How do we include **3D Magnetotelluric** Data into Joint **Probabilistic Inversions?**

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In collaboration with Geoscience Australia, Geological Survey of South Australia, Geological Survey of NSW, Northern Territory Geological Survey

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Overview

- **Introduction** to inverse problem and joint inversion
- Deterministic and **probabilistic inversion**
- **Reduced Basis** Method
- **RB+MCMC** approach

- Joint 3D MT+SW probabilistic inversion
  - **Structure** of the code
  - **Parameterisation**
  - **Sampling** strategy
  - **Synthetic** example

- Conclusions

What is **M**agneto**T**ellurics (**MT**)?

Passive technique

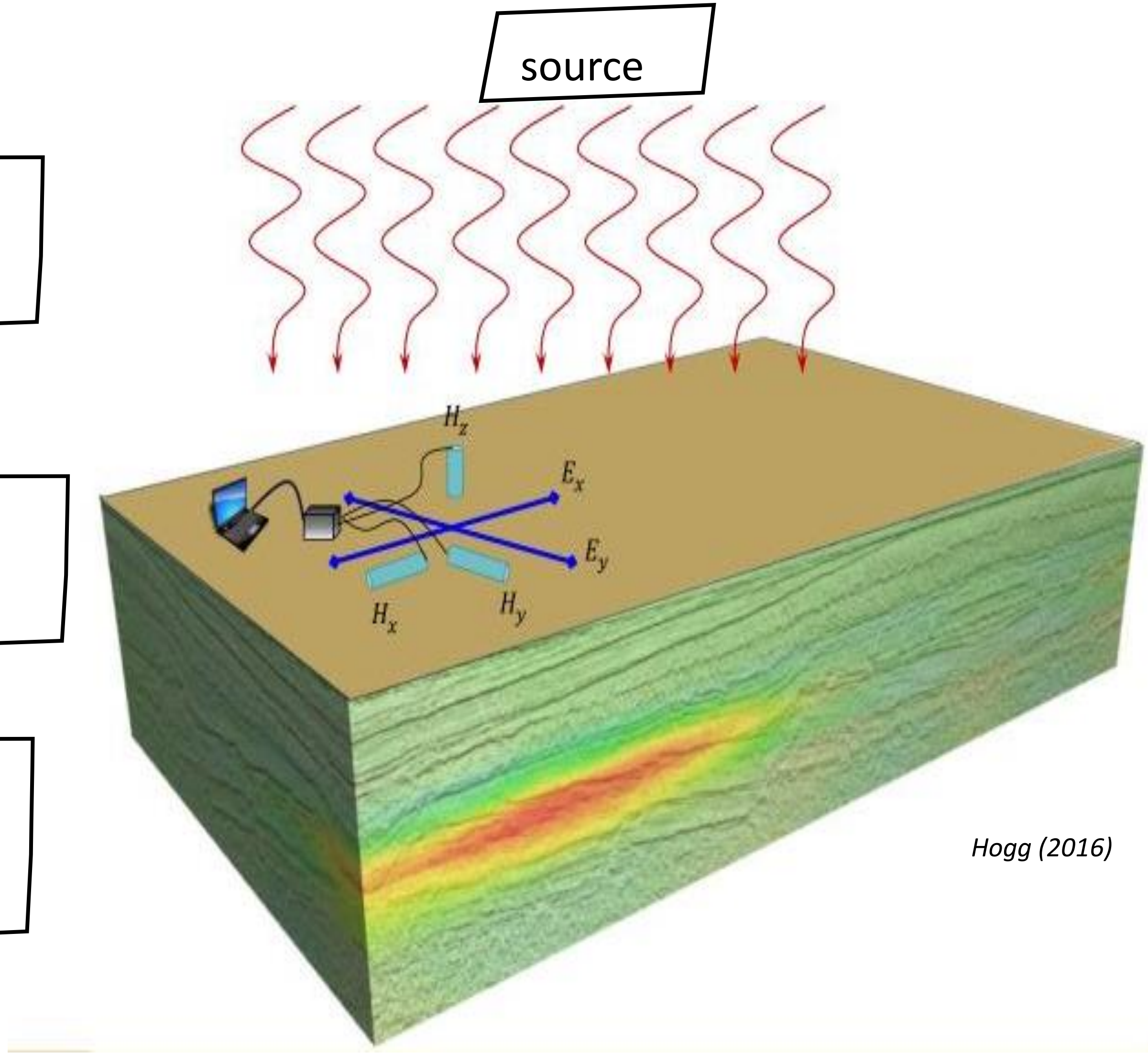
measures variations of the Earth's **electric** and **magnetic** field.

Objective

determine **electrical conductivity** distribution below the surface.

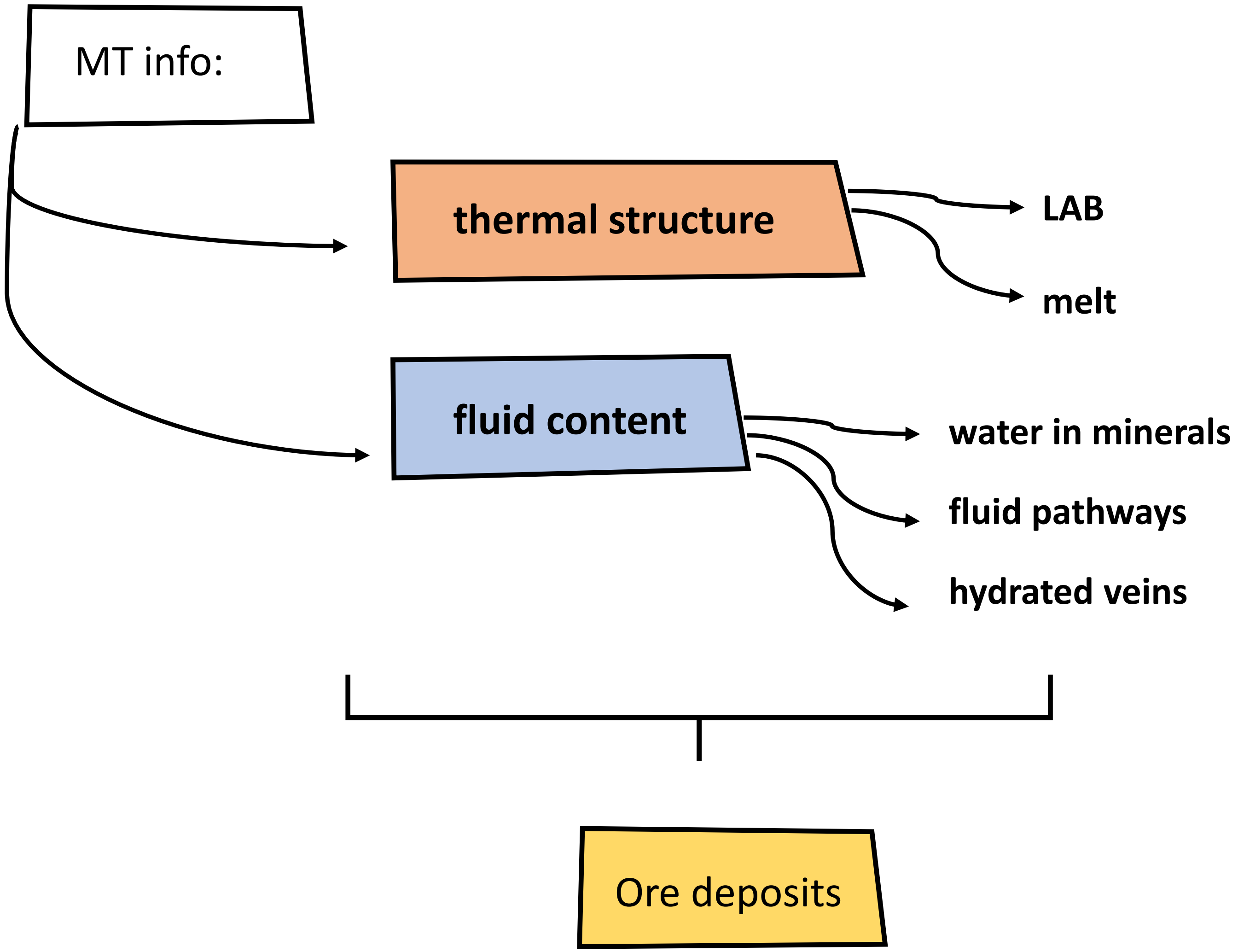
Electrical conductivity (σ)

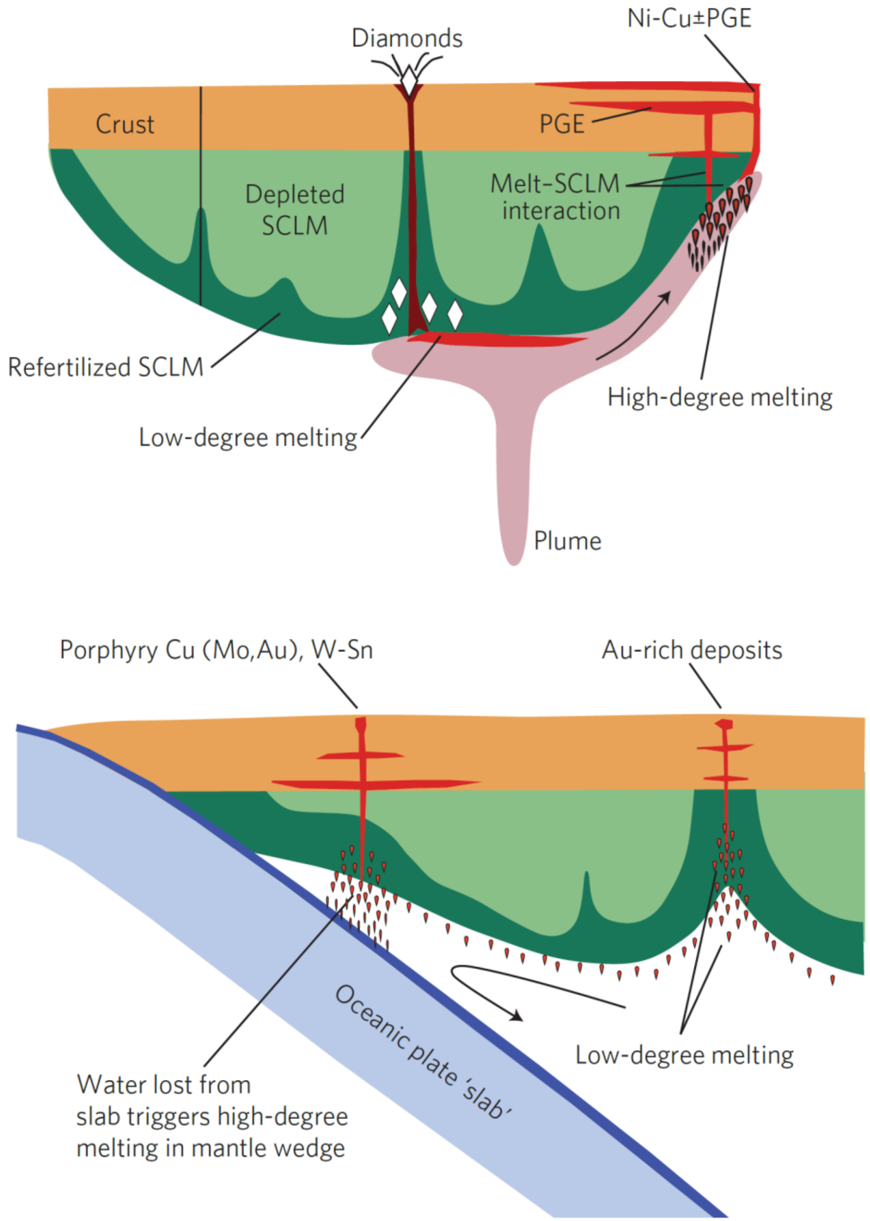
Depends on: composition, temperature, **water content**, **melt content**



Hogg (2016)

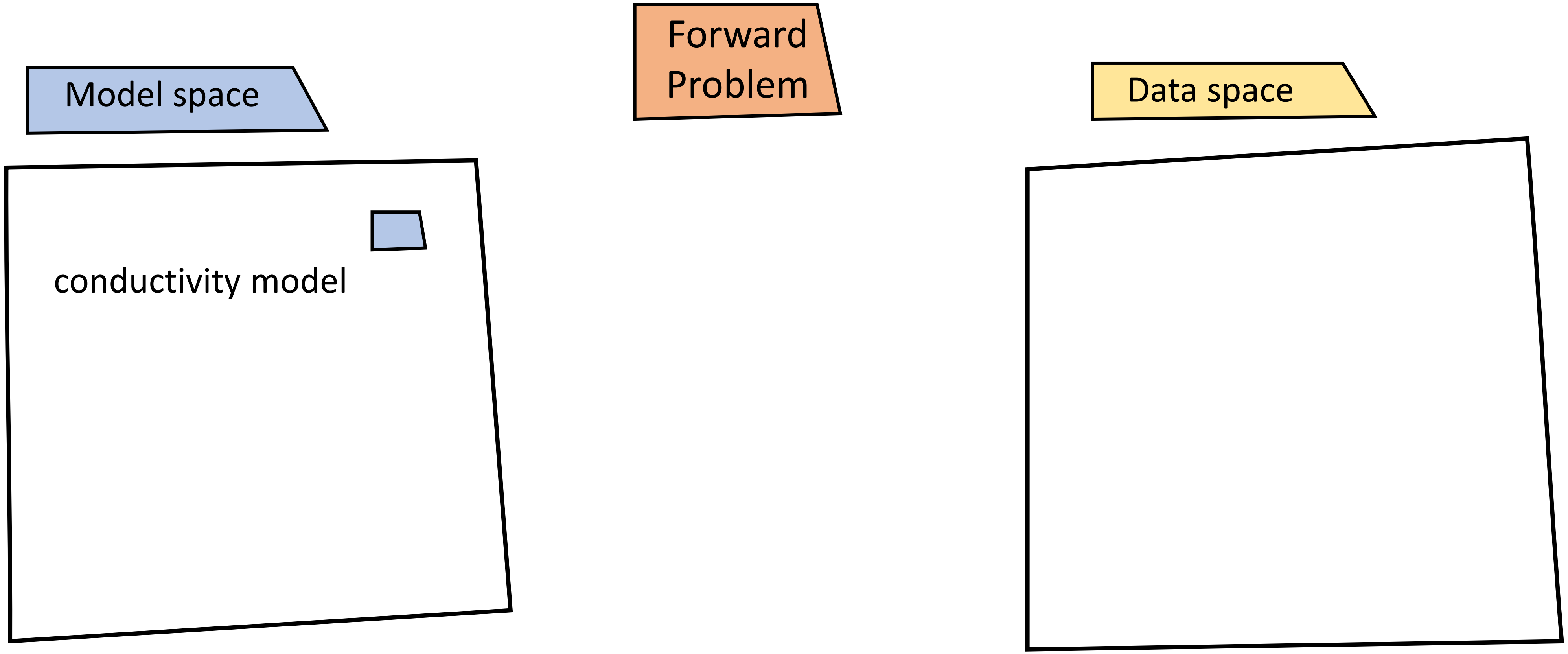
Importance of MT





Griffin et al. (2013)

