Physics of the Earth and Planetary Interiors, 10 (1975) 201–208

DC RESISTIVITY METHODS FOR DETERMINING RESISTIVITY IN THE EARTH'S CRUST

GEORGE V. KELLER
Department of Geophysics, Colorado School of Mines, Golden, Colo. (U.S.A.)

(Received May 12, 1975)

The use of Schlumberger and dipole arrays for crustal-scale resistivity soundings is considered. Advantages and disadvantages of the two methods are described. The depth to which resistivity may be determined from field measurements is discussed as well as the determination from the sounding curves of various parameters associated with layered structure. The interpretation of experimental data using reference curves as well as two approaches used in computer assisted interpretation are discussed.

1. Introduction

Methods for measuring electrical conductivity such as the magneto-telluric method (MT) and the geomagnetic deep sounding method (GDS) have been used with considerable effectiveness in recent years in detecting zones of high conductivity in the outer reaches of the mantle (Caner et al., 1967; Keller, 1971b). These methods have been developed to detect highly conductive zones at depths ranging from 10s to 100s of kilometers in the earth, but the problem of determining the conductivity profile in the more-resistant parts of the earth's crust has so far defied all attempts at solution. This is disappointing because information about this part of the crust may be particularly important in understanding the brittle tectonic behavior of the earth, as opposed to the viscous tectonic behavior in deeper regions. For example, it may be that the strain that accumulates to cause earthquakes is associated particularly with crustal layers that are likely to have high electrical resistivity as well. Therefore, this review has as its objective a survey of the attempts that have been made to define the electrical-conductivity profile through the resistant part of the earth's crust.

2. Direct-current sounding method

The direct-current sounding method is the best known of the electrical probing methods. Many specific field techniques have been used in dc soundings, but for crustal-scale surveys, only two of these have found favor – the Schlumberger technique and the dipole technique. The Schlumberger electrode array consists of four colinear electrode contacts (Kunetz, 1966). In making a sounding, the outer two electrodes, which are used to supply current to the ground, are moved progressively away from the center of the spread. The inner two electrodes, at the center of the array, are used to measure the voltage drop developed by the current. In discussing the Schlumberger array, the assumption is made that the separation between the inner measuring electrodes is small compared to the separation between the outer current electrodes. If this assumption is valid, the ratio of voltage drop to electrode separation can be said to be approximately equal to the electric field intensity.

With the dipole array, four electrodes are used also (Al'pin et al., 1966), but they are not arranged geometrically in the same manner as with the Schlumberger array. Current is supplied to the ground with one pair of electrodes, usually fixed in location, while a component of the electric field is mapped as a function of distance from this current source with a second pair of electrodes. If the separation between the current electrodes is much less than the distance to the location at which the electric field is being detected, then it is possible to characterize the source solely in terms of its dipole moment – current intensity times electrode separation – rather than by these two parameters individually.

Normally, the electric field is mapped away from the dipole source along one of the principal directions. When the component of electric field parallel to the source axis is mapped outwards on the equatorial axis, this sounding is termed an equatorial dipole sounding. For a horizontally layered earth, this procedure produces the same apparent resistivity curve as a function of separation as does the Schlumberger array, though this similarity does not hold for any other earth geometry. When the electric field is measured at locations along the axis of the source, the sounding is termed a polar dipole sounding, and the apparent resistivities behave quite differently from those measured with a Schlumberger or equatorial dipole array. Other locations of the receiving electrodes may be used also (see Al'pin et al., 1966; Keller, 1966), and the electric field may even be mapped in detail in the plane about the source dipole (Furgerson and Keller, 1974). However, in the latter case, the patterns of resistivity which are computed from the field data are very complicated and may not be of much value in crustal-scale studies.

