.05 Hz (Figure 1c). However, the static effect corresponds to particular situations (see Section V), and when present, generally exists in a limited frequency range. Outside this range, the $\rho$-curves are neither identical nor parallel and the $\Phi$-curves are divergent (domains (c) in Figures 1a, 1b and 1d). This situation indicates the presence of 2-D or 3-D effects which may involve inductive phenomena or galvanic coupling between the heterogeneous upper crust and the deep conductive mantle through the resistive lower crust (see Sections III and IV).

In the past five years, the development of efficient 3-D numerical models permitted numerical studies of the effect on the MT sounding curves of a 3-D heterogeneous body embedded in a layered substratum. These numerical studies of synthetic situations are very interesting, because they allow the estimation of the bias in the sounding curve due to the presence of a heterogeneous body. Park et al. (1983) considered an L-shaped conductive valley in the resistive upper layer of a stratified Earth, and computed the sounding curves at four sites (A, B, C, and D, Figure 2a). The results, given in Figure 2b, show that an effect is observed on the $\rho$-curves at all sites, even at site A located more than 100 km away from the hetero-
Fig. 1. Examples of observed crustal effects in apparent resistivity and phase sounding curves (Andrieux, personal communication). The frequency ranges (a), (b) and (c) are defined according to the behaviour of the apparent resistivity and phase curves (see text for further explanations).
3. MT sounding for site A.

4. MT sounding for site B.

5. MT sounding for site C.

6. MT sounding for site D.
Fig. 2. Effects of an L-shaped sedimentary basin on magnetotelluric sounding curves (after Park et al., 1983). (a) Sketch of the 3-D model geometry. Maximum and minimum apparent resistivities were computed for sites A, B, C, and D. (b) Comparison between the computed sounding curves and 1-D local sounding curves for the four sites in (a). (c) Comparison between the computed sounding curves and 2-D sounding models for sites B, C, and D. In each case, the 2-D model geometry is indicated below the sounding curves.

geneous body. Figure 2b also compares the minimum or maximum apparent resistivity with the 1-D curves corresponding to the four stations. It shows that significant errors of interpretation can be made if 3-D effects are disregarded. At sites B, C, and D, these authors also compared the computed 3-D curves and 2-D curves corresponding to a model which is a 2-D cross section through the long axis of the valley of Figure 2a, extending to infinity away from the plane of the section. The results are shown in Figure 2c. They indicate that outside the valley, at site B, the 2-D data closely resemble the 3-D data. Inside the valley, however, the strong anisotropy in the 2-D sounding curves is very different from the 3-D data at site C; at site D, the 3-D data set resembles that of the 2-D case, but with a drop in anisotropy of about 50%. Therefore, interpretation of the 3-D curves using 2-D models would lead to serious errors in the results for site C; for sites B and D, it would lead
to a cross-section which resembles the actual cross section of the valley, but with inaccurate depth and resistivity values. Furthermore the site C curves show that in a complex structure, isotropic resistivity data appearing as 1-D can occur near points of approximate symmetry.

Wannamaker et al. (1984) using their own algorithm, made similar computations for a sedimentary basin in a four-layered Earth (Figure 3a) and compared 2-D and 3-D sounding curves at four sites located in and around the basin. Their results, shown in Figure 3b, lead to the same conclusions as those drawn from Park et al. (1983) results.

Finally, both Wannamaker et al. (1984) and Park showed that the inductive phenomena become negligible in the lower part of the frequency range. The results at these lower frequencies can therefore be accounted for in terms of the deflection of currents by the galvanic charges that accumulate because of conductivity contrasts. This situation will be discussed in greater detail in the next sections.
Fig. 3. Effects of a plate-like 3-D body on magnetotelluric sounding curves (after Wannamaker et al., 1984). (a) Sketch of the 3-D model geometry. The sounding curves were computed at sites A, B, C, and D. (b) Principal apparent resistivity and impedance arguments $\rho_{xy}$ and $\Phi_{xy}$ (dotted curves) and $\rho_{xy}$ and $\Phi_{xy}$ (dashed curves) for sites A, B, C, and D. The 1-D curves correspond to the response of either a basin without lateral bounds (site A) or the regional host (sites B, C, and D).
The question of the distortion of telluric currents by conductivity heterogeneities has been dealt with by many authors in relation to the most common situations (e.g. Berdichevski et al., 1973; Weidelt, 1975; Berdichevski and Dimitriev, 1976; Le Mouel and Menvielle, 1982; Jones, 1983; Wannamacker et al., 1984; Andrieux and Wightman, 1984; Park, 1985) and in the case of a heterogeneous thin sheet overlying a stratified substratum (e.g. Price, 1949; Vasseur and Weidelt, 1977; Weaver, 1979; McKirdy et al., 1985; Menvielle and Tarits, 1986). In this section, we summarize the basic theoretical results concerning this problem. For the sake of clarity, we shall give successively the results obtained using poloidal and toroidal modes, when the depth of penetration of the field in the heterogeneous upper layer exceeds the thickness of this layer, and the results obtained using scalar and vector potentials.