Inverse theory



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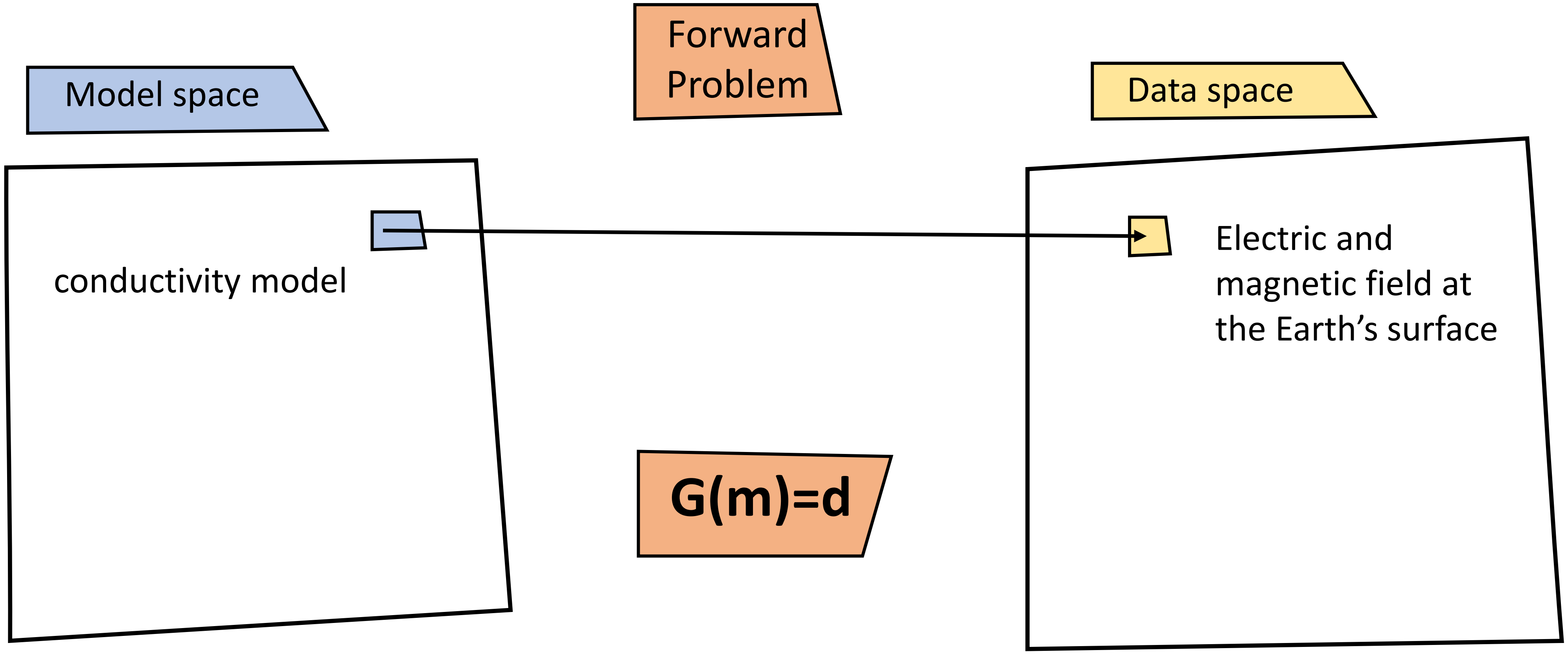
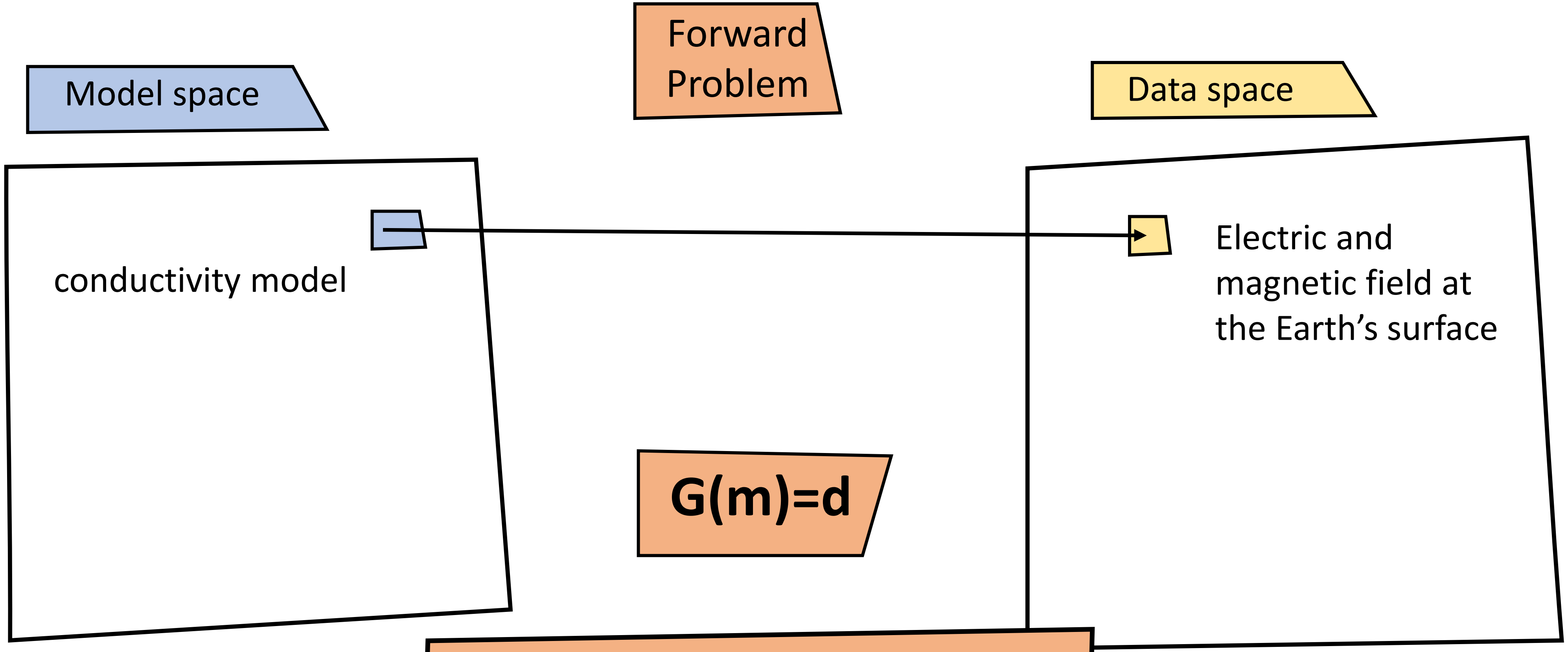
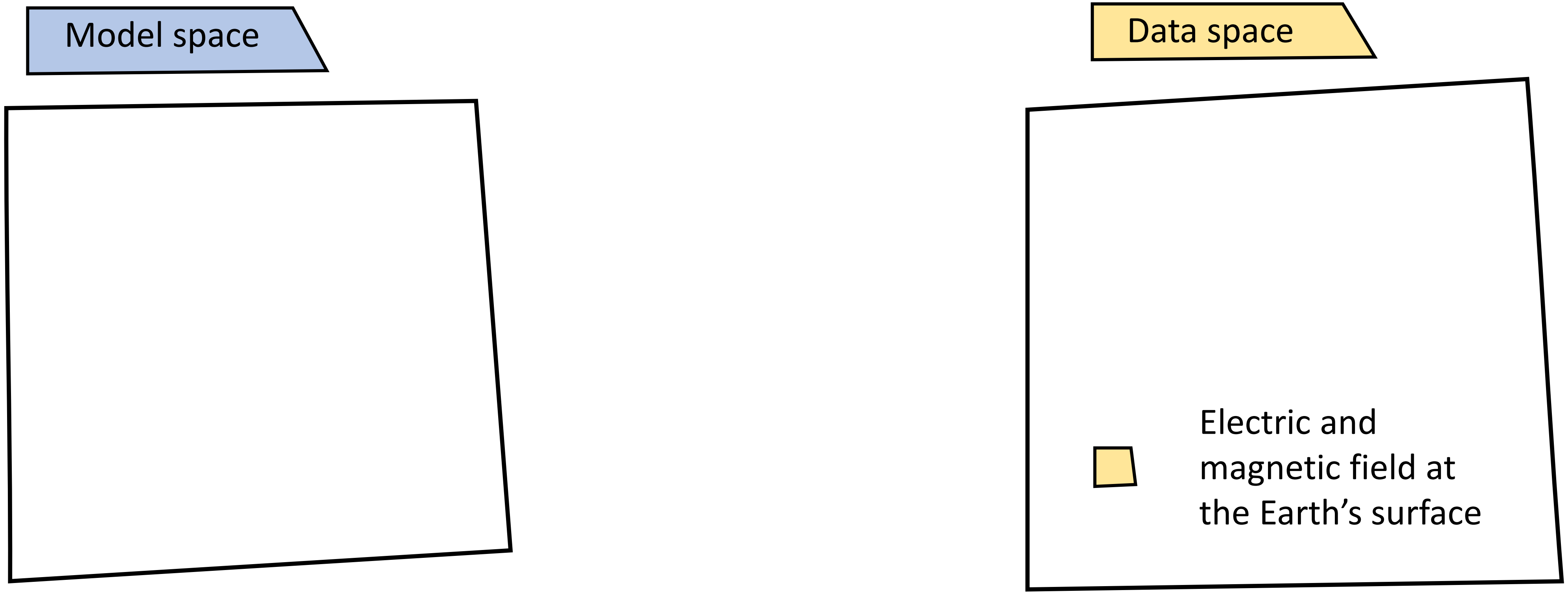




In MT the forward is fast in 1D and 2D, but computationally expensive in 3D!!

Inverse theory



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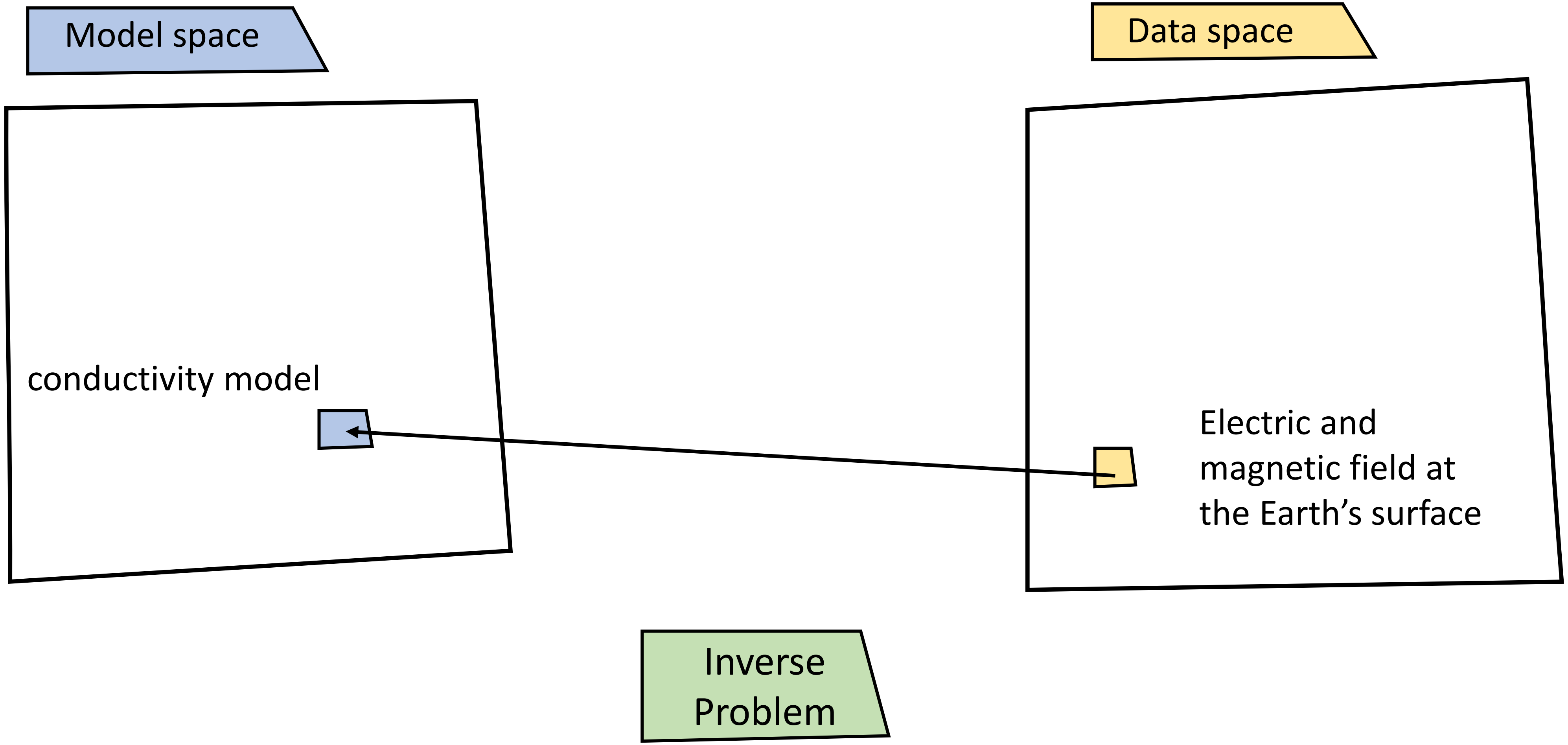