Both the Schlumberger and dipole arrays have advantages and disadvantages relative to one another when they are used for crustal-scale resistivity soundings, and so the choice of one or the other is not always clearcut. The principal disadvantage of the Schlumberger array is that the span between current electrodes must be increased to 100–200 km to provide information about the lower crust and upper mantle, even under favorable conditions. Inasmuch as currents of several ampères are required to provide a detectable signal at the receiving electrodes at such spacings, the voltages applied to the current line must be of the order of hundreds to several thousands of volts. Extensive precautions must be taken so that such a length of current-carrying wire does not comprise a hazard to life. One satisfactory solution to this problem is the use of out-of-service power lines, which are already protected. In recent years, considerable work has been done during grounding tests of high-voltage direct-current transmission lines. A limitation of such an approach is the fact that soundings must be made at locations where there are available power lines, rather than at locations which may be the most interesting from the geological point of view.

Advocates of the use of dipole arrays for crustal-scale resistivity soundings apparently feel that the operational ease of the method is the chief advantage over the Schlumberger array. With a dipole source length of 1–10 km, it is usually possible to make soundings even in densely inhabited areas, with judicious choice of the source location. Very large currents, at least 100 A, are required to provide measurable signals at the maximum spacings. It is not as inconvenient to provide these large currents for a dipole array as for a Schlumberger array because the source electrodes for a dipole array remain fixed in location. Therefore, it is practical to spend considerable effort in lowering contact resistance at the source electrodes. Quite commonly, the source electrodes used in dipole surveys consist of 50–100 m of drill pipe in specially drilled holes, or in other cases, hundreds of metal stakes driven into the ground.

A very serious disadvantage of the dipole array is its sensitivity to errors caused by relatively small lateral inhomogeneities in the near-surface resistivity (Furgerson and Keller, 1974). Because of this, it is necessary to make a great many more measurements to establish the shape of a sounding curve – that is, the manner in which apparent resistivity varies with spacing.

In both the Schlumberger and dipole arrays, the depth to which resistivity may be determined from field measurements is very crudely related to the maximum spacing between electrodes (Keller, 1966; Frolich, 1967). For the very simple case of a conductive sequence of rocks underlain by highly resistant basement, definition of the probing depth of an electrode array is straight-forward. The resistivity sounding curve for this case has two asymptotes when plotted to logarithmic scales, as shown in Fig. 1. The left-hand asymptote is a horizontal line, yielding an average conductivity for the surface layers – those above the resistant basement rock. The right-hand asymptote is a line rising with a slope of +1, characterized by the equation:

$$a/\rho_a = S \quad (1)$$

where a is the electrode spacing, taken to be half the distance between current electrodes for the Schlumberger array, or the total distance between dipole centers in the dipole array, ρ_a is the apparent resistivity value computed at the spacing a, and S is the actual longitudinal conductance of the sequence of rocks above basement,

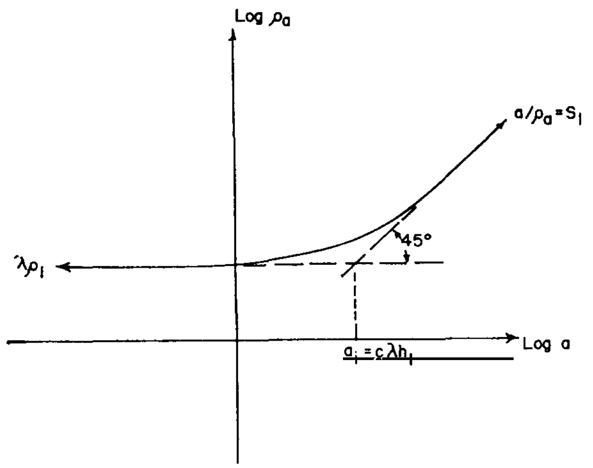

Fig. 1. The asymptotes of a two-layer dc resistivity sounding curve in which the second layer is highly resistive.

defined as:

$$S = \int_0^h \sigma(z)\mathrm{d}z \qquad (2)$$

where $\sigma(z)$ is the conductivity as a function of depth, z, and h is the thickness of the conductive sequence of rocks.