1. Decomposition in terms of the Poloidal and Toroidal Modes

Electric currents can always be expressed as the sum of the poloidal and toroidal modes which respectively drive the vertical and horizontal current systems (e.g. Weaver, 1973; Vasseur and Weidelt, 1977; Backus, 1986). The poloidal and toroidal electric fields behave differently, especially at low frequencies, and their characteristic scale lengths are also different (Vasseur and Weidelt, 1977; Ranganayaki and Madden, 1980). The vertical and horizontal current distortion situations considered by Park (1985) correspond to limiting cases where the anomalous electric field (defined as the difference between the actual electric field and the field which would be observed without heterogeneities) is purely poloidal or toroidal.

a. The Poloidal Mode

The lower crust is usually a poor conductor, and when considering overall problems such as induction in the oceans, it is customary computational efficiency to assume that this crust is perfectly non-conducting (e.g. Bullard and Parker, 1970; Hobbs, 1971; Hewson-Browne et al., 1973). However, to assume the existence of a perfectly insulating lower crust leads to the cancellation of the vertical currents inflow and outflow at the bottom of the non-uniform surface sheet. This assumption no longer works in many geophysical cases in which there is resistive coupling, across the resistive lower crust, between the heterogeneous upper crust and the conductive mantle. Vertical currents inflow and outflow do exist at the bottom of the surface sheet, and are driven by the poloidal electric field. For low frequencies, the anomalous electric field is almost purely poloidal (Vasseur and Weidelt, 1977).

Berdichevski and Dimitriev (1976) pointed out that an important parameter of the problem of resistive coupling through a resistive screen is the value $\tau = 1/\lambda_v$ where $\lambda_v$ is the square root of the product of the total conductance of the upper
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layer by the total resistance of the intermediate resistive layer. This problem was discussed at length by Ranganayaki and Madden (1980). Extending Price's thin sheet analysis, these authors considered an Earth model made up of a stratified substratum topped by an anisotropic layer with a horizontal conductivity \( \sigma_s \) and a vertical resistivity \( \rho_s \), which are not reciprocal so that \( \rho_s \sigma_s > 1 \). In practice, this anisotropic layer is a combination of a conductive heterogeneous sheet, which has a conductivity \( \sigma \) and a thickness \( \Delta z_1 \), and represents the heterogeneous upper crust, on top of a resistive layer which has a resistivity \( \rho \) and a thickness \( \Delta z_2 \), and represents the resistive lower crust. In this case, Price's conventional boundary conditions have to be replaced by (Ranganayaki and Madden, 1980):

\[
[E_z]_+ = -i\omega \mu \Delta z_2 z \times H_x^+ + \nabla \cdot \left[ \rho \Delta z_2 \left( \nabla \times H^+_x \right) \cdot z \right]
\]

\[
[H_x]_+ = -\sigma \Delta z_1 z \times E_z^- + \left( \Delta z_1 / i\omega \mu \right) \nabla \cdot \left[ \left( \nabla \times E_z^- \right) \cdot z \right],
\]

where \( E_z \) and \( B_z \) are respectively the horizontal electric field and magnetic induction, \( z \) the vertical unit vector (positive downward) and \( [\Gamma]^+_z = \Gamma(z=+0) - \Gamma(z=-0) = \Gamma^+ - \Gamma^- \) the change occurring in \( \Gamma \) across the anisotropic thin sheet. An \( \exp(-i\omega t) \) time dependance is assumed.

The above equations allowed these authors to estimate the distance required by the crustal current levels to readjust to changes in crustal conductance. This distance, called the adjustment distance, is expressed by the Berdichevski and Dimitriev's parameter \( \lambda_v \)

\[
\lambda_v = \left( \sigma_s \rho_s \right)^{1/2} = \left( \sigma \Delta z_1 \rho \Delta z_2 \right)^{1/2}.
\] (1)

Dawson et al. (1982) also used anisotropic thin sheets to study the electromagnetic field in the B-polarization mode, close to the boundary of two adjacent half planes with distinct conductivities \( \sigma_1 \) and \( \sigma_2 \). For this 2-D situation, they obtained an exact analytical solution and showed that the exponential decay of the anomalous field on either side of the boundary between the two half planes is governed by the attenuation constants \(-I_1(\nu_1)\) and \(-I_1(\nu_2)\) where:

\[
\nu_j = \left[ \frac{1}{2 \sigma_0^2 \rho_s^2} \left[ 1 - \left( 1 - \frac{4 \sigma_0^2 \rho_s^2}{\sigma_j \rho_s^2} + 4 i\omega \mu \sigma_0^3 \rho_s^3 \right)^{1/2} \right] - \frac{1}{\sigma_j \rho_s^2} \right]^{1/2}
\] (2)