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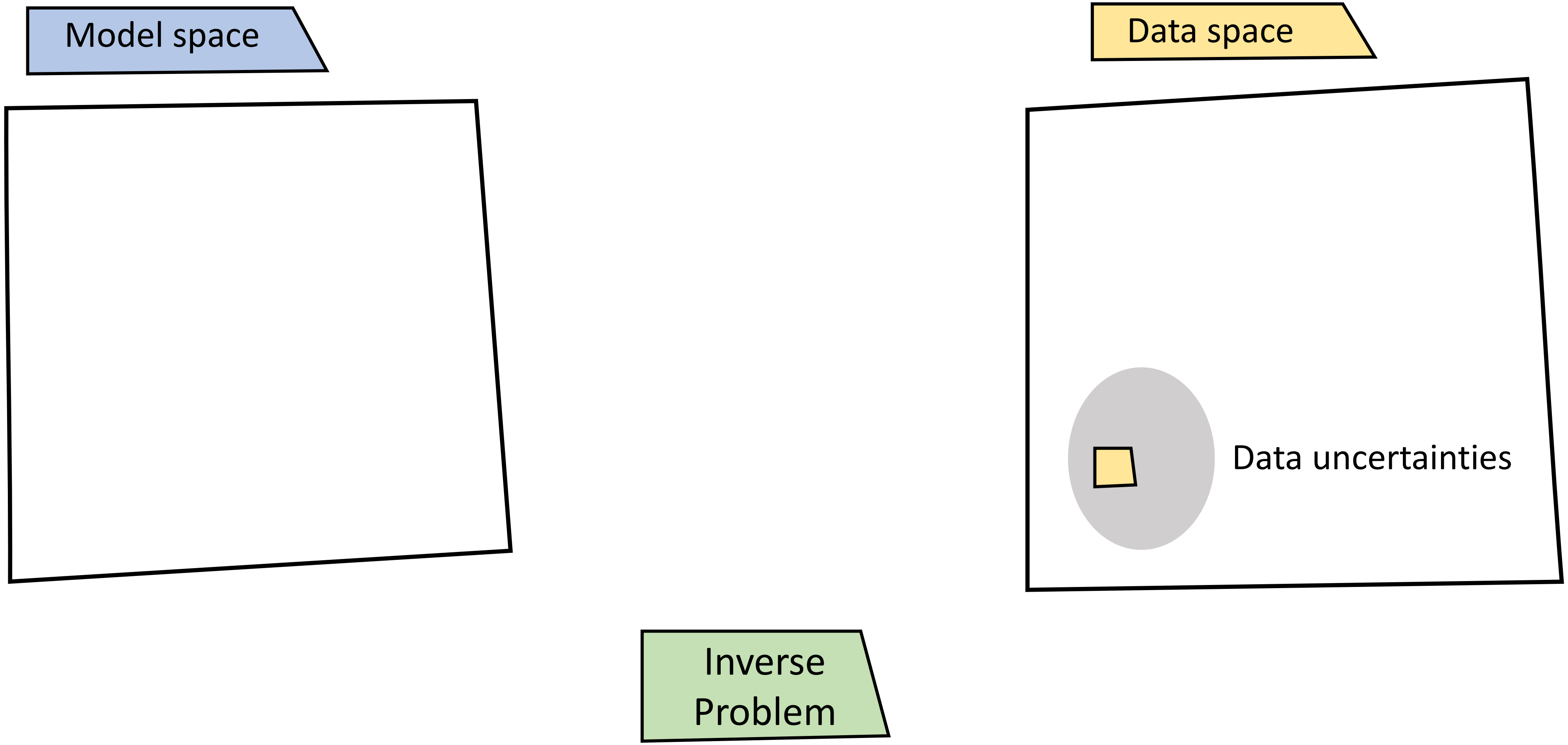
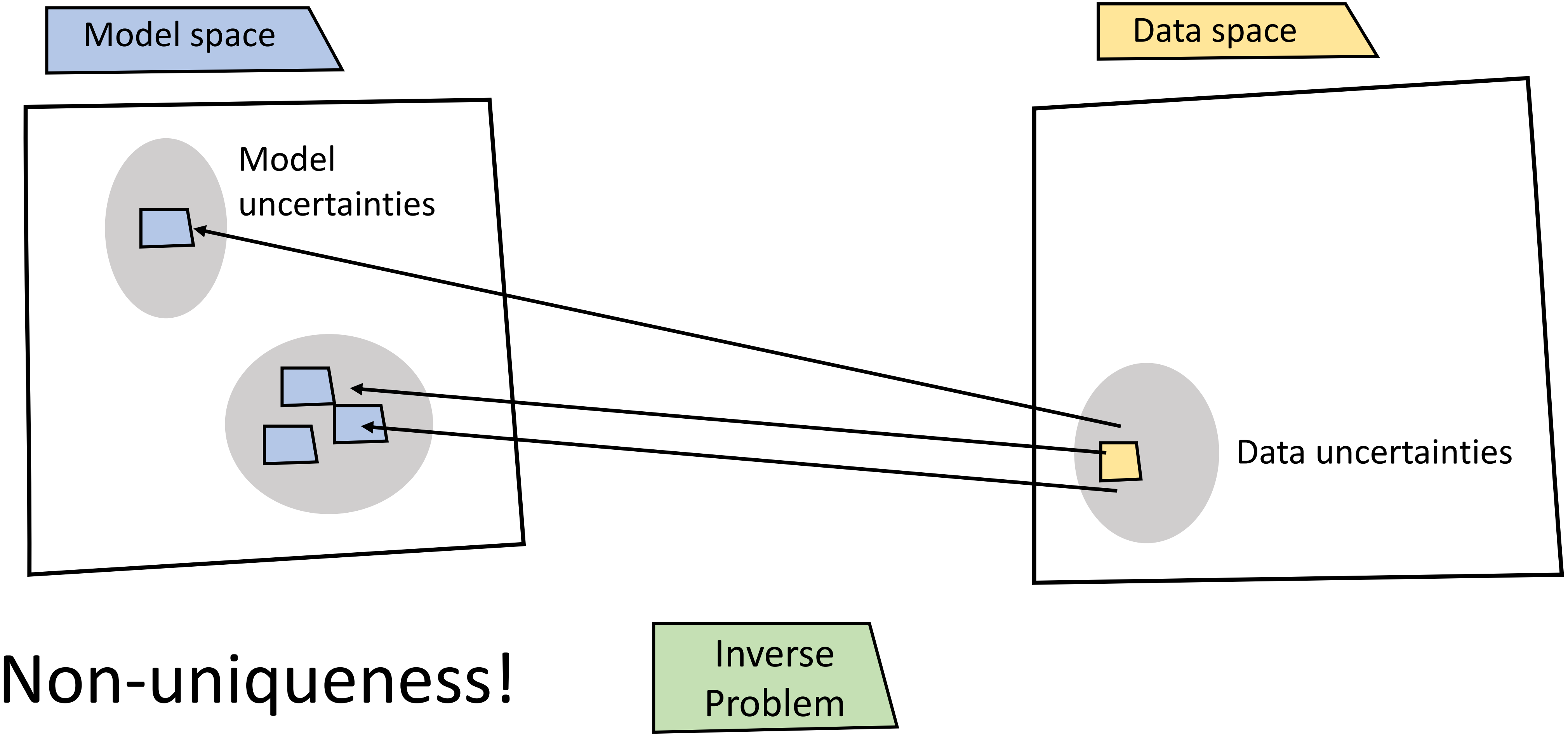
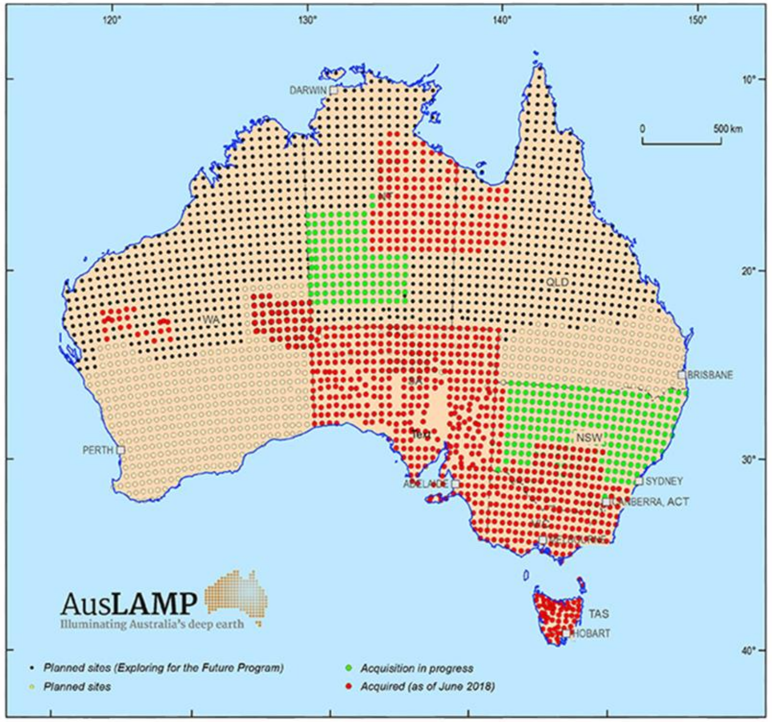




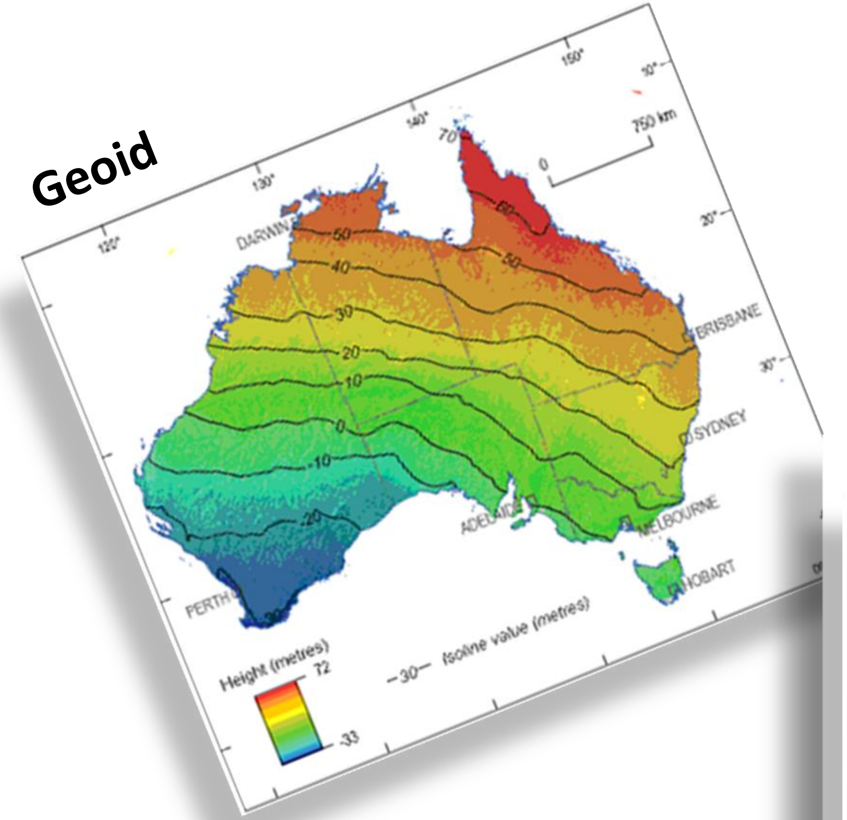
Non-uniqueness!

3D MT Data


120°
130°
140°
150°
10°
20°
30°
40°
0
500 km
DARWIN
NT
QLD
WA
SA
NSW
BRISBANE
PERTH
SYDNEY
TAS
HOBART
AusLAMP
Illuminating Australia's deep earth
Planned sites (Exploring for the Future Program)
Planned sites
Acquisition in progress
Acquired (as of June 2018)

Joint inversion

Geoid



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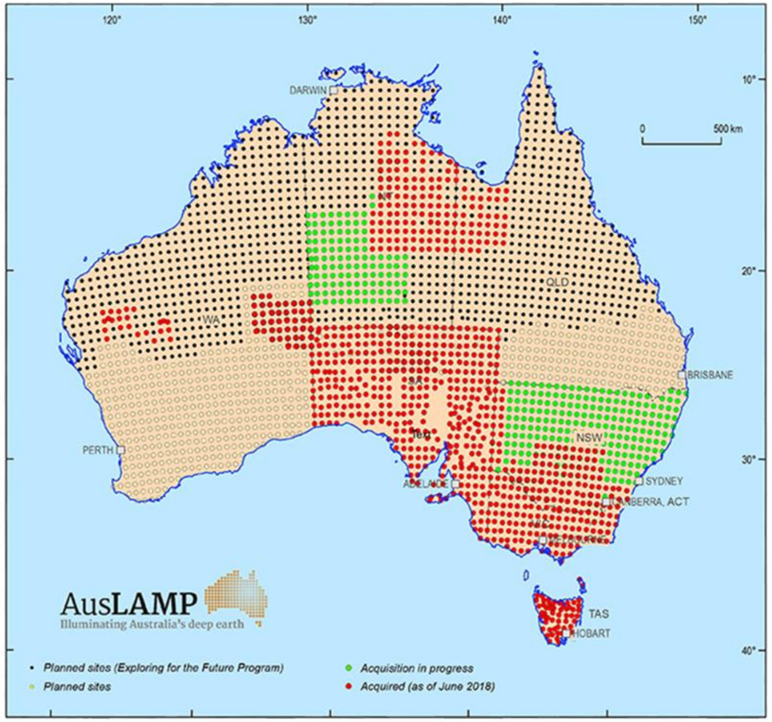

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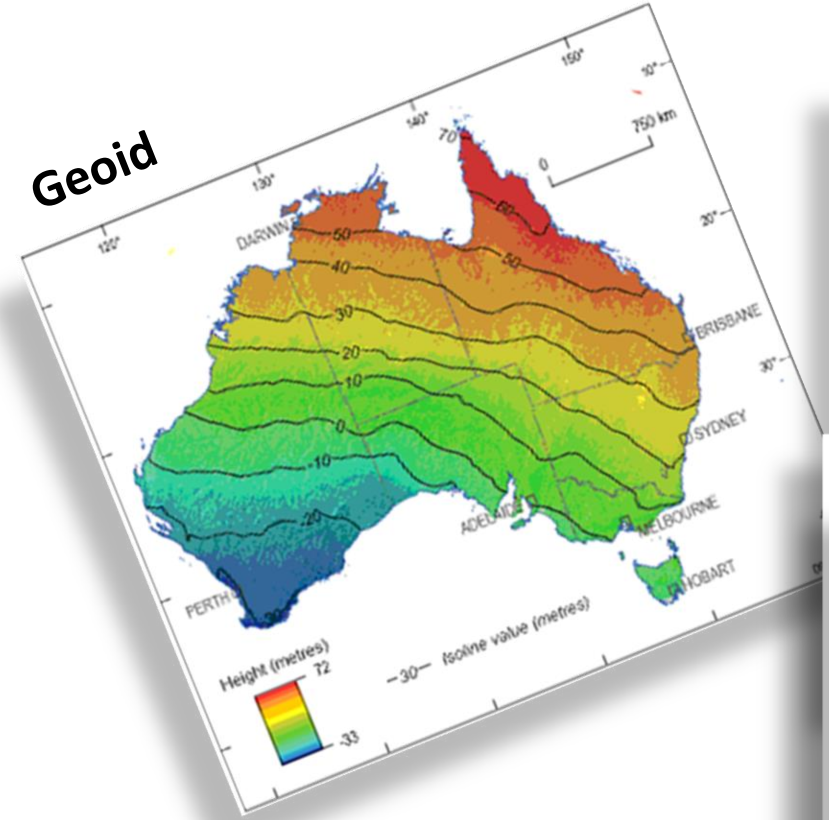
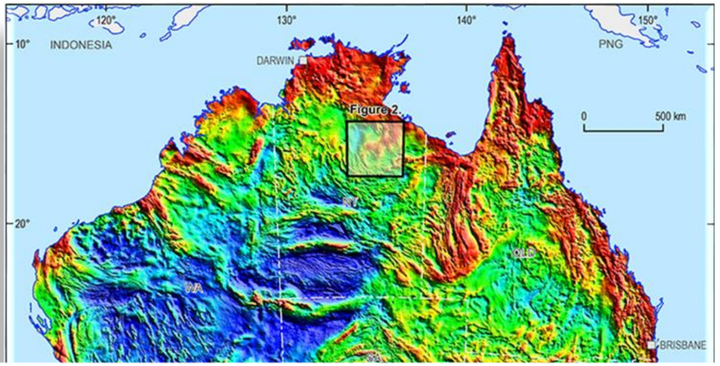