The intercept of these two asymptotes occurs at a spacing:

$$a_i = c\lambda h \qquad (3)$$

and so, can be used to define the probing depth of an array. Here, λ is a coefficient for vertical-to-horizontal anisotropy in the sequence of conductive rocks, defined as:

$$\lambda = \frac{(TS)^{1/2}}{h} \qquad (4)$$

where T is the transverse resistance of the sequence of conductive rocks, defined in a manner similar to longitudinal conductance:

$$T = \int_0^h \rho(z)\mathrm{d}z \qquad (5)$$

but using resistivity, $\rho(z)$, as a function of depth rather than conductivity.

The constant, c, in eq. 3 varies depending on the array being used. It is unity for the Schlumberger and equatorial dipole arrays, and 2 for the polar dipole array. Thus, for a section with no vertical-to-horizontal anisotropy, apparent resistivity values would begin to depart significantly from the resistivity of the surface layer at a spacing equal to the depth of basement, if the Schlumberger or equatorial dipole arrays were being used, but not until a spacing twice as large was reached, if the polar dipole array were being used.

Most rocks are moderately anisotropic, with values for λ commonly running from 1.1 to 2.0 (Keller, 1968a). In such cases, the depth of reach of an array is reduced by 10–100%, depending on the character of the rocks above the basement contact.

The error in depth estimates caused by anisotropy is not the greatest uncertainty when dc methods are applied to the crustal-scale problem; rather, it arises from a phenomenon known as T-equivalence. The crust and upper mantle can be thought of grossly as a three-layer sequence, in which the middle layer has the highest resistivity. In all probability, the contrast in resistivity between the middle layer and its neighbors is quite large, being at least several orders of magnitude. When the resistivity of a layer is much higher than that of the layer beneath, current flow lines in the upper of the two layers will be essentially vertical, and the flow of current will be controlled by the leakage resistance, T, rather than by the resistivity, Thus, two layers, one twice as thick as the other but with only half the resistivity, may have precisely the same effect on apparent resistivity values measured at the surface. The two cases would then be termed T-equivalent.

In the crustal model, because the surface layers are much more highly conductive than the crystalline basement, apparent resistivity values would first begin rising with a slope of nearly +1, with minor amounts of current leaking through the second layer to deeper, more conductive rocks. At larger spacings, the area available for vertical leakage increases, until finally, the loss of current in the surface layer becomes significant. The apparent resistivity reaches a maximum when half the total current has leaked out of the surface layer, and then decreases rapidly at larger spacings. For the Schlumberger and equatorial dipole arrays, the spacing at which the maximum occurs is (Keller, 1968a):

$$a_{\max} = \left(\frac{S_1 T_2}{2}\right)^{1/2} \qquad (6)$$

where S_1 is the longitudinal conductance of the sur-

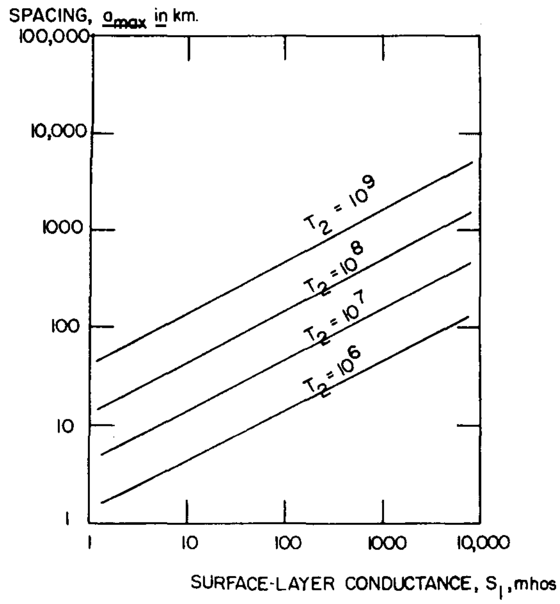

Fig. 2. The spacing for which the maximum apparent resistivity is recorded for a three-layer sequence with the middle layer being highly resistive.

face layers and T_2 is the transverse resistance of the resistant portion of the crust.