\( \sigma_0 \) being the conductivity of the half space under the sheet and \( \delta_0 \) the depth of penetration of the electromagnetic field in this medium. Clearly (2) is equivalent to (1) when:

\[
1 - \left( 1 - \frac{4 \sigma_0^2 \rho_s^2}{\sigma_j \rho_s^2} + 4 i\omega \mu \sigma_0^3 \rho_s^3 \right)^{1/2} \ll \frac{2 \sigma_0^2 \rho_s}{\sigma_j}.
\]

More recently, Fainberg and Singer (1987) investigated this problem using asymptotic expressions for the components of the electromagnetic field of a grounded electric dipole. They showed that the effect of resistive coupling in rela-
tion to a surface anomaly can be disregarded for distances $r$ from the edge of the anomaly such as:

$$r \ll \lambda_v/\sqrt{1 + \sigma Z_0^2}$$

where $Z_0$ is the Tikhonov-Cagniard impedance for the medium under the surface layer. This estimate reduces to $\lambda_v$ when $|\sigma Z_0^2| \ll 1$, i.e. when:

$$\Delta z l \left(\rho_e \sigma\right)^{1/2} \ll 1$$

where $\Delta z$ and $\sigma$ are respectively the thickness and conductivity of the upper layer, $\delta$ the depth of penetration of the electromagnetic field in the medium of conductivity $\sigma$ and $\rho_e$ the apparent resistivity of the medium under the surface layer. For frequencies below $10^{-2}$ Hz this condition is fulfilled in most geophysical situations. In the low frequency domain, we shall therefore retain $\lambda_v$ as estimate of the adjustment length for galvanic coupling through the resistive crust.

The physical meaning of this adjustment length has been discussed by Ranganayaki and Madden (1980, p. 450). These authors introduced the concept of equilibrium current distribution, namely the current distribution which would prevail if the conductivity structures extended uniformly far enough in one or two horizontal directions to be considered as 2-D or 1-D structures. In that case, one would expect the actual current distribution to be the same as that predicted by a one- or two-dimensional model with the same conductivity structure as the local region. If the current comes from another part of the crust, with a different equilibrium current distribution, one cannot expect the current levels to be equilibrium levels unless there is either current flow to or from the upper mantle, or inductive processes related to the anomalous flow of current. When the inductive phenomena can be disregarded, $\lambda_v$ is the scale length for this readjustment. In other words, $1/\lambda_v$ expresses the degree of galvanic connexion between the upper layer and the conductive mantle. The thinner the intermediate resistive layer and the lower its resistivity, the better the galvanic connection between the upper layer and the mantle. $\lambda_v^{-1}$ is therefore sometimes called the galvanic constant (Berdischevski and Dimitriev, 1976).

b. The Toroidal Mode

The horizontal currents, driven by the toroidal electric field will now be considered. In a heterogenous layer, the geometrical characteristics of these currents are quite simply related to the geometry of the conductivity heterogeneities. On the boundaries of these heterogeneities, there is indeed an accumulation of volumic density charges such that:

$$\rho = \varepsilon \sigma E \cdot \nabla (1/\sigma).$$

These boundary charges are responsible for the deflection of telluric currents by
local heterogeneities of conductivity. Equation (3) clearly shows that, in the vicinity of sharp conductivity contrasts, the Coulombian field associated with these charges may be of the same order of magnitude as the normal electric field which would be observed if there were no heterogeneous body (e.g. Le Mouël and Menvielle, 1982); this Coulombian field may severely distort the MT tensor and apparent resistivity curves, even when the heterogeneous bodies are small (e.g. Wannamaker et al., 1984).

Park (1985) attempted to estimate the anomalous electric field in such situations for an elliptical conductive body embedded in a resistive medium. He established that outside this body the electric field perturbations due to the horizontal currents driven by the toroidal field vary with distance in the same way as a low order polynomial. Consequently, the characteristic decreasing length of the anomalous field is of the same order as the characteristic dimension of the heterogeneous conductive body.

2. Decomposition in Terms of Vector and Scalar Potential

As shown by many authors, description of the electromagnetic field using scalar and vector potentials simplifies the analysis of the frequency range for which the inductive effects can be disregarded (e.g. Le Mouel and Menvielle, 1982; Andrieux and Wightman, 1984; Counil et al., 1984; Jones, 1984; Wannamaker et al., 1984; Park, 1985; Counil et al., 1986; Menvielle and Tarits, 1986). Furthermore, this description is well adapted to the study of static distortion of current situations, as it permits easy elimination of the crustal effects of the observations made in such situations.

a. Basic Results

Consider an Earth model composed of a substratum \((z > h, z\) being positive downwards), whose conductivity \(\sigma_s(z)\), depends solely on the depth \(z\), except in a given number of layers with conductivity heterogeneities. Assume further that the upper layer is heterogeneous. In order to describe the effects due to lateral heterogeneities of conductivity in the heterogeneous layers, let us write:

\[
\sigma(M) = \sigma_s(z) + \sigma_a(M)
\]

\(\sigma_a(M)\) being zero except in the heterogeneous layers.