Gravity anomalies



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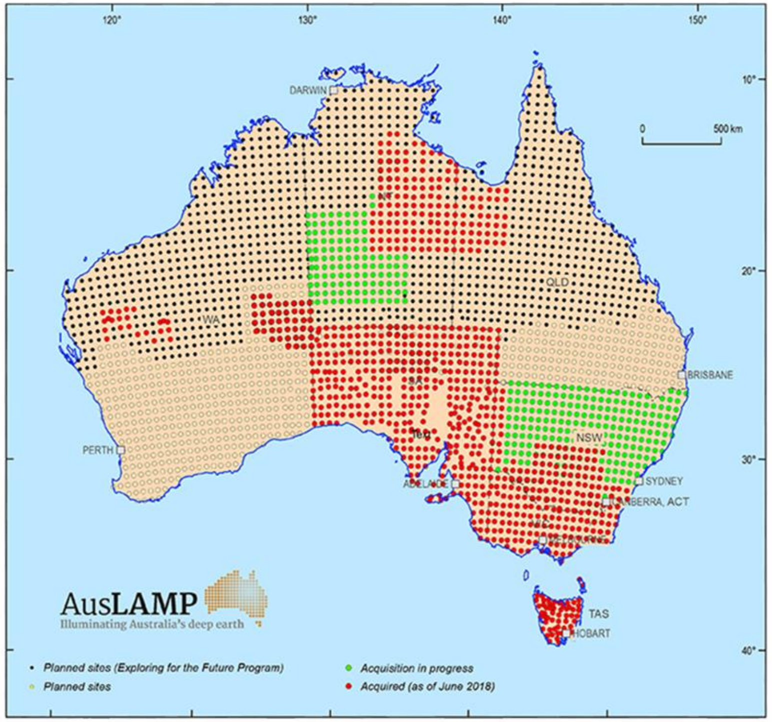

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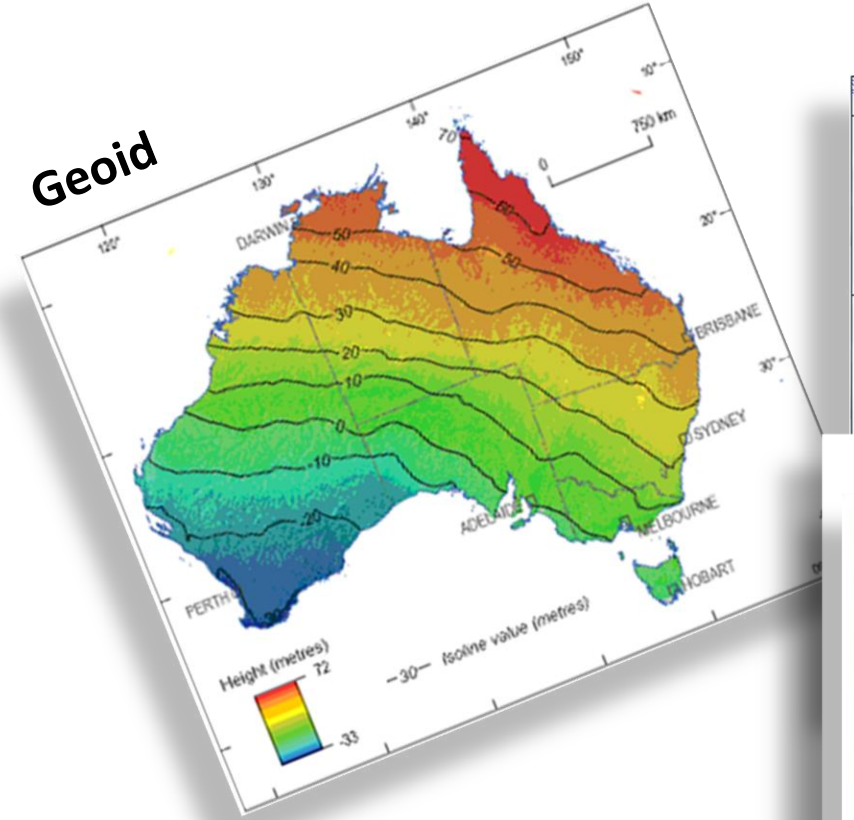

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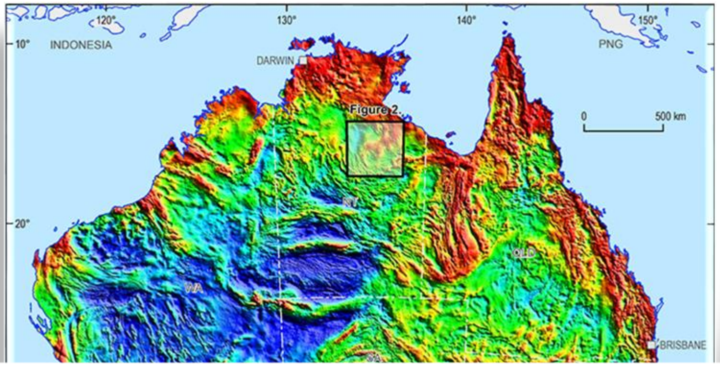
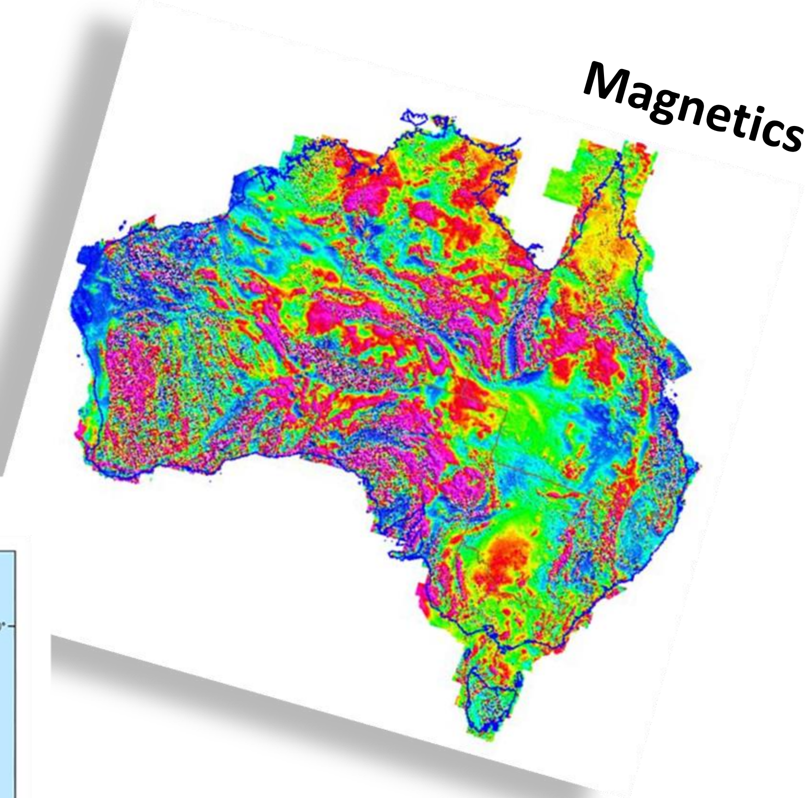
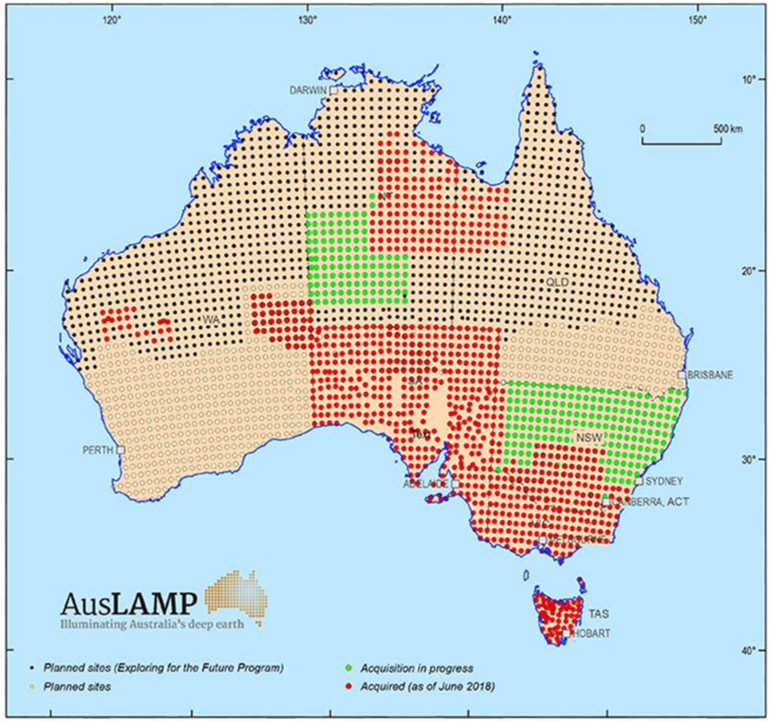


Magnetics

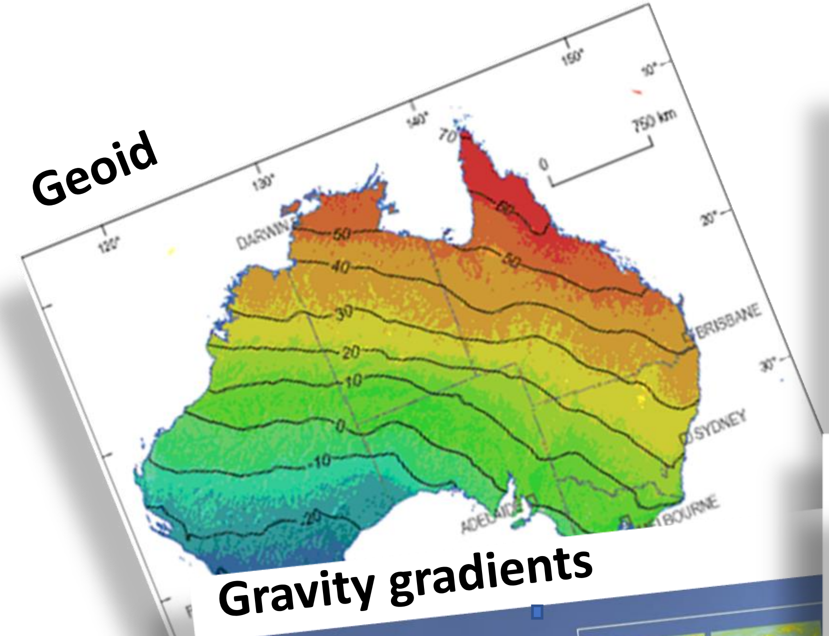


3D MT Data



Joint inversion

Geoid



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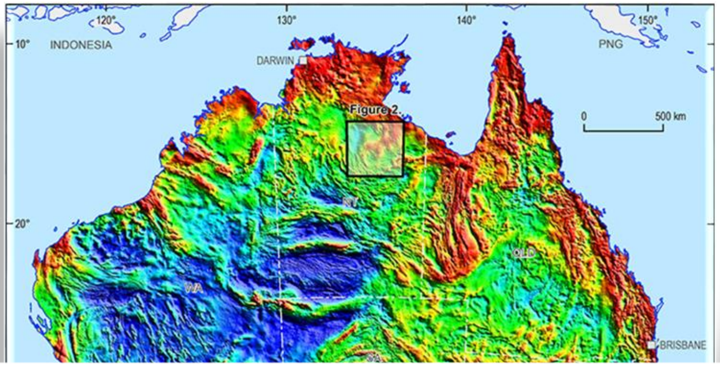

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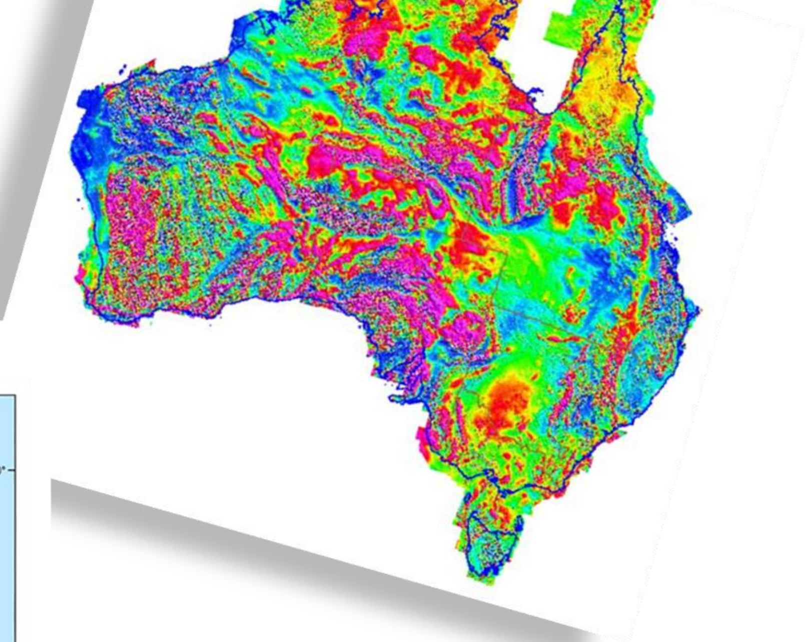
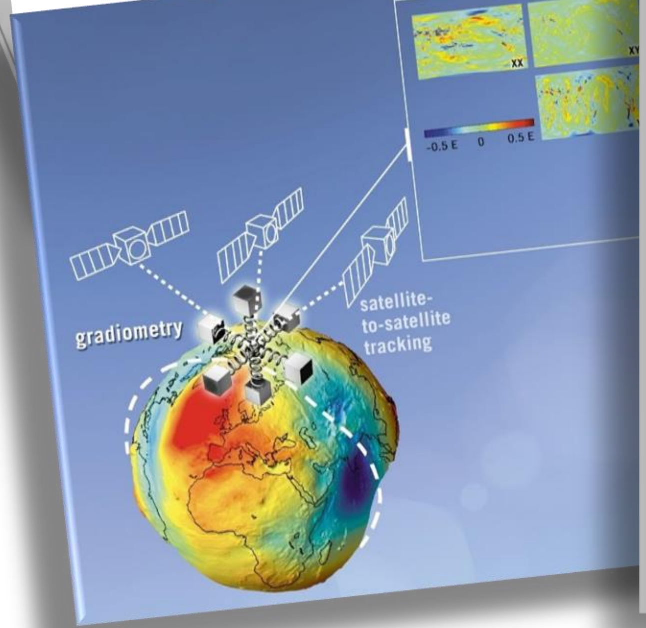
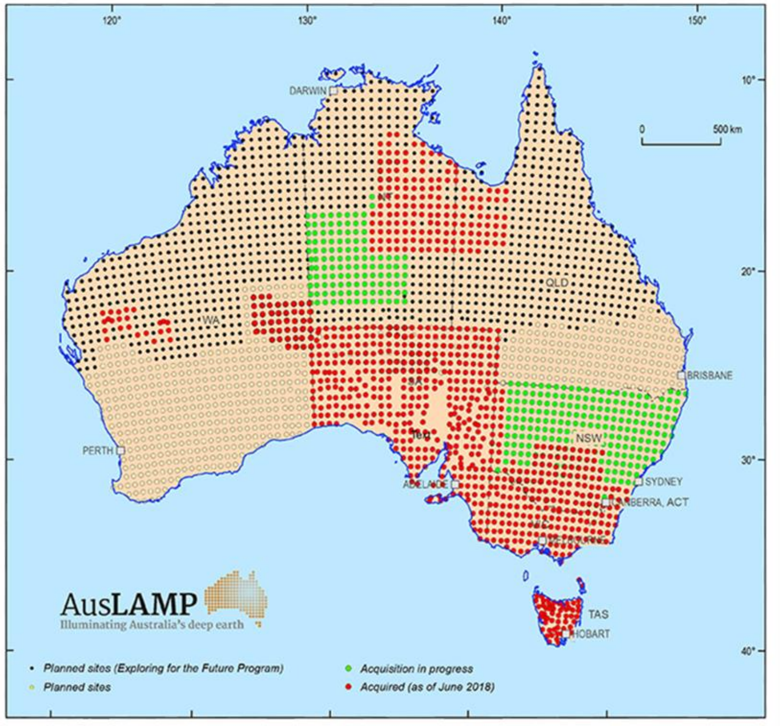