It is readily apparent that the spacing to which one must go to establish the maximum on the apparent resistivity curve for a crustal-scale sounding increases both with the conductance of the surface layers and with the transverse resistance of the crust. The results of such soundings which have been reported in the literature provide values for T_2 ranging from 10^6 to 10^9 Ωm^2. The conductance of surface rocks may range from a few mhos where they consist only of weathered crystalline basement to some thousands of mhos in deep sedimentary basins. The spacings required to measure maximum apparent resistivity are shown graphically in Fig. 2 as a function of S_1 and T_2.

The value of apparent resistivity observed at the maximum depends on S_1 and T_2, and is essentially independent of the resistivity in the second layer, and is approximately:

$$\rho_{a,\max} = \left(\frac{2T_2}{S_1}\right)^{1/2} \tag{7}$$

The maximum value for apparent resistivity in a crustal-scale sounding is shown graphically in Fig. 3 as a function of S_1 and T_2.

A final factor needs to be considered in evaluating dc methods: the effect of gross lateral changes in the electrical properties of near-surface rocks. When dc soundings are expanded to spacings of a hundred or more kilometers to determine the maximum of the sounding curve, there is need for a method to determine whether that maximum is caused by lateral changes in resistivity, or truly by the presence of more conductive rocks at depth. *Without this cross-check, the value for transverse resistance of the crust determined from dc soundings can only be considered to be a minimum possible value.* This is because lateral changes in the resistivity of surface rocks can cause only a minor elevation of apparent resistivity along the rising branch of a sounding curve, but can cause very large reductions in apparent resistivity from this asymptote (see Al'pin et al., 1966). A particularly striking example of a false effect is that which occurs with the equatorial dipole array expanded over a gently-dipping basement surface. The apparent resistivity curves for measurements made over such a structure pass through a maximum, which would be interpreted in a field survey as indicating a finite transverse resistance for the second layer. This is true for dips as

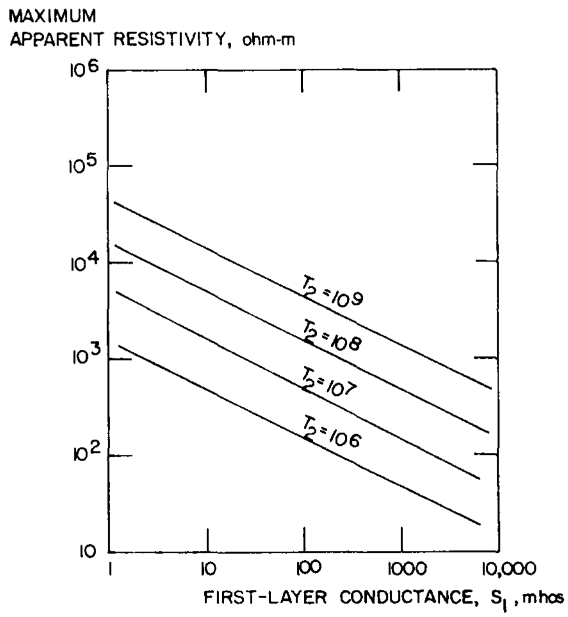

Fig. 3. The maximum apparent resistivity which is observed over a three-layer sequence with the middle layer being highly resistive.

slight as 1°, which might reasonably persist over distances of 100 km. Recently, van Zijl and Joubert (1975) recommend that repeat measurements be made with an offset receiver spread in Schlumberger sounding to test for the presence of lateral effects.

3. Direct-current sounding results

Early attempts to determine the conductivity profile through the crust and mantle have been reviewed previously (Keller, 1966). Because of scatter in such data, interpretation of a single dipole sounding is uncertain, though when groups of soundings made in the same geological province are combined, more tractable results may be obtained.

Another extensive set of dc sounding data has been obtained in the western United States as a result of grounding tests of the Northwest-Southwest Intertie – a high-voltage direct-current transmission line connecting the northwest Pacific with the southern California and Nevada region (Cantwell et al., 1966; Keller, 1968b). During the grounding tests, currents of up to several hundred ampères were passed over the line and returned through the ground at terminals near Portland, Oregon, Tracey, California, and Boulder City, Nevada. Many hundreds of measurements of electric field strength were made about each terminal at distances ranging from a few tens of meters to several hundreds of kilometers, with measurements being made by teams from Bonnecille Power Administration, the Bureau of Reclamation, the U.S. Geological Survey, and Geoscience, Inc. The purpose of the tests was to establish the rate at which the electric field decreased with distance from a grounding station. This could be established quite well on a statistical basis because of the large volume of data obtained.