Introduce the vector potential \(A\) and the scalar potential \(\Phi\). In the whole space, the electric field \(E\), the magnetic induction \(B\) and the electric current density \(J\) may be written as (in the quasisteady state approximation):

\[
E = -(\partial A / \partial t) - \nabla \Phi
\]

\[
B = \text{curl} \ A
\]

\[
J = 1/\mu \text{curl} \ B
\]
\[ \Delta \Phi + \rho / \varepsilon = 0 \]  
\[ \Delta \mathbf{A} + \mu \mathbf{J} = 0 \]  
\[ \text{div} \mathbf{A} = 0 \]

\( \rho \) being the volumic density of charge. For a given external excitation, let \( \mathbf{A}_n \) be the vector potential corresponding to the stratified normal model \( (\sigma = \sigma_n (z) \text{ for } z > 0) \). The corresponding normal solution is then:

\[ \mathbf{E}_n = - (\partial \mathbf{A}_n / \partial t) \]  
\[ \mathbf{B}_n = \text{curl} \mathbf{A}_n \]  
\[ \mathbf{J}_n = 1/\mu \text{ curl } \mathbf{B}_n \]

where \( \mathbf{A}_n \) and \( \mathbf{J}_n \) satisfy Equations (5-2) and (5-3); the anomalous field is then defined as the difference between the actual and normal electromagnetic fields, thus:

\[ \mathbf{A}_a = \mathbf{A} - \mathbf{A}_n \]  
\[ \mathbf{E}_a = \mathbf{E} - \mathbf{E}_n = - (\partial \mathbf{A}_a / \partial t) - \nabla \Phi \]  
\[ \mathbf{B}_a = \mathbf{B} - \mathbf{B}_n = \text{curl} \mathbf{A}_a \]  
\[ \mathbf{J}_a = \mathbf{J} - \mathbf{J}_n = \sigma \mathbf{E}_n + \sigma \mathbf{E}_a. \]

It is clear from Equation (7-2) that the electromagnetic field is the sum of two terms, which correspond to two different physical processes. The first, \( \mathbf{E}_a = - (\partial \mathbf{A}_a / \partial t) \), corresponds to the self and mutual induction processes linked to the anomalous flow of currents; the second, \( \mathbf{E}_e = - \nabla \Phi \) is the Coulombian electric field due to the boundary charges.

\( \mathbf{E}_e \) and \( \mathbf{E}_a \) do not vary with frequency in the same way: at low frequencies, \( \mathbf{E}_a \) is of the order of \( \omega (\mathbf{E}_n + \mathbf{E}_e) \), and the anomalous field \( \mathbf{E}_a \), if any, reduces to the Coulombian field \( \mathbf{E}_e \) (e.g. Le Mouel and Menvielle, 1982). The conditions for which this approximation is valid are discussed in detail in the next section.

b. The Electromagnetic Field in Static Distortion Approximation

Consider now the case where (1) the anomalous electromagnetic field is mostly galvanic and (2) the galvanic coupling with a deeper conductive layer through a resistive screen is negligible, and consider the geophysical situations, for which (a) the heterogeneous layers can be considered as thin sheets, and (b) the geometry of the external excitation is locally that of one elementary surface harmonic (or the sum of two elementary surface harmonics of the same degree). This situation corresponds to the static distortion of currents, as has been described by Le Mouel and Menvielle (1982). These authors proved that, in such situations, there is no
phase shift between the anomalous electric and magnetic fields related to a heterogeneous 3-D body and to the normal electric field in the heterogeneous layer where this body is located. The anomalous electric field $E_a$ displays a D.C like behaviour everywhere and the anomalous magnetic induction is related to $E_a$ through the Biot-Savart law. Wannamaker et al. (1984) obtained the same results using their own notation.

Consider first the case of a stratified substratum topped by one heterogeneous thin layer. At the surface of this model, $E_a$ and $B_a$ would be related to $E_n$ and $B_n$ through very simple tensorial relationships (Le Mouel and Menvielle, 1982):

$$E_a(P, t) = E_n(P)E_n(t)$$ (8-1)

$$B_a(P, t) = B_n(P)E_n(t)$$ (8-2)

where $E_n$ and $B_n$ are real tensors which depend only on the observation point $P$. Introducing Cagniard's magnetotelluric impedance between $E_n$ and $B_n$ leads, in the frequency domain, to:

$$E_a(P, \omega) = 1/\mu Z_c(\omega)E_n(P)C B_n(\omega)$$ (9-1)

$$B_a(P, \omega) = 1/\mu Z_c(\omega)B_n(P)C B_n(\omega)$$ (9-2)

with

$$E_n = (1/\mu) Z_c C B_n$$ (10)

$Z_c$ being the impedance of the normal stratified substratum and $C$ the ($\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$) matrix.