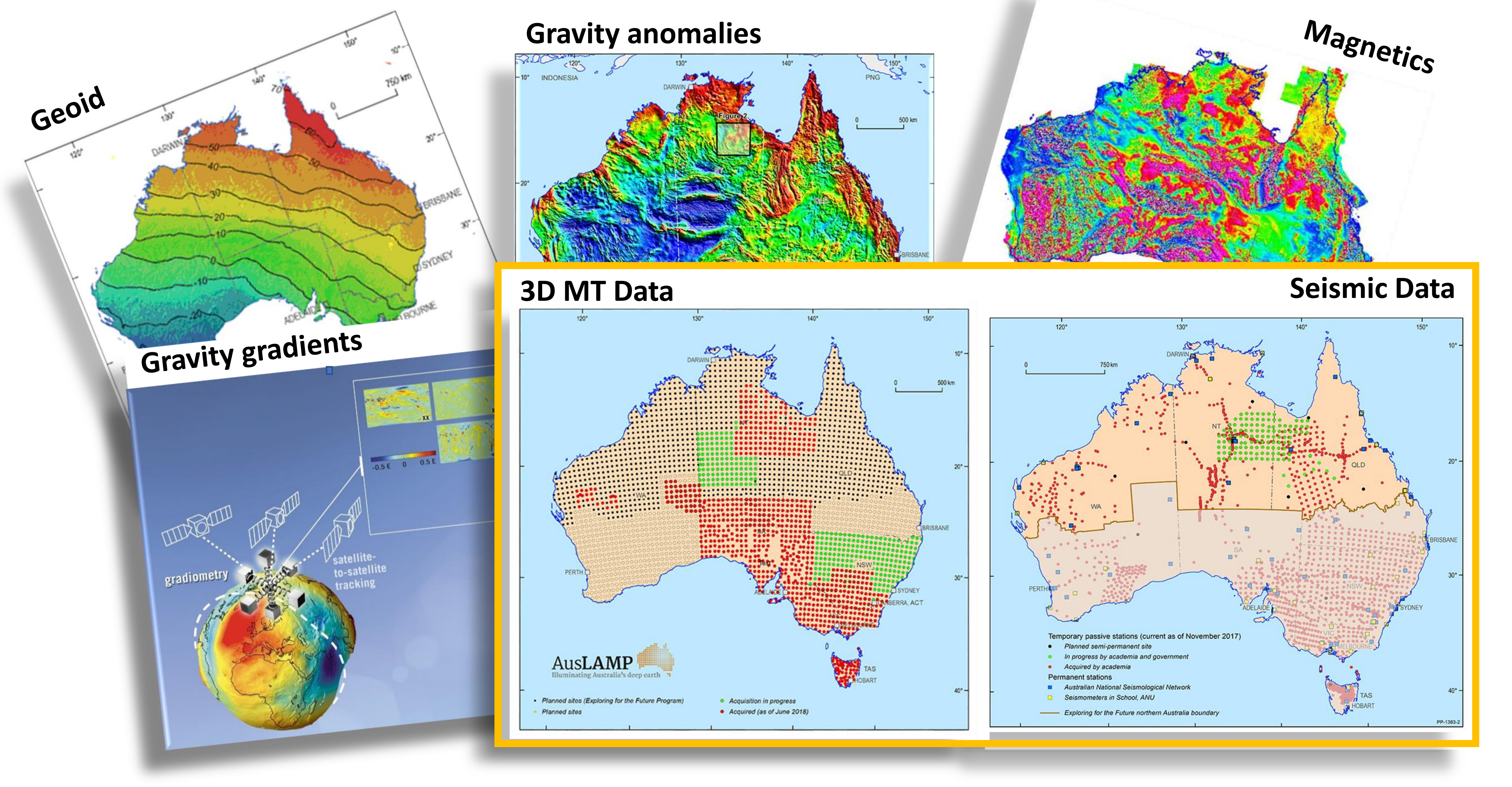
Gravity gradients



3D MT Data



Joint inversion



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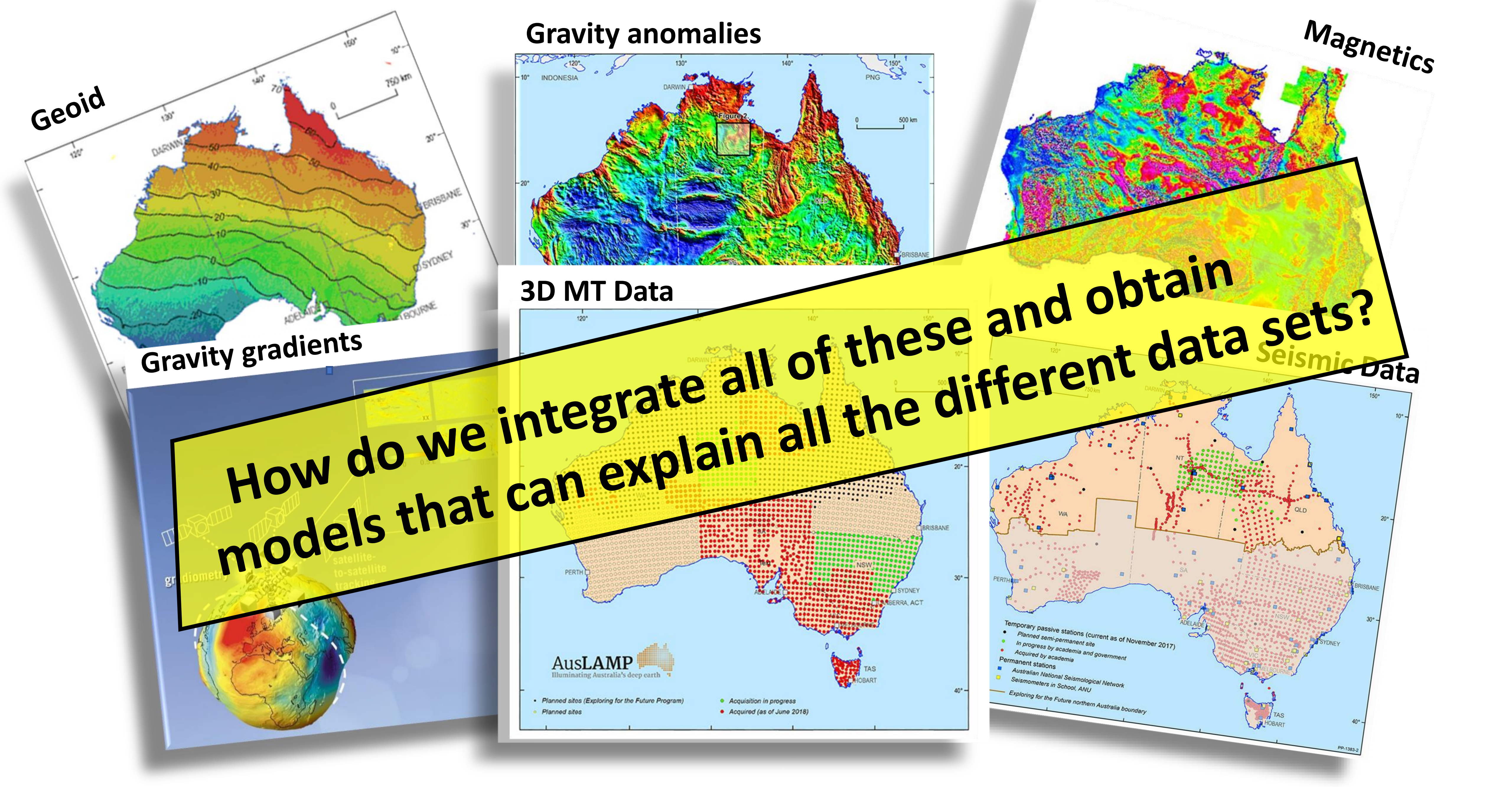
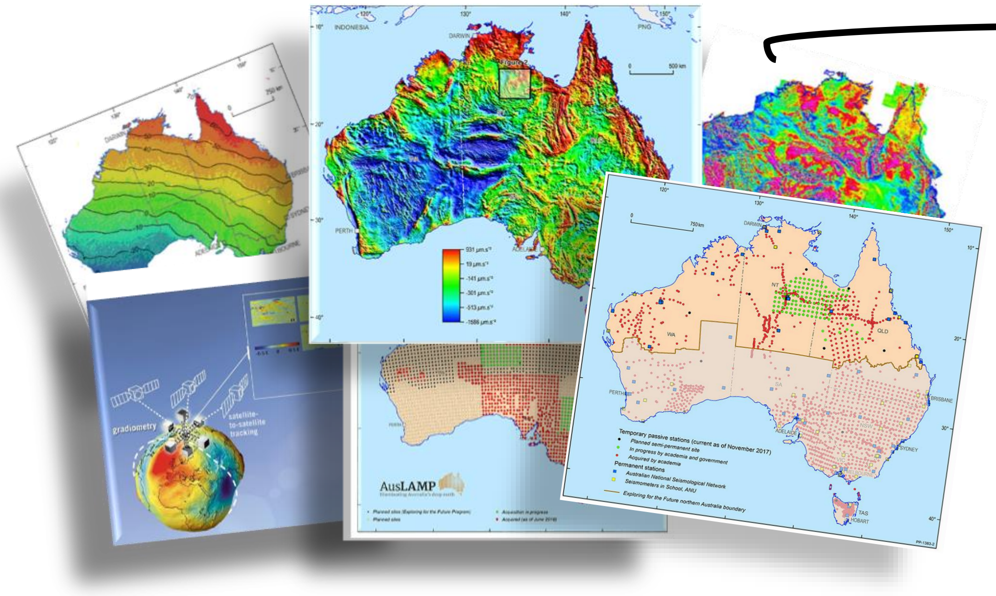




Deterministic or Traditional Inversion

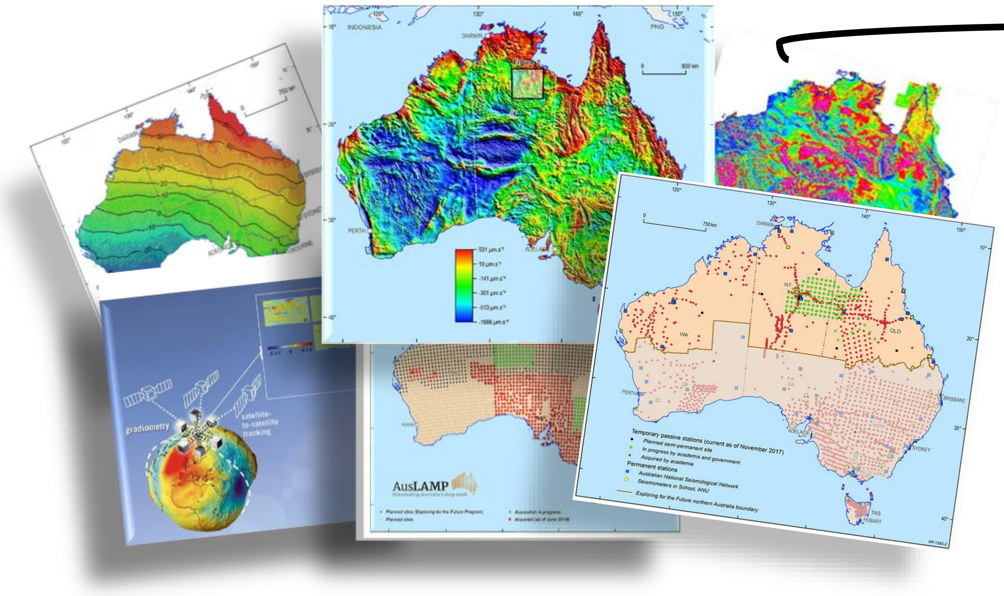
Advantages

- Successfully used for inverting a large number of unknowns

Disadvantages

- Solution depend on the **choice** of regularizations and parameterizations
- **limited uncertainty information** on the estimated parameters
- **'bias':** features of the true model may not be resolved

Only one ' best solution' per inversion!



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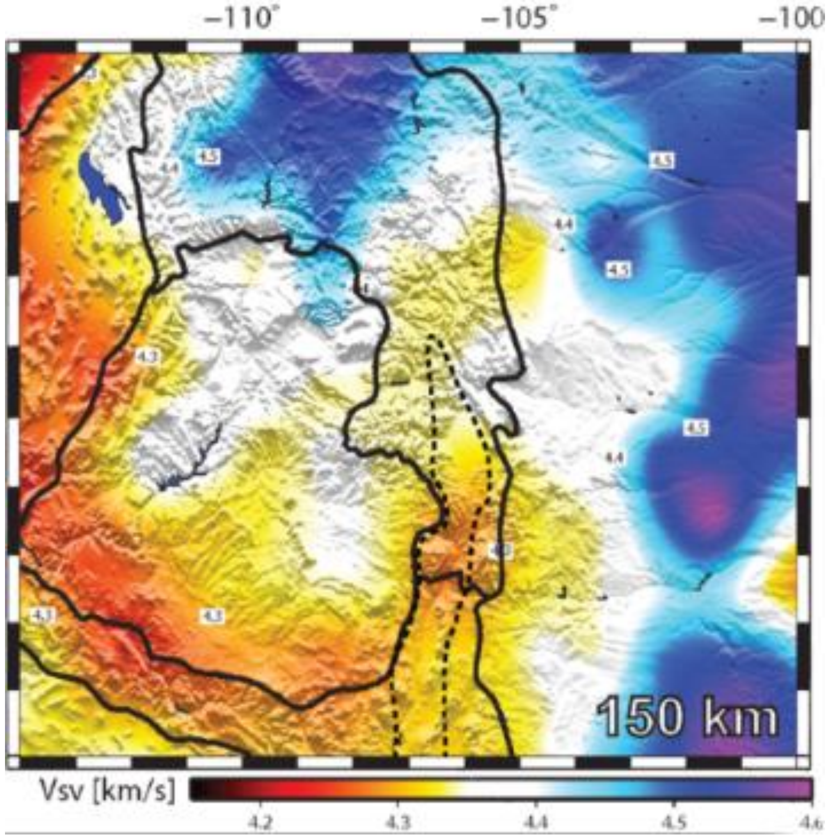

Afonso et al. (2016)

Single and 'best' model
based on a single physical parameter
(vs,vp, etc)

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- Successfully used for inverting a large number of unknowns