Such data may be used to construct equivalent Schlumberger sounding curves. A long power line may be treated as supplying current to the ground at a single point if measurements are made at distances from the end of the line which are small compared to the length of the line. Then, in theory and for a layered earth only, the computed apparent resistivity is exactly that which one would get with a Schlumberger array having a half-spacing between current electrodes equal to the distance from the end of the line to the site at which a measurement is made.

Van Zijl (1969) and Van Zijl et al. (1970) have published dc sounding curves on a crustal scale obtained using the true Schlumberger array in South Africa. These curves clearly demonstrate the advantage of the Schlumberger array over the dipole array in terms of producing a sounding curve with little or no scatter. Other crustal-scale resistivity surveys have been described by Flathe (1967), Blohm and Flathe (1970), Antonov et al. (1969), and Keller (1971a).

Fig. 4 shows the location of the maximum points on the available dc sounding curves, along with lines showing the values for T_2 corresponding to these maximum points. Values for S_1 and T_2 taken from these data are listed in Table I, along with a brief description of the geological setting. However, it must be stressed that these values for transverse resistance are of dubious reliability. In many cases, the maximum of a sounding curve is just barely established at the maximum spacing for which field measurements were made. It is quite possible that some of the maximums are generated prematurely by the effects of lateral changes in resistivity, or that in other cases, a maximum may exist at larger spacings than were used in the field surveys.

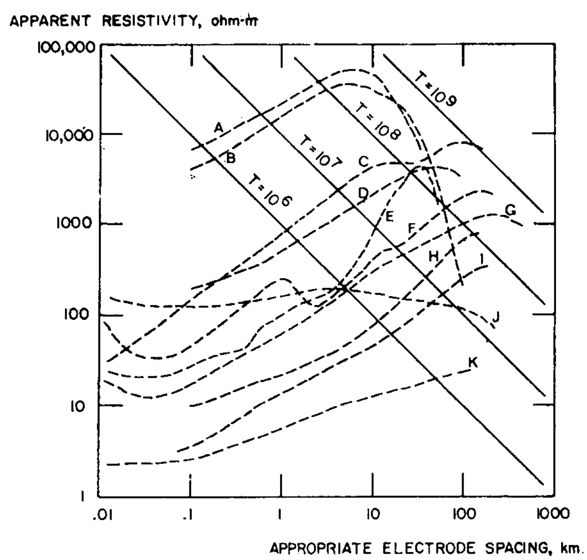

Fig. 4. Summary of deep dc resistivity soundings. A = Maine; B = Adirondacks; C = South Africa; D = Nebraska–Iowa; E = Siberia; F = South Africa; G = Nevada; H = Colorado plateaus; I = California Basin and Range; J = Columbia River plateau; K = California Central Basin.

TABLE I

Summary of parameters from crustal-scale resistivity soundings

Location of survey	Conductance in the upper part of crust (Ω^{-1})	Transverse resistance (Ωm^2)
Maine	0.07	$5 \cdot 10^8$
Adirondacks	0.1	$4 \cdot 10^8$
South Africa	20–40	$(0.5-1.0) \cdot 10^9$
Siberia	10	$5 \cdot 10^8$
Nevada	50	$3 \cdot 10^8$
Colorado plateaus	120	$>2 \cdot 10^8$
California	500–1,000	$>1 \cdot 10^8$
Columbia River Basin	–	$>2 \cdot 10^7$

4. Interpretation

The classic method of interpreting direct-current soundings is by comparison of an experimentally obtained field curve with a pre-calculated catalog of reference curves. Several such catalogues have been published, including those by Compagnie Générale de Géophysique (1963) and by Rijkswaterstaat (1969). Graphical techniques for interpolating between curves in such catalogues have been the subject of study for many years (Kalenov, 1957; Zohdy, 1965; Keller and Frischknecht, 1966). However, the curve comparison method for interpreting resistivity sounding data is rapidly being replaced by computer-assisted interpretation techniques which permit more reliable results to be obtained with less effort.