Tarits and Menvielle (1983; 1986) investigated the more general case corresponding to intralithospheric heterogeneities located in fairly thin layers. They described the uppermost tens of kilometers of the earth as a stratified medium including one or more heterogeneous thin sheets topped by another heterogeneous thin sheet, which account for the very superficial contrasts of conductivity. In this case, the anomalous electromagnetic field does not display such a simple behavior at the surface of the model. Nevertheless, simplifications occur in the magnetic field behaviour when the coupling between the anomalous flows of electric currents linked to the different heterogeneous layers becomes negligible: thus, the anomalous magnetic field observed at the surface of the model is the sum of the anomalous magnetic fields corresponding to each of the heterogeneous layers taken separately. For instance, in the case of two heterogeneous layers, separated by a resistive medium, the anomalous magnetic field corresponding to the deeper layer is the difference between the anomalous field observed and the anomalous magnetic field which would correspond to the superficial heterogeneous layer overlying the normal substratum (Tarits and Menvielle, 1983; 1986).
IV. Inductive Effects

The inductive effects constitute the other class of topographic effects. In Section III-2, we showed that inductive effects are described by the anomalous vector potential $\mathbf{A}_a$.

Substituting Equation (7) in Equation (5) yields:

$$\Delta \mathbf{A}_n = -\mu \sigma_n \mathbf{E}_n$$

$$= \mu \sigma_n \partial \mathbf{A}_n / \partial t$$

$$\Delta \mathbf{A}_a = \mu \sigma \partial \mathbf{A}_a / \partial t = -\mu \sigma \mathbf{E}_e - \mu \sigma_a \mathbf{E}_n$$

$$= \mu \sigma \nabla \Phi + \mu \sigma_a \partial \mathbf{A}_n / \partial t$$

$$\Delta \Phi - \mathbf{v} \cdot \nabla \Phi = -\mathbf{v} \cdot \mathbf{E}_e - \mathbf{v} \cdot \mathbf{E}_n$$

$$= \mathbf{v} \cdot \partial \mathbf{A}_a / \partial t + \mathbf{v} \cdot \partial \mathbf{A}_n / \partial t$$

where $\mathbf{v} = \sigma \cdot \nabla (1/\sigma)$; $\mathbf{E}_n$, $\mathbf{E}_a$ and $\mathbf{E}_e$ are defined as in Section III-2.

Consider now Equation (11-2). In the normal medium ($\sigma = \sigma_n$; $\sigma_a = 0$); it reduces to:

$$\Delta \mathbf{A}_a = \mu \sigma_n \partial \mathbf{A}_a / \partial t = \mu \sigma_n \nabla \Phi$$

This equation clearly shows that, in the normal medium, the source term is the ohmic dissipation of the galvanic field $\mathbf{E}_e = -\nabla \Phi$. This field, due to the charge accumulation on lateral contrasts of conductivity, depends in turn on $\mathbf{A}_a$ through Equation (11-3). Far from the heterogeneity of conductivity, the $\mathbf{E}_e$ field behaves like $(L/r)^2$, where $L$ is the characteristic dimension of the conductivity heterogeneity and $r$, the distance from the centre of the anomalous body. When the heterogeneous body is embedded in a homogeneous half space, $L$ obviously scales the lateral extent of the domain around the body where $\mathbf{A}_a$ may be significant. When $\sigma_n$ varies with depth, the existence of a coupling between the heterogeneous body and a deep conductive layer through a resistive screen may drastically increase the size of this domain ($L$ has to be replaced by $\lambda_v = (\rho \sigma_s)^{1/2}$; see Section III.1).

The source term varies in the same way as $\sigma_n$, and therefore $\mathbf{A}_a$ varies too. In the resistive layer of normal medium, $\mathbf{A}_a$ may exist and it dissipates according to both diffusive and ohmic processes. Typically, the diffusive term varies like $A_a/S_1$ where $S_1$ is the characteristic section of the source current $-\sigma_n \nabla \Phi$; the ohmic term varies like $\omega \mu \sigma_n A_a$, where $\omega$ is the pulsation. When $\omega \mu \sigma_n \ll 1/S_1$, the ohmic dissipation process becomes negligible and there are no anomalous inductive effects in the normal medium.

In the heterogeneous body, one has to consider Equation (II-2). The source term constitutes its right hand part. It is the sum of the ohmic dissipation of the galvanic field in a medium of conductivity $\sigma = \sigma_n + \sigma_a$ on the one hand, and of the normal field in a medium of conductivity $\sigma_a = \sigma - \sigma_n$ on the other hand. A resistive heterogeneous body in a conductive host corresponds to $\sigma \approx 0$, i.e. $\sigma_a \approx -\sigma_n$; a
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conductive body in a resistive host corresponds to $\sigma \approx \sigma_a$. In both cases, the source term varies in the same way as $\sigma_a$ and therefore $A_o$ varies too.

In the heterogeneous body, $A_o$ may again dissipate according to both diffusive and ohmic processes. Typically, the diffusive term $\Delta A_o$ varies like $A_o / S_2$ where $S_2$ is a characteristic section of the heterogeneous body; the ohmic term varies as $\omega \mu \sigma A_o$. When $\omega \mu \sigma << 1/S_2$, the ohmic dissipation process becomes negligible inside the body and the inductive effects disappear.