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- Solution depend on the **choice** of regularizations and parameterizations
- ➘ **limited uncertainty information** on the estimated parameters
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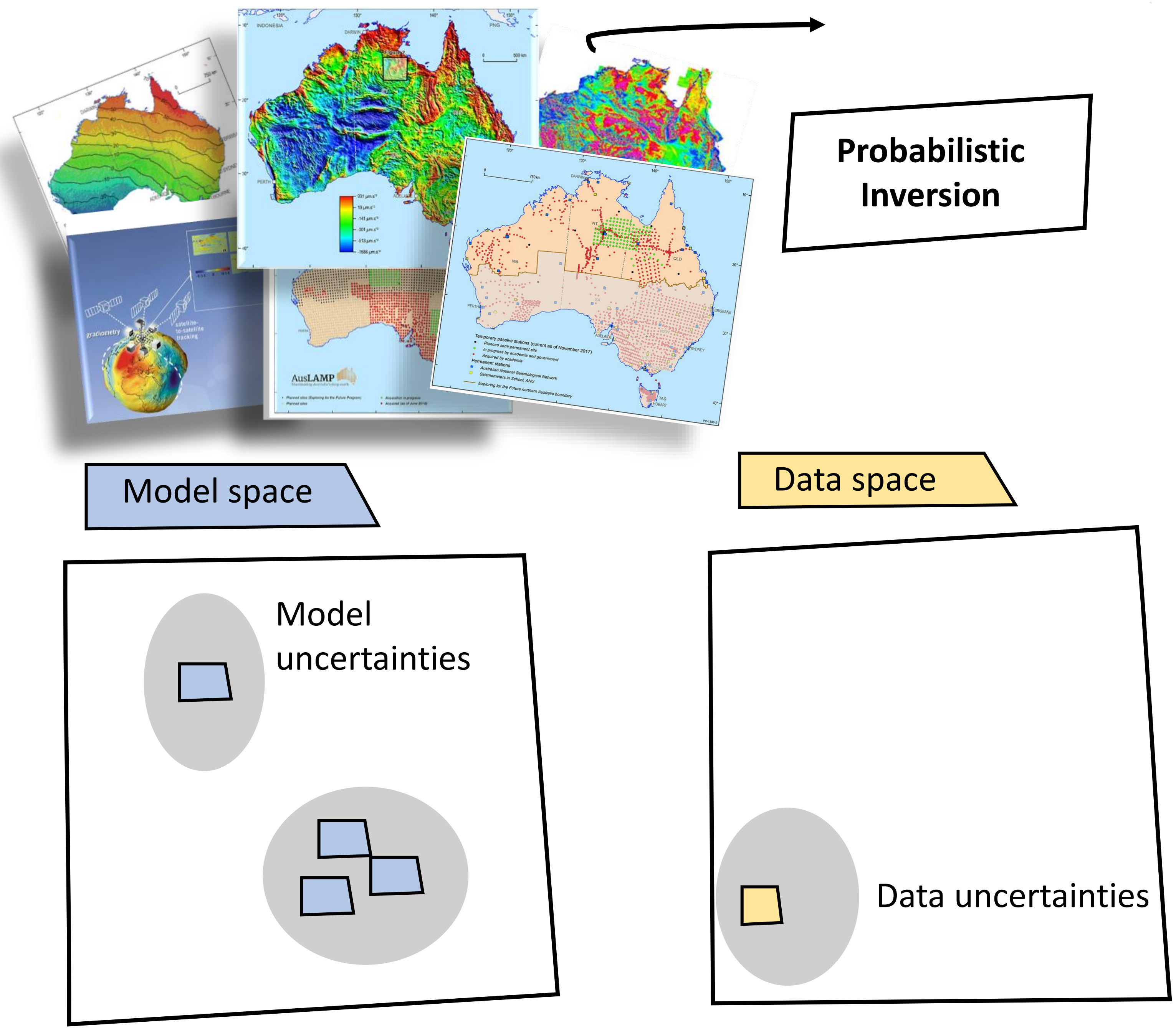
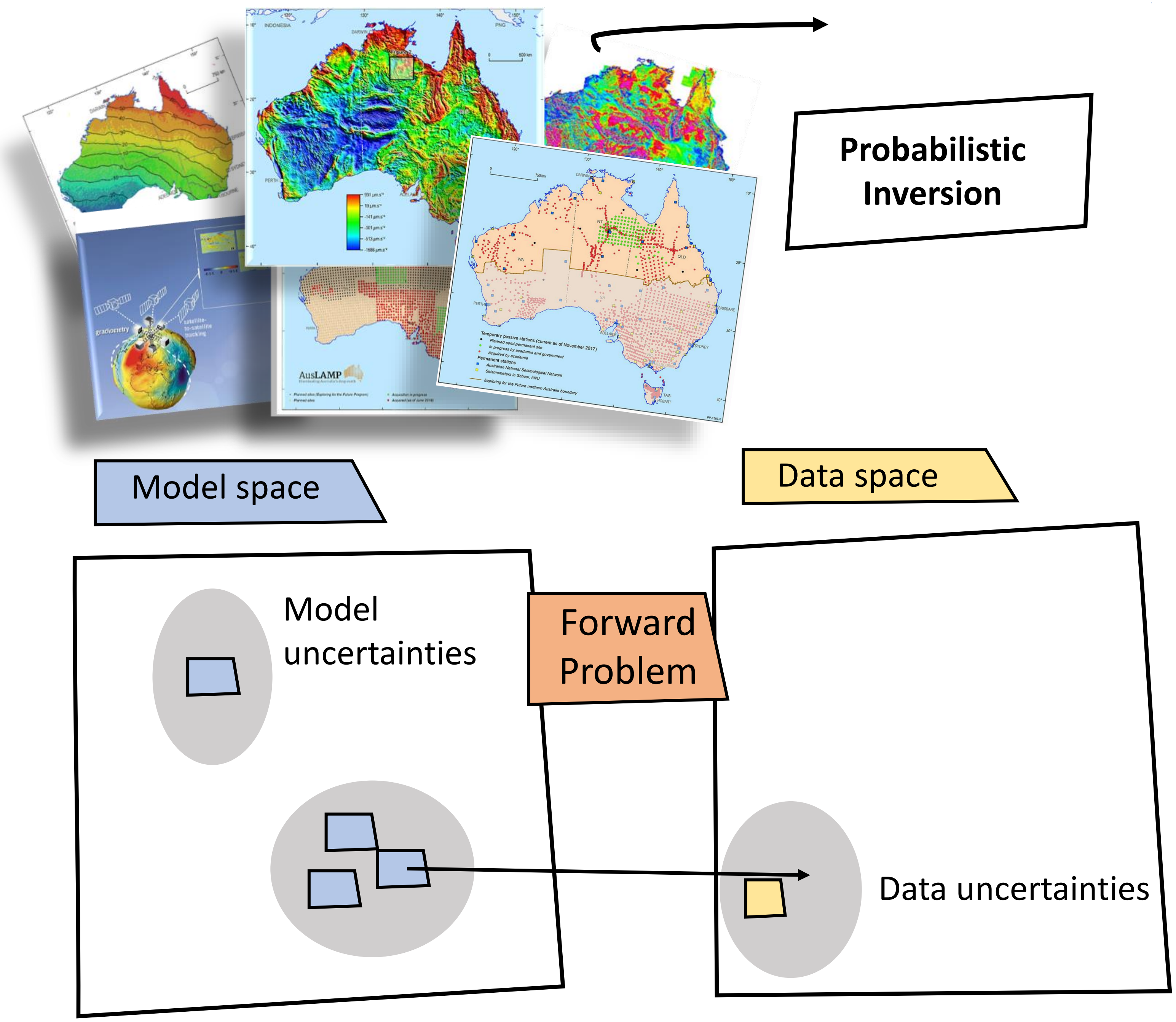
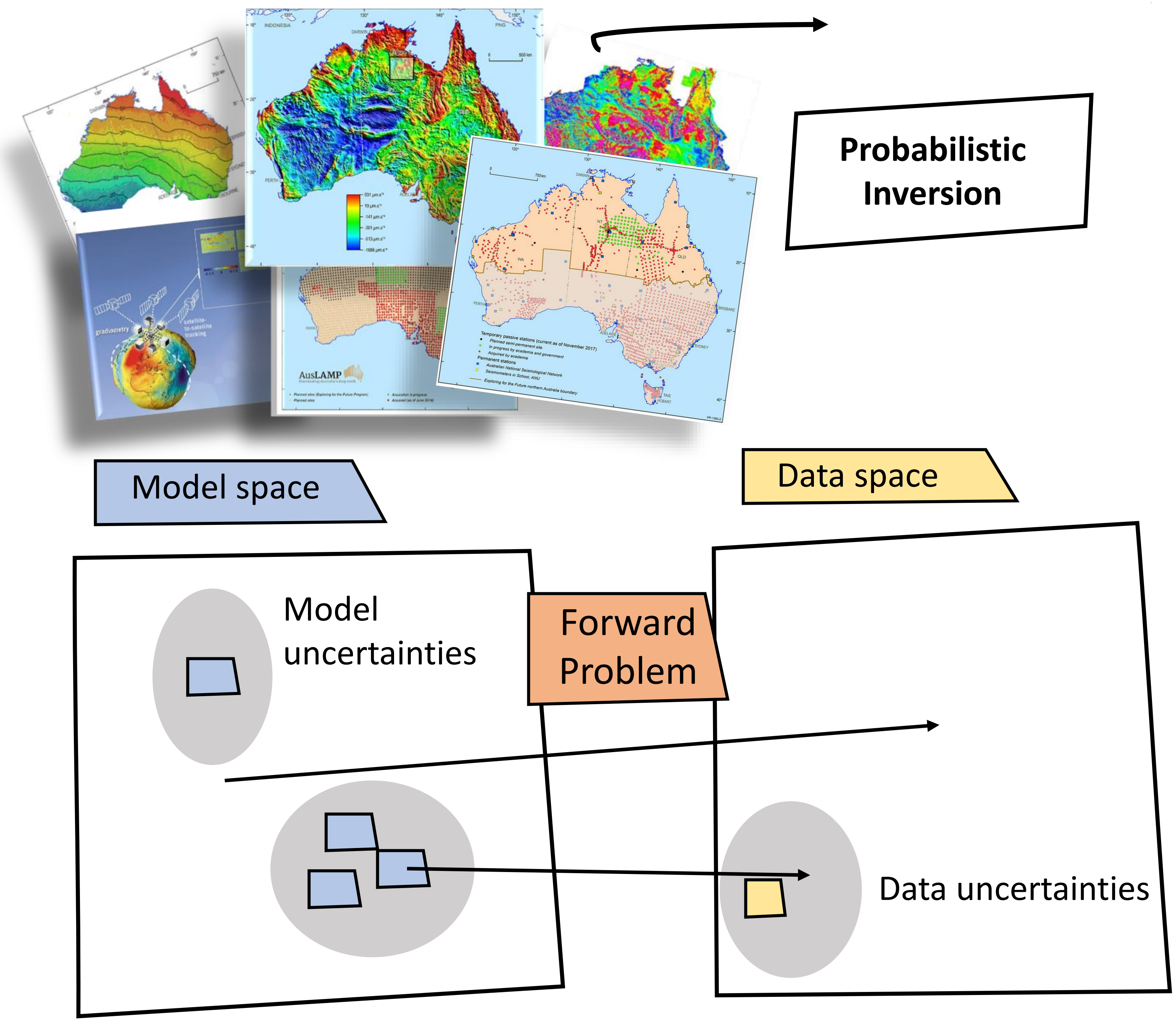

Probabilistic Inversion
Model space
Data space
Model uncertainties
Data uncertainties
AusLAMP



Probabilistic Inversion
AusLAMP
Model space
Data space
Model uncertainties
Forward Problem
Data uncertainties



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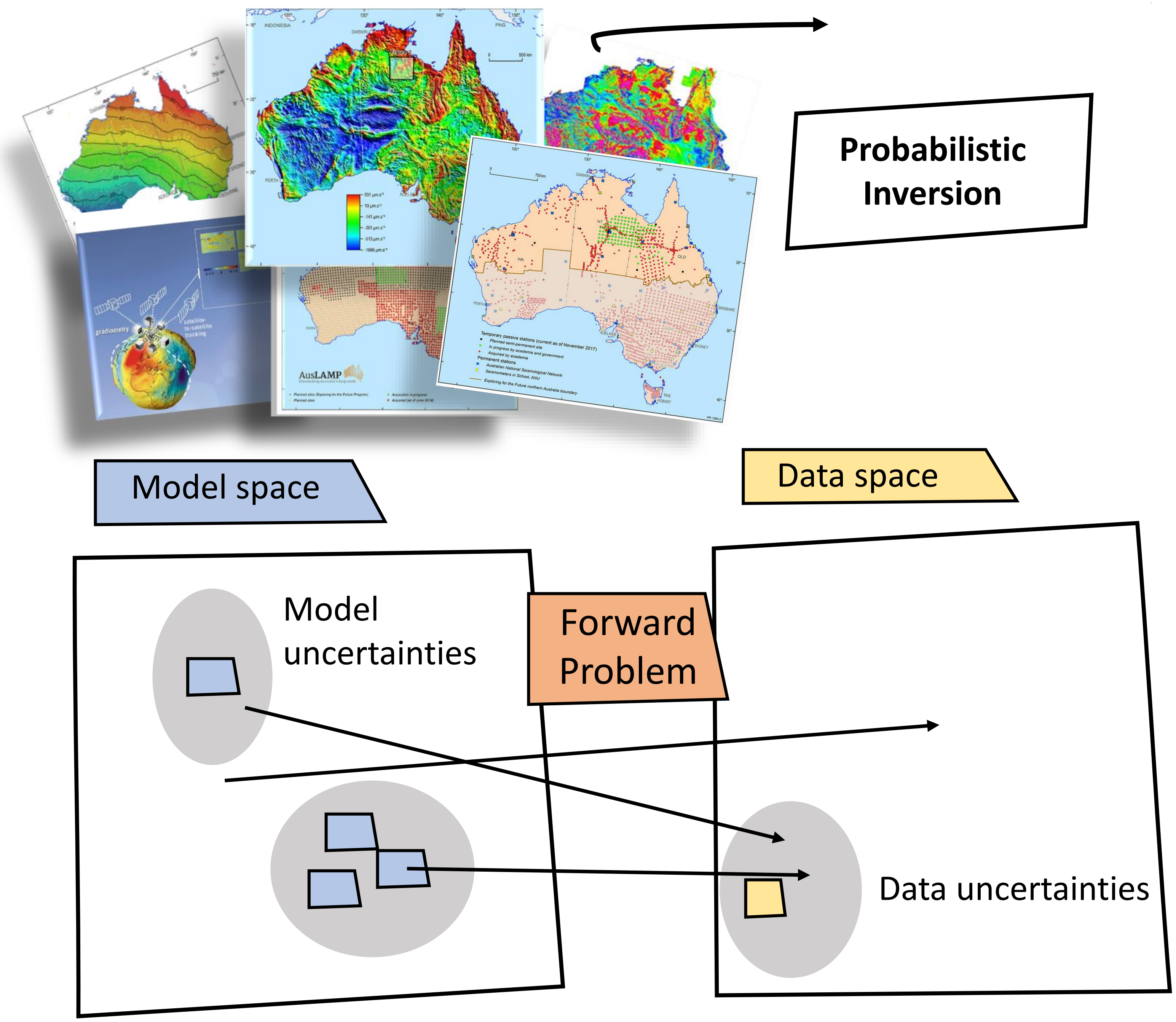