Two approaches can be used in computer-assisted interpretation. One consists of generating a series of model curves until a curve is found which closely matches the field curve (Meinardus, 1967; Crous, 1971; Depperman, 1973; and Inman et al., 1973), while the other consists of a direct extraction of a resistivity vs. depth function from the field curve, as proposed by Pekeris (1940) and Onodera (1970). The indirect method using computer simulation has been used much more extensively than the direct method.

The expression for apparent resistivity for a horizontally layered earth is usually derived in the form of a Hankel transform integral. For example, for the Schlumberger array, apparent resistivity is given by:

$$\rho_a = \rho_1 a^2 \int_0^\infty K(m,\rho_i,h_i) m J_1(ma)\mathrm{d}m \qquad (8)$$

where ρ_1 is the resistivity of the first layer, ρ_i and h_i are the resistivity and thickness of the ith layer, a is the array spacing, m is the separation parameter needed in obtaining a solution to the Laplace equation by separation of variables, $K(m,\rho_i,h_i)$ is a "kernel" function containing all the available information about the resistivity vs. depth function, and J_1 is the Bessel function of the first kind of order one.

The first step in interpretation consists of inversion of the Hankel transform to obtain an expression for the kernel function, K:

$$2K(m,\rho_i,h_i) = \int_0^\infty \frac{1}{a}\left(\frac{\rho_a}{\rho_1} - 1\right) J_1(ma)\mathrm{d}a \qquad (9)$$

The integration indicated to the right in eq. 9 must be accomplished numerically. Several techniques have been used to minimize errors caused by the oscillatory nature of the integrand, including a modified form of Gaussian quadrature (Meinardus, 1967), and a spline-fitting method (Crous, 1971). Recently, a method of converting the integral in eq. 9 to a convolution form has come into use (Ghosh, 1971; Das and Ghosh, 1973; Anderson, 1973; Daniels, 1974). By using the algebraic transformations $x = \ln a$ and $y = \ln(1/m)$, eq. 9 becomes:

$$\int_{-\infty}^{\infty} \frac{\rho_a}{\rho_1} J_1(e^{x-y})\mathrm{d}x \qquad (10)$$

Being a convolution integral, the integrand can be thought of as a set of filter weights and a sampled function to be filtered. Anderson (1973) has published sets of filter weights for the various convolution integrals that arise in the theory of electrical prospecting. The advantage of the convolution form of integration is the rapidity which may be realized on a digital computer.

Once the kernel function is obtained by numerical inversion of the Hankel transform, the second stage of computation is simulation of the kernel function with an assumed resistivity vs. depth profile. A first guess at this profile is made, and the resultant kernel compared with that obtained from the field curve. The error between the two is reduced by means of a least-squares technique; that is, the derivative of the error with respect to each of the parameters describing the resistivity–depth profile is computed, and a set of simultaneous equations which minimize the error is set up. This set of equations will have a unique solution only if no equivalence, T or S, exists in the resistivity–depth profile. Various methods of forcing convergence to obtain a solution for minimum error have been used, including the Backus and Gilbert algorithm (Backus and Gilbert, 1967; Inman et al., 1973) and the Marquardt algorithm (Marquardt, 1963; Crous, 1971; Daniels, 1974). In either case, convergence is obtained only when minimization is obtained with respect to the equivalence parameters T or S for beds which exhibit equivalence, rather than with respect to resistivity and thickness in those beds.

Computer-assisted interpretation has proved to be highly effective in interpreting resistivity soundings. Typically, the rms error in matching a field curve is an order of magnitude less for computer assisted methods than for graphical methods. Moreover, interpretations are consistent, and the cost is low, being of the order of 1 US$. An important benefit of computer-assisted interpretations is that the various derivatives which are computed can be used to estimate the resolution of the interpretation. If a derivative with respect to the resistivity or thickness of a particular layer is large, then the resolution for that parameter is high.