Therefore, the anomalous inductive effects are in any case significant when the characteristic section of the anomalous flow of current is not significantly smaller than the square of $\delta$ the depth of penetration of the electromagnetic field in the medium where the currents are flowing.

Le Mouël and Menvielle (1982) first established this result for the anomalous currents in the normal medium, and Wannamaker et al. (1984) demonstrated it in general situations. The particular situation of a 2-D conductive body in a resistive host was investigated by Park (1985) and Menvielle and Tarits (1986) who obtained the same result.

V. Some Typical Effects of Crustal Conductivity Heterogeneities

We shall now use the results established in Sections III and IV to interpret the main features of the crustal effects observed and to discuss ways of eliminating them.

1. The Case of the Fault

One of the simplest lateral contrasts of conductivity is a fault separating two layers with different conductivities but the same thickness $h$. For frequencies high enough for the penetration depth in each medium to be smaller than $h$, the situation can be modelled by two quarter spaces of differing electrical conductivity juxtaposed in such a way that the conductivity does not vary along the x-axis. The electromagnetic field has been numerically or experimentally studied in such situations by many authors in cases of both H- and E-polarization respectively corresponding to the magnetic field (H-pol.) and the electric field (E-pol.) parallel to the fault (e.g. Jones and Price, 1970; 1973; Jones, 1973; Price, 1973; Berdichevski and Dimitriev, 1976; Fischer et al., 1983; Schnegg et al., 1986). For E-polarization, the adjustment length has been found to be of the order of several skin depths, and for H-polarization, of several tenths of skin depths. Such differences in the behaviour of the electromagnetic field result from the difference between the boundary conditions for E- and H-polarization. In the case of E-polarization, the electric field is parallel to the fault. Its continuity results in a very gradual change from the left-hand to the right-hand 1-D structure. In H-polarization, the electric field is perpendicular to the fault; the continuity of the electric currents results in an adaptation over very short distances to the left or right hand 1-D structure. Figure 4a is an illustration of this situation.
For low frequencies, the behaviour is the same, as long as there is no resistive layer cut in the conductive formations, because such layers would prevent the currents from adjusting quickly to a changing structure, and one would then have to take $\lambda_0$ as the adjustment length for H-polarization, as discussed in Section III-1-a. For E-polarization, the adjustment length remains of the order of $(\rho_{ap}/\omega\mu)^{1/2}$ where $\rho_{ap}$ is the apparent resistivity of the layered medium (Ranganayaki and Madden, 1980). Figure 4b illustrates this situation.

Thus, when analysing field data, the use of 1-D models is only possible for stations fairly far from the fault, and it must be carefully considered in relation to the geoelectric structure of the area under study. In general, some extraneous information about the deep conductivity is necessary, on which to base any data interpretation in terms of 1-D modelling. However, 2-D and/or 3-D modelling is also necessary to check any interpretation derived from the 1-D approximation (e.g. Schnegg et al., 1986).
2. The Conductive Dyke Effect

Another crustal effect can be encountered in areas close to very elongated conductive structures embedded in a more resistive host. In such structures, the electric field is characterized by a large anisotropy between $E_\parallel$ and $E_\perp$, where $E_\parallel$ and $E_\perp$ are respectively the components of $E$, parallel and perpendicular to the axis of the structure. At low frequencies, the related anomalous magnetic field is linearly polarized at the surface of and above the host.

As far as we know, the first example reported in the literature dealt with measurements made at the Ebro observatory (Spain). Using hourly mean values for both telluric and magnetic variations during quiet days, Bauer (1922) observed...
strong polarization of the electric field in a direction whose azimuth is N23°W (Figure 5a). More recently, in the region of Sopron (Hungary), Adam and Veró (1961) observed an $E_\parallel / E_\perp$ ratio of about 100 at a station located just above a fault. In this case, the magnetic field was less affected because the conductive structure responsible for the current concentration was not very extended, since the $E_\parallel / E_\perp$ ratio decreased by one order of magnitude at stations located 100 m away from the centre of the anomaly.

When the cross section of such a structure is large enough, the anomalous variations in the magnetic field become significant. Adam et al. (1986) reported strong distortion of this field along the Periadriatic lineament, in connection with an elongated fault system. Mosnier and Planson (1982) also reported a similar effect in the central part of French Pyrenees. Using differential techniques, they showed that for periods of about 10 min. the anomalous magnetic field was twice as high as the normal one, and that for periods of about one hour, it was 100% as high, at some stations. They also showed that, for these periods, the anomalous field is linearly polarized. The fact that the maximum of the anomaly coincided with the trend of graphite veins (see Figure 5b) clearly indicated that the anomalous field observed was linked to the high concentration of telluric currents in these geological structures (Thera and Dupis, 1983).