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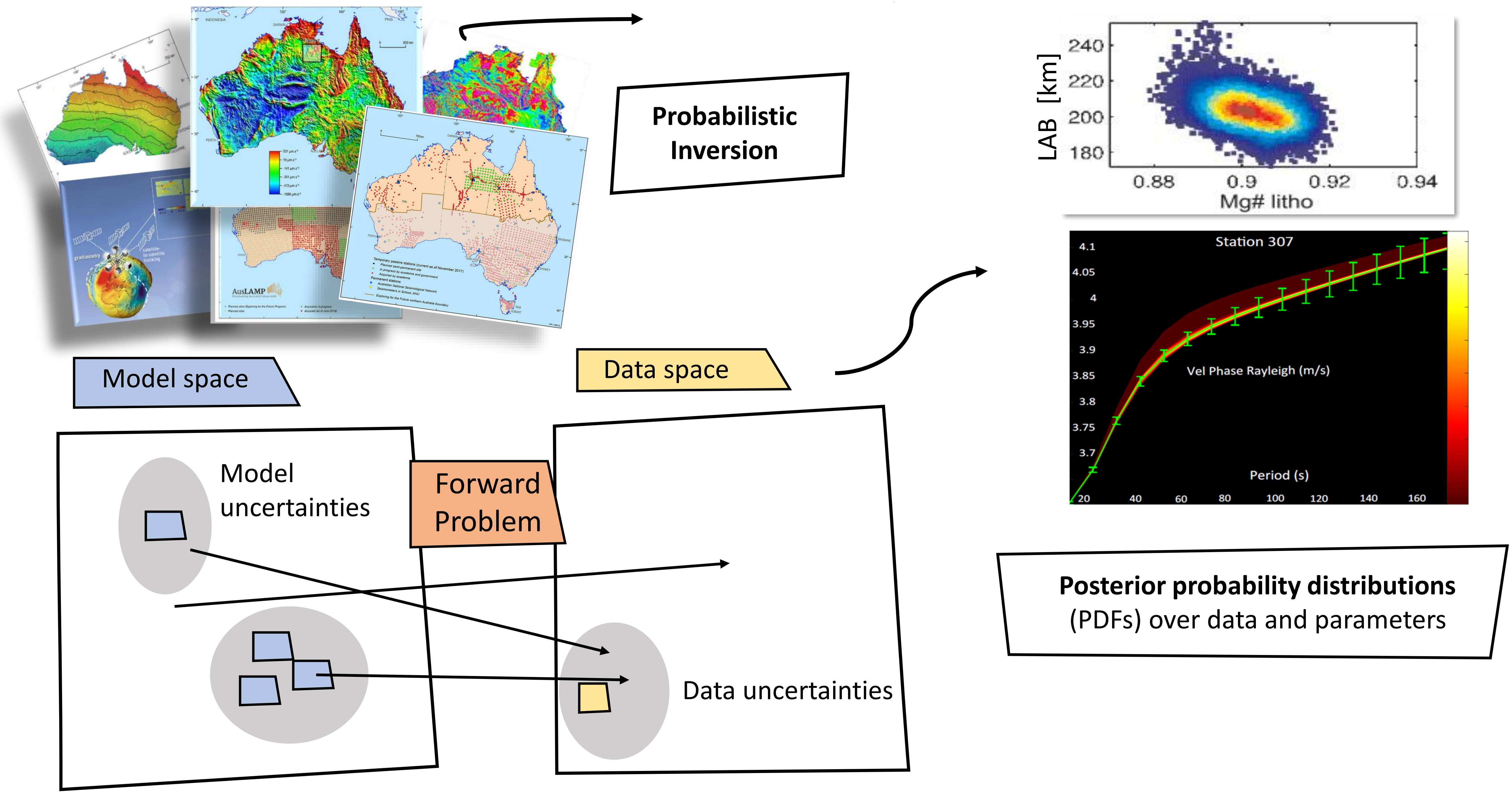
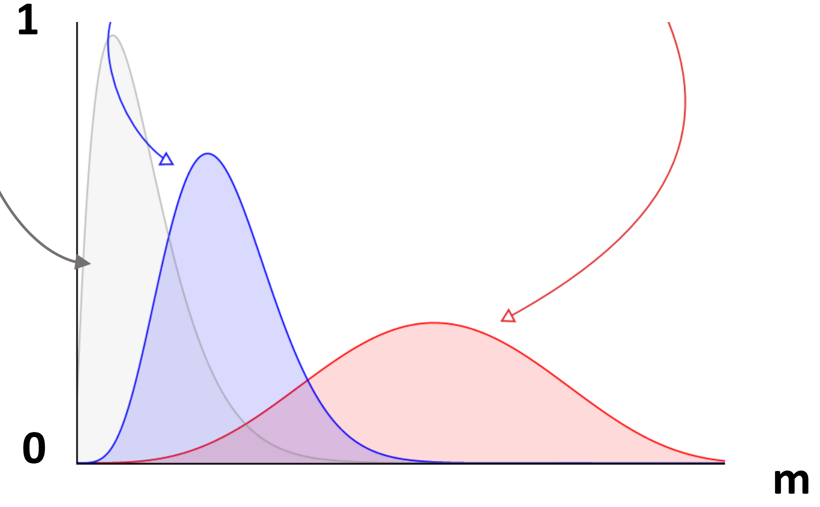


Probabilistic Inversion
AusLAMP
Model space
Data space
Model uncertainties
Forward Problem
Data uncertainties
LAB [km]
240
220
200
180
0.88
0.9
0.92
0.94
Mg# litho
Station 307
4.1
4.05
4
3.95
3.9
3.85
3.8
3.75
3.7
Vel Phase Rayleigh (m/s)
Period (s)
20
40
60
80
100
120
140
160
Posterior probability distributions
(PDFs) over data and parameters

3D MT into Multi-Observable **Probabilistic Inversion**

What is a Probabilistic Inversion?

Most general solution



3D MT into Multi-Observable **Probabilistic Inversion**

What is a Probabilistic Inversion?

Most general solution

Posterior probability density function (PDF) over **m** sought via the **Bayes' rule:**

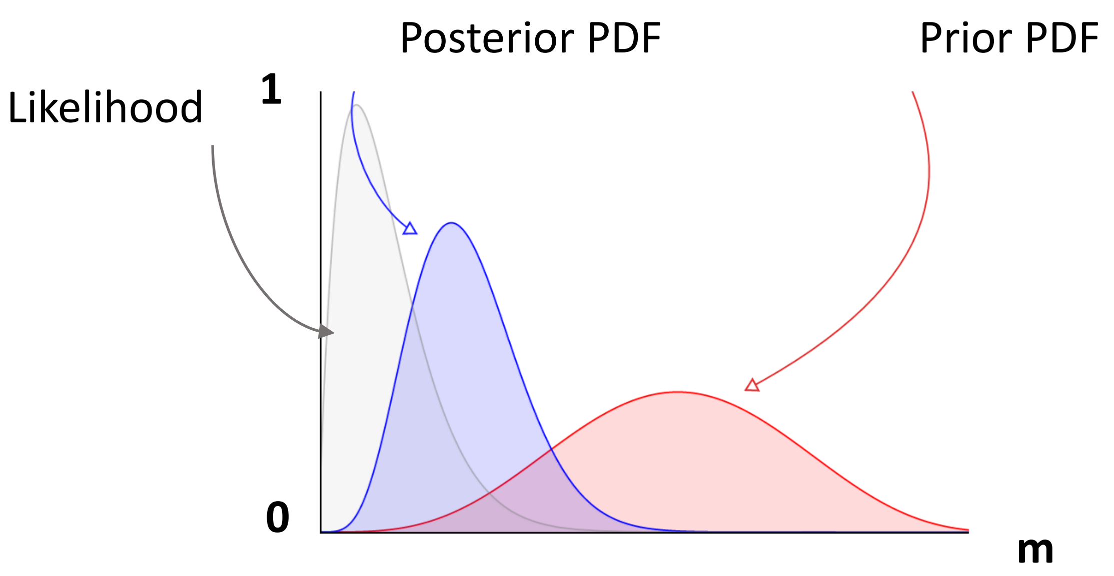
$$P(\mathrm{m}|\mathrm{d}) = \frac{\mathcal{L}(\mathrm{d}|\mathrm{m})P(\mathrm{m})}{P(\mathrm{d})}$$

$$P(\mathrm{m}|\mathrm{d}) \propto \mathcal{L}(\mathrm{d}|\mathrm{m})P(\mathrm{m}).$$

$P(\mathrm{m}|\mathrm{d})$ the posterior conditional PDF of m given the data **(d).**

$P(\mathrm{m})$ is the prior PDF over **m**

$P(\mathrm{d})$ is the prior PDF over **d**



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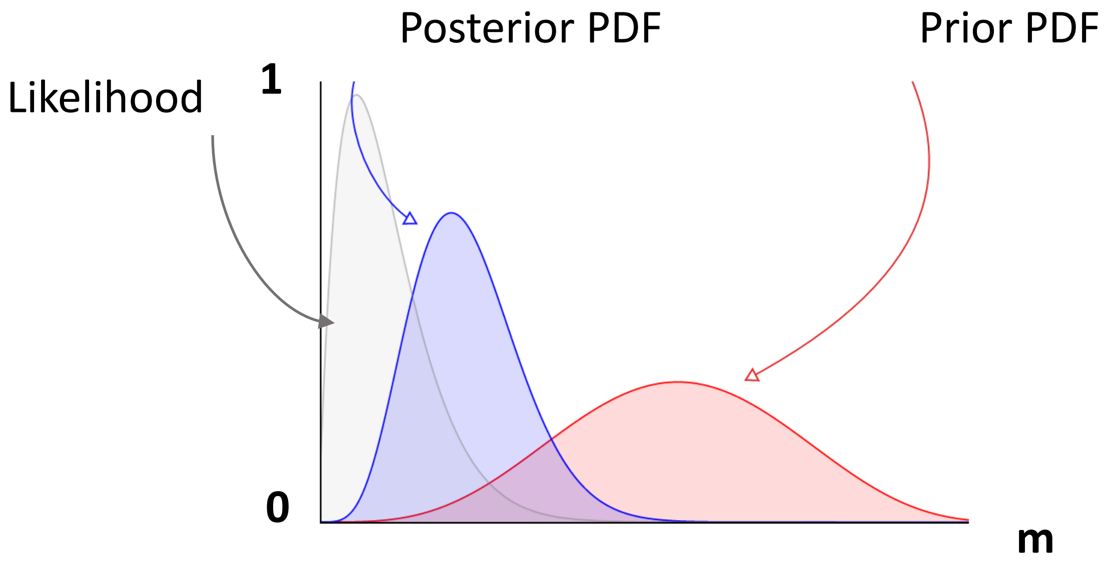

Likelihood

$\mathcal{L}(\mathrm{d}|\mathrm{m})$: conditional PDF of **d** given **m**.

Assuming data errors following a Gaussian distribution: $\mathcal{L}(\mathrm{d}|\mathrm{m}) \propto \exp\left[-S(\mathrm{m})\right],$

$S(\mathrm{m})$ is the error function or misfit: $S(\mathrm{m}) = \frac{1}{2}(\mathrm{d}-\mathrm{d}_{calc})^t\, \mathbf{C_D}^{-1}\,(\mathrm{d}-\mathrm{d}_{calc}).$

Joint inversions of uncorrelated datasets:

$$\mathcal{L}(\mathrm{m}) = \prod \mathcal{L}_j(\mathrm{m}).$$

3D MT into Multi-Observable **Probabilistic Inversion**

What is a Probabilistic Inversion?

MCMC

MCMC algorithms produce approximations of the true posterior by repeatedly drawing models $\mathbf{m}_t$ and evaluating their posterior probability

$$P(\mathbf{m}|\mathbf{d}) \propto \mathcal{L}(\mathbf{d}|\mathbf{m})P(\mathbf{m}).$$

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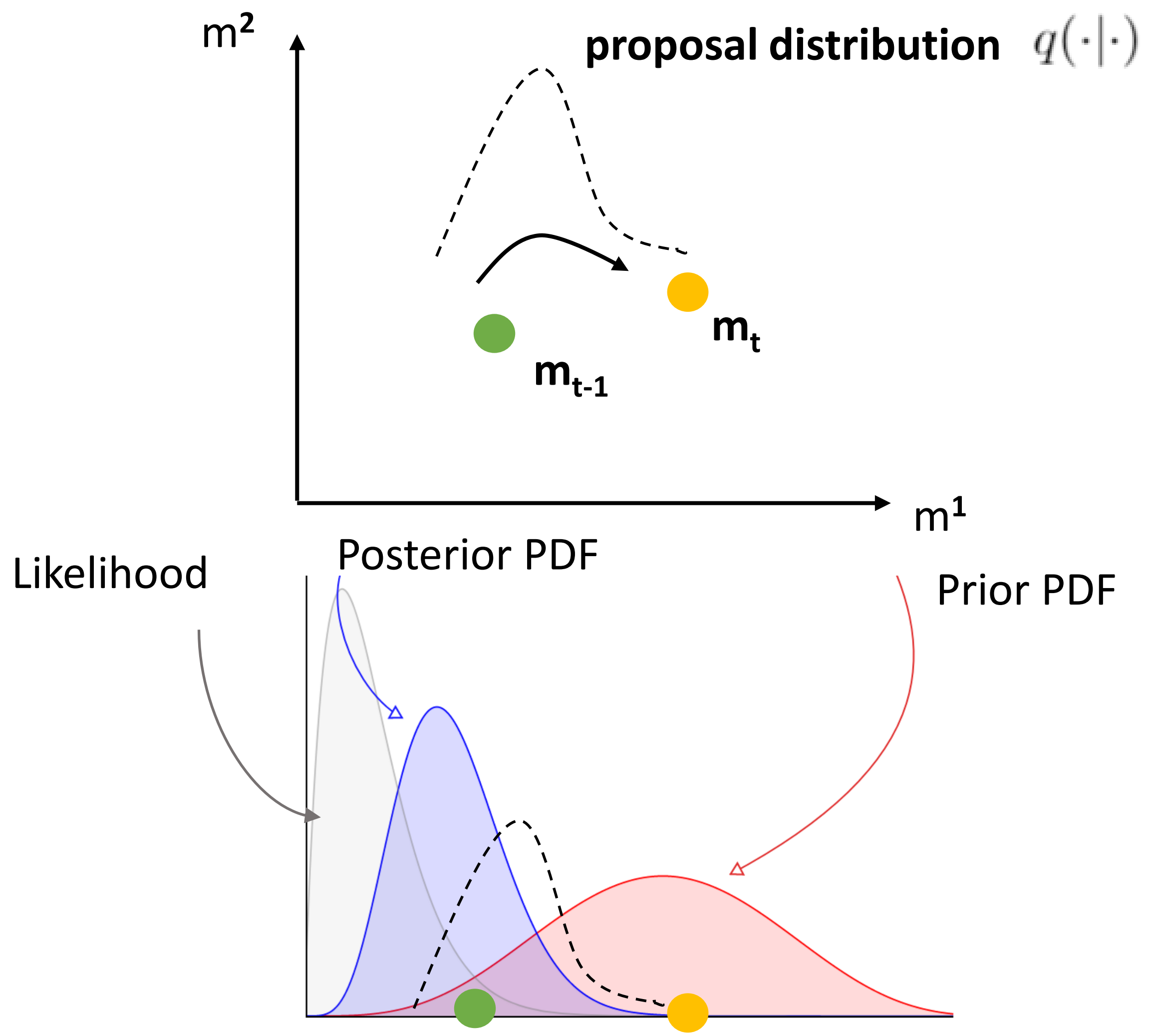
Metropolis-Hasting



m=(m¹, m²)