5. Comments

Crustal-scale resistivity surveys are an expensive undertaking, in view of the large electrode arrays which must be used, and the high power which is required. Because of the screening effect of the resistant part of the crust, unreasonably large electrode arrays must be used to detect the presence of conductive regions in the outer mantle. I would like to call your attention to a seeming paradox with respect to the relationship between depth of penetration of current in the earth and frequency. If the crust of the earth were perfectly insulating, for the sake of simplicity, we can readily see that direct current (frequency = 0) would never provide information about conductive zones below the crust. However, as the frequency is raised from zero, some magnetic induction will take place below the insulating crust, and perhaps the effect can be detected at the earth's surface. In fact, the inductive effects will increase as the frequency is raised until the point is reached where skin effects in the surface layers become important. In my opinion, controlled-source electromagnetic methods offer more prospect for studying resistivity in the outer mantle than do direct-current methods. So far, no crustal-scale electromagnetic induction surveys have been reported in the literature, but such efforts are under way (Keller, 1971a; Heacock, 1971).

References

Al'pin, L.M., Berdichevskiy, M.N. Vedrintsev, G.A. and Zagarmistr, A.M., 1966. Dipole Methods for Measuring Earth Conductivity. Consultants Bureau, New York, N.Y.

Anderson, W.L., 1973. FORTRAN IV programs for the Determination of the Transient Tangential Electric Field and Vertical Magnetic Dipole for an M-layered Stratified Earth by Numerical Integration and Digital Linear Filtering. U.S.G.S. Publication PB-226 240/5, Denver, Colo.

Antonov, Yu.N. and Izyumov, I.F., 1969. On deep soundings with direct current in western Siberia. Geol. Geofiz., 8: 98–101.

Backus, G.E. and Gilbert, J.F., 1967. Numerical applications of a formalism for geophysical inverse problems. Geophys. J.R. Astron. Soc., 13: 247–276.

Blohm, E.K. and Flathe, H., 1970. Geoelectrical deep sounding in the Rhine-graben. In: J.H. Ellies and St. Mueller (Editors), Graben Problems, Schweizerbart Stuttgart, pp. 239–241.

Caner, B., Cannon, W.H. and Livingston, C.E., 1967. Geomagnetic depth sounding and upper mantle structure in the Cordillera region of western North America. J. Geophys. Res., 72 (24): 6335–6352.

Cantwell, T., Nelson, P., Webb, J.E., Orange, A., Kinyon, A.L., Stevens, R.F. and Waugh, C.L., 1966. Ground current in high voltage transmission I-preliminary report on Columbia Basin tests. Inst. Electr. Electron. Eng. Trans. Power Appl. Syst., PAS 85 (3): 240–253.

Compagnie Générale de Géophysique, 1963. Master Curves for Electrical Sounding. European Association of Exploration Geophysicists, The Hague.

Crous, C.M., 1971. Computer-assisted Interpretation of Electrical Soundings. Thesis T-1363, Colorado School of Mines Golden, Colo., 108 pp.

Daniels, J.J., 1974. Interpretation of Electromagnetic Soundings Using a Layered Earth Model. Thesis T-1627, Colorado School of Mines, Golden, Colo., 84 pp.

Das, U.C. and Ghosh, D.P., 1973. A study of the direct interpretation of dipole sounding resistivity measurements over layered earth. Geophys. Prospect., 21 (2): 379–400.