As discussed by many authors (Hebert, 1983; Fischer, 1984; Menvielle and Tarits, 1986), the linear polarization of the anomalous magnetic field indicates that...
there is static distortion of currents. The anomalous currents flowing in the conductive structure display a DC-like behaviour, and are connected with the anomalous magnetic field, in accordance with the Biot-Savart law. It is therefore easy, in such cases, to derive accurate information about the local geometry of the structure from the geometry of the anomalous magnetic field (e.g. Fischer, 1984). However, complete interpretation of the characteristics of the electromagnetic field in these situations makes it necessary to consider the characteristic horizontal dimension $\lambda$ of the problem of the induction process. As stated in Section III, the value of $\lambda$ depends on the conductive structure of the crust in the neighborhood of the observation point $P$. However, determination of $\lambda$ is beyond the scope of this section.

For the high frequencies (i.e. when $\lambda$ is smaller than half the width of the structure), the electromagnetic field depends only on the conductivity in the dyke. As the frequency decreases, the $\lambda$ parameter increases and the problem becomes a 2-D one. For low frequencies, the surrounding area influencing the observed electromagnetic field may be much larger than the resistive blocks limiting the channel, and serious errors in determining the resistivity at depth may result if these low frequency data are interpreted using 2-D models.

Accordingly, the study of such elongated structures might be conducted by pursuing a strategy involving magnetotelluric and differential geomagnetic sounding. Throughout the entire field experiment, it might then be necessary to record the transient variations in the geomagnetic field simultaneously at a fixed reference station, located far from the dyke, and at the mobile sounding stations. As a first step, study of the anomalous magnetic field at frequencies at which the approximation of static distortion of currents holds supplies determinations of the depth, width and integrated conductivity of the dyke. In addition, the study, at each sounding station close to the dyke, of the frequency range, if any, for which the layered approximation holds, provides unshifted estimates of the conductivity profile in the embedding medium. This makes it possible to build a rough but correct approximation of the cross section of the structure, and a more refined cross section accounting for inductive effects can then be deduced. Many algorithms have already been proposed, which are based on finite difference techniques (e.g. Jones and Vozoff, 1978) or finite elements (e.g. Weidelt, 1975; Reddy et al., 1977; Ting and Hohmann, 1981; Wannamaker et al., 1984) or the Rayleigh-FFT approach (e.g. Jiracek, 1973). An example of the interpretation of electromagnetic soundings over a wide 2-D structure through numerical modelling can be found in Zhdanov et al. (1986).

3. The static effect

As illustrated in Section II, MT sounding curves do not generally display a simple behaviour at observation points located near three dimensional heterogeneities. However, one particular case is worth discussing further: it is the static effect, described by Andrieux and Wightman (1984) on the basis of a concrete observa-
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In many cases, a significant frequency range exists for which the \( \Phi \)-curves corresponding to two electric cross components are superimposed whereas the \( \rho \)-curves are parallel and distinct. These authors made a thorough analysis of this effect and showed that it can be interpreted in terms of static distortion situations when \( \mathbf{B}_a \) is negligible.

Their results can easily be demonstrated using the simple Earth model introduced in Section III.2. Consider the magnetotelluric tensor as observed at an observation point \( P \) located on the Earth's surface, and write:

\[
\mathbf{E} = \frac{1}{\mu} \mathbf{Z} \mathbf{B}.
\]

When (1) the static distortion of currents approximation holds, (2) the effects of deeper conductivity heterogeneities can be neglected and (3) \( ||\mathbf{B}_a|| \ll ||\mathbf{B}_n|| \), Equations (9-1) and (9-2) in Section III and (12) above lead to:

\[
\begin{align*}
\mathbf{Z} &= \mathbf{Z}_c (\mathbf{I} + \mathbf{E}_a) \mathbf{C} \\
&= \mathbf{Z}_c \mathbf{D} \mathbf{C}
\end{align*}
\]

where \( \mathbf{D} \) is the deviating tensor (Counil et al., 1984; see also Larsen, 1975). \( \mathbf{D} \) is real and depends only on the observation point.

Thus, throughout the whole frequency range in which these approximations are valid, knowledge of \( \mathbf{Z} \) provides accurate estimates of the argument of \( \mathbf{Z}_c \) but not of its modulus, thus giving the apparent resistivities to within a real constant which does not depend on the frequency. Conversely, relations (13) only hold when these particular conditions are fulfilled. It is indeed easy to show that the existence of either anomalous electric fields related to heterogeneous bodies located at significantly different depths, or of anomalous magnetic fields, leads to a frequency dependent relation between \( \mathbf{Z} \) and \( \mathbf{Z}_c \). The static effect is then characteristic of static distortion situations in which \( \mathbf{E}_a \) is related to the conductivity heterogeneities in the superficial layer and \( \mathbf{B}_a \) is negligible.