Depperman, K., 1973. An interpretation system for geoelectrical sounding graphs. Geophys. Prospect., 21 (3): 424–463.
Flathe, H., 1967. The determination of the electrical resistivity of the crust within the region of the Rhinegraben. Abh. Geol. Landesamtes Baden-Württemberg, 6: 96–97.
Frolich, R.K., 1967. The depth penetration of dipole arrays compared with the Schlumberger arrangement. Geoexploration, 5: 195–203.
Furgerson, R.B. and Keller, G.V., 1974. Computed Dipole Resistivity Effects for an Earth Model with Vertical and Lateral Contrasts in Resistivity. Tech. Rep. Off. Nav. Res., NR 081-275, Colorado School of Mines, Golden, Colo., 194 pp.
Ghosh, D.P., 1971. The application of linear filter theory to the direct interpretation of geoelectrical resistivity sounding measurements. Geophys. Prospect., 19 (2): 192–217.
Heacock, J.G., 1971. Intermediate and deep properties of the earth's crust, a possible electromagnetic wave guide. In: J.G. Heacock, The Structure and Physical Properties of the Earth's Crust. Am. Geophys. Union Monogr. 14, Washington, D.C., pp. 1–10.
Inman, J.R., Ryu, F. and Ward, S.H., 1973 Resistivity inversion. Geophysics, 38 (6): 1088–1108.
Kalenov, E.N., 1957. Interpretation of Vertical Electric Sounding Curves. Gos. Nauchno-Tekh. Izd., Moscow, 473 pp.
Keller, G.V., 1966. Dipole method for deep resistivity studies. Geophysics, 31 (6): 1088–1104.
Keller, G.V., 1968a. Electrical prospecting for oil: Colo. Sch. Mines Q., 63 (2).
Keller, G.V., 1968b. Statistical study of electric fields from earth-return tests in the western states compared with natural fields. Inst. Electr. Electron. Eng. Trans. Power Appl. Sys., PAS 87 (4): 1050–1057.
Keller, G.V., 1971a. Electromagnetic survey of the central volcanic zone, North Island, New Zealand, N.Z. Dep. Sci. Ind. Res. Bull. (in preparation).
Keller, G.V., 1971b. Natural field and controlled source electromagnetic prospecting methods. Geoexploration, 9 (2/3): 99–148.
Keller, G.V. 1971c. Electrical studies of the crust and upper mantle. In: J.G. Heacock, The Structure and Physical Properties of the Earth's Crust. Am. Geophys. Union Monogr. 14, Washington, D.C., pp. 107–126.
Keller, G.V. and Frischknecht, F.C., 1966. Electrical Methods in Geophysical Prospecting. Pergamon, Oxford, 526 pp.
Keller, G.V., Anderson, L.A. and Pritchard, J.I., 1966. Geological Survey investigations of the electrical properties of the crust and upper mantle. Geophysics, 31 (6): 1078–1087.
Kunetz, G., 1966. Principles of Direct Current Resistivity Prospecting. Bornträger, Berlin, 103 pp.
Marquardt, D.W., 1963. An algorithm for least-squares estimation of nonlinear parameters. J. Soc. Ind. Appl. Math., 11 (2): 432–441.
Meinardus, H.A., 1967, The kernel function in direct-resistivity sounding. Thesis T-1103, Colorado School of Mines, Golden, Colo., 151 pp.
Onodera, S., 1970. An analytic interpretation of apparent resistivity sounding curve for a multiple layered earth. Mem. Fac. Eng. Kyushu Univ., 29 (2): 107–120.
Pekeris, C.L., 1940. Direct method of interpretation in resistivity prospecting. Geophysics, 5 (1): 31–42.
Rijkswaterstaat, 1969. Standard graphs for resistivity prospecting. European Association of Exploration Geophysicists, The Hague.
Van Zijl, J.S.V., 1969. A deep Schlumberger sounding to investigate the electrical structure of the crust and upper mantle in South Africa. Geophysics, 34: 450–462.
Van Zijl, J.S.V. and Joubert, S.J., 1975. A crustal geoelectrical model for South African Precambrian granitic terrains based on deep Schlumberger soundings. Geophysics (in press).
Van Zijl, J.S.V., Hugo, P.L.V. and De Bellocq, J.H., 1970. Ultradeep Schlumberger sounding and crustal conductivity structure in South Africa. Geophys. Prospect., 18 (4): 615–634.
Zohdy, A.A.R., 1965. The auxiliary point method of electrical sounding interpretation and its relationship to the Dar Zarrouk parameters. Geophysics, 30: 644–660.