Relations (13) also show that, in a given area, close to the boundaries of a very superficial heterogeneity the intensity of the electric field may vary significantly from one station to the other, within a few kilometers or less. Such variations were observed by Bahr (1983) in the Harz mountains of West Germany where significant changes in the modulus of the impedance occurred between MT stations 500 m apart. Similar observations have been made by Andrieux and Wightman (1984) in Wyoming (U.S.A.).

As illustrated in Section II, problems arise when trying to interpret a 3-D MT tensor in terms of the mean variation of conductivity with depth in the area described by the station. In the case of static effects, the uncertainty in the determination of the actual apparent resistivity leads to uncertainties in the determination of resistivities at depth, that might prevent reliable interpretation of MT soundings. The derivation of accurate variations of resistivity with depth therefore requires
some extraneous information, to permit the correction of the variations observed in the apparent resistivity.

For this correction, the $Z_c$ must be known for a given frequency (or for a limited frequency range) in which the static effect is observed, since the deviation process is not frequency-dependant. This correction allows determination of the deviating $D$ tensor for the whole period range in which this effect is observed. Among others, the “Compagnie Generale de Geophysique” and Geoconsult suggested deriving $Z_c$ from the variation of resistivity with depth in the uppermost layers of the crust as deduced from in loop transient soundings close to a MT station. This method, that we will call hereafter the CGG/Geoconsult correction method, is very efficient in most cases where the static effect is observed, because the Time domain Electro-Magnetic sounding (TDEM) has a good lateral and vertical resolution in the uppermost hundreds of meters of the Earth. The description of the conductive structures provided by TDEM is then used to compute the MT apparent resistivity at the sounding station in the high frequency domain. Comparison of this computed true apparent resistivity and the observed resistivities leads to the multiplicative real constant due to the Static effect. The CGG/Geoconsult method was successfully applied in Wyoming to study a volcanic covered area (Geoconsult, 1983); the results of this study were reported by Andrieux and Wightman (1984). Figure 6 gives an illustration of the results obtained by these authors using the CGG/Geoconsult method.

Another solution for the interpretation of MT curves with a static effect is the use of numerical (e.g. Chouteau et Bouchard, 1986) or analog models, but it would only give accurate results if the crustal conductive structure in the neighborhood of the station can be accurately derived from other geological or geophysical investigations.

Fig. 6. Example of the cancelling out of the static effect (from Andrieux and Wightman, 1984). The actual apparent resistivity is deduced from the observed resistivities $\rho_{xy}$ and $\rho_{yx}$ using the apparent resistivity $\rho_T$ provided by TDEM soundings.
VI. Conclusion

In this review, we deal with the development of economical interpretation methods which allow advantage to be taken of the real quality of electromagnetic data.

Such methods have already been proposed in 1-D or 2-D situations. However, there are already few genuine 1-D or 2-D situations, and using 1-D or 2-D methods to interpret magnetotelluric measurements in 3-D situations has now been proved to be misleading. The real challenge therefore lies in the development of interpretative methods that work in general cases and provide accurate results without involving unduly heavy computations.

As shown by Park (1985), these methods should be based upon rigorous analysis of the physical processes governing the behaviour of the electromagnetic field at the sounding stations. One of the key parameters to be estimated is the scaling length of the horizontal area that the station represents. This length is generally called the induction or adjustment length. It depends on both the frequency and conductivity distribution around the station.

We have reviewed here the published determinations of the adjustment length, which only deal with limiting cases. Clearly, the actual distributions of conductivity rarely correspond to these limiting cases. Nevertheless, in most cases it is possible to define frequency bands in which the behaviour of the electromagnetic field observed at a given station can be accounted for in terms of one of these limiting cases. For instance, the anomalous magnetic field (i.e. the difference between the observed field and the field which would be observed without heterogeneities) is the result of both the accumulation of galvanic charges on the lateral contrasts of conductivity and of self and mutual induction processes. Both galvanic and inductive processes are present in the whole frequency range but, at a given station, there are generally frequency bands in which inductive phenomena become negligible in relation to galvanic phenomena.

Thus, the interpretation of electromagnetic data might begin by the recognition of the physical phenomena which govern the behaviour of the electromagnetic field at a given station in a given frequency range. So far few criteria have been proposed for such recognition. We reviewed these criteria here and believe that the determination of other simple criteria making it possible to discriminate between the driving physical phenomena, when they exist, should enormously increase the efficiency of magnetotelluric, and more generally of electromagnetic investigations.

Finally, the aim of an electromagnetic investigation is to produce a model of conductivity distribution whose likelihood can be estimated. We suggest starting by the determination of the range of frequencies for which the static distortion of current approximation holds. In general, it is possible to cancel out crustal effects in these frequency bands (see example in Section V). Interpretation of the corresponding data will give the most reliable information on the conductive structure of the area studied.

A model deduced from this preliminary analysis can then be constructed to
interpret the whole data set. One has to keep in mind that, in many cases, this model cannot be deduced from simply extrapolating to infinity the conductive structure in the area under study, but should include the conductivity distribution in a zone restricted by the adjustment length determined at the observation point for the frequency considered.

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