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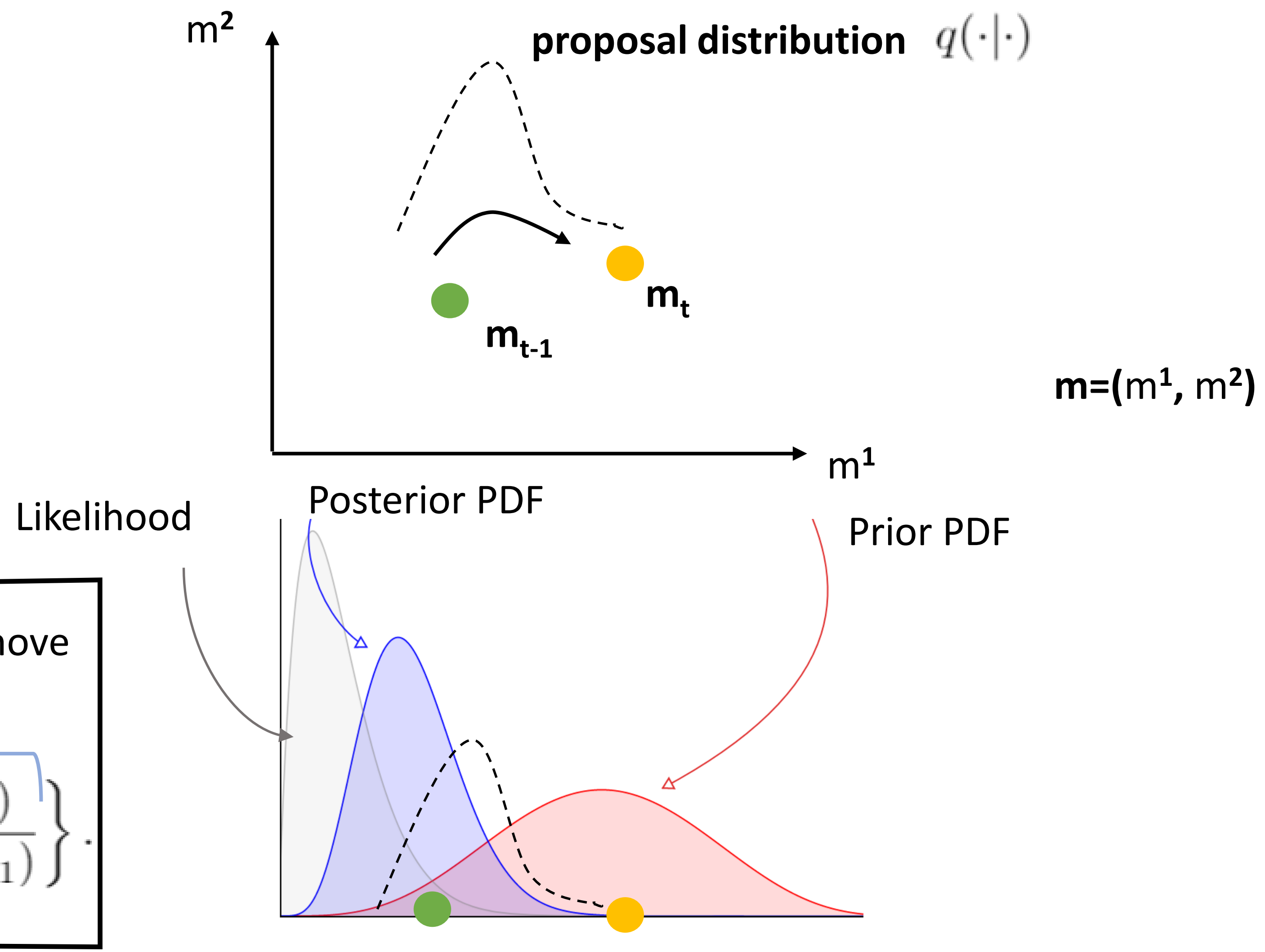

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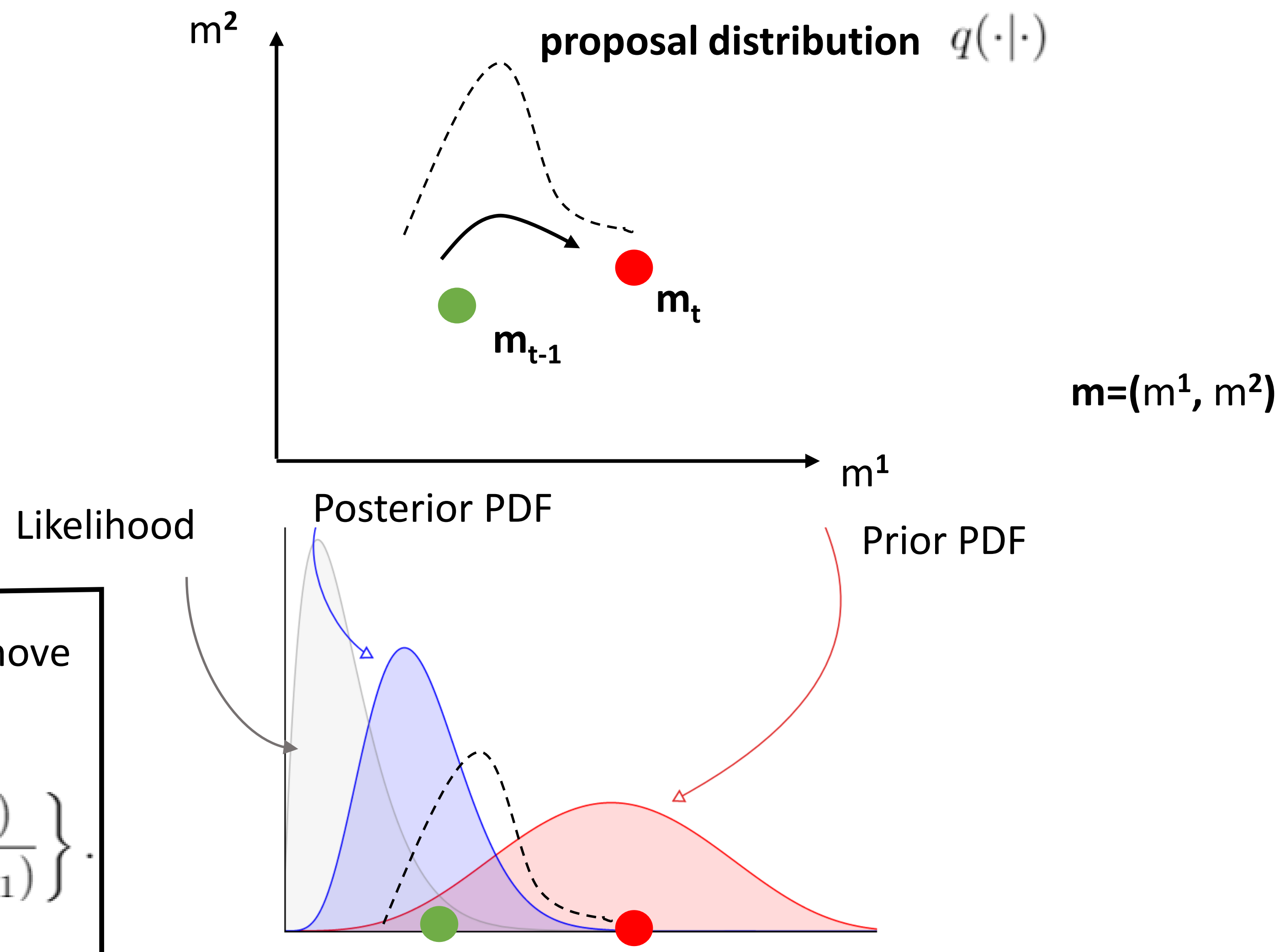

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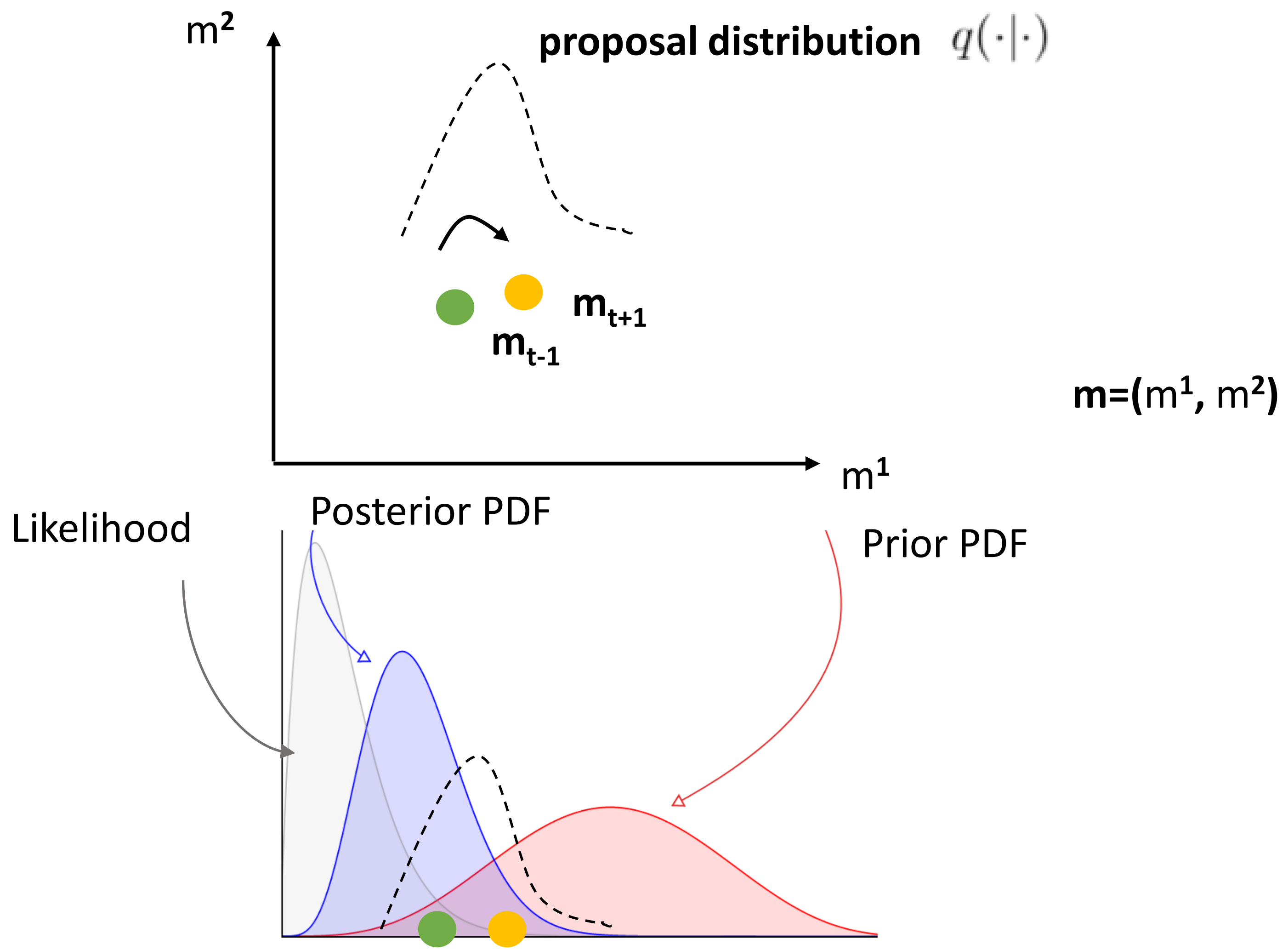

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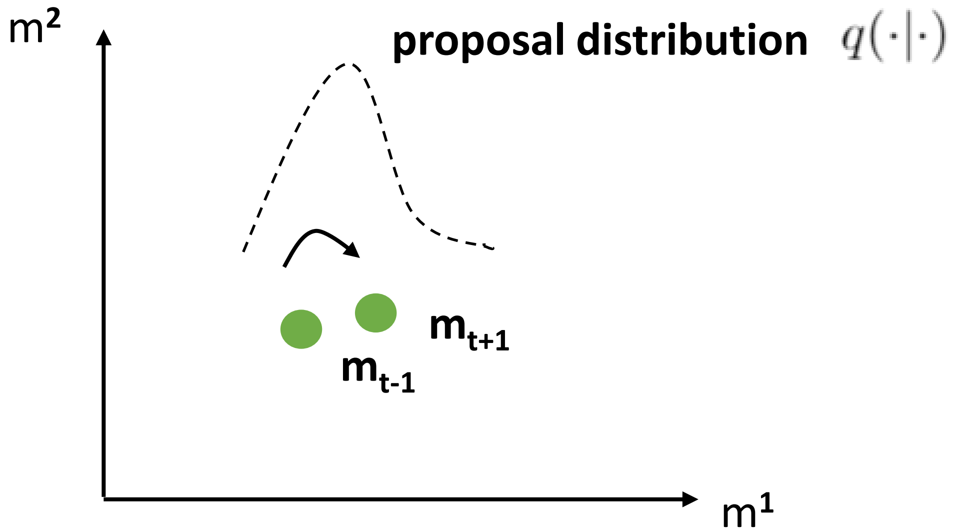
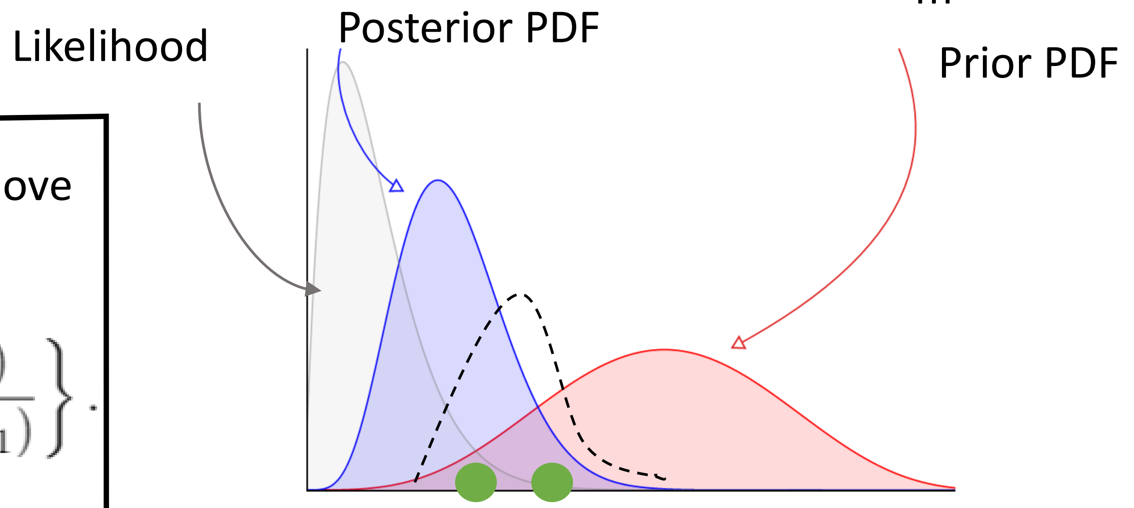


m=(m¹**,** m²**)**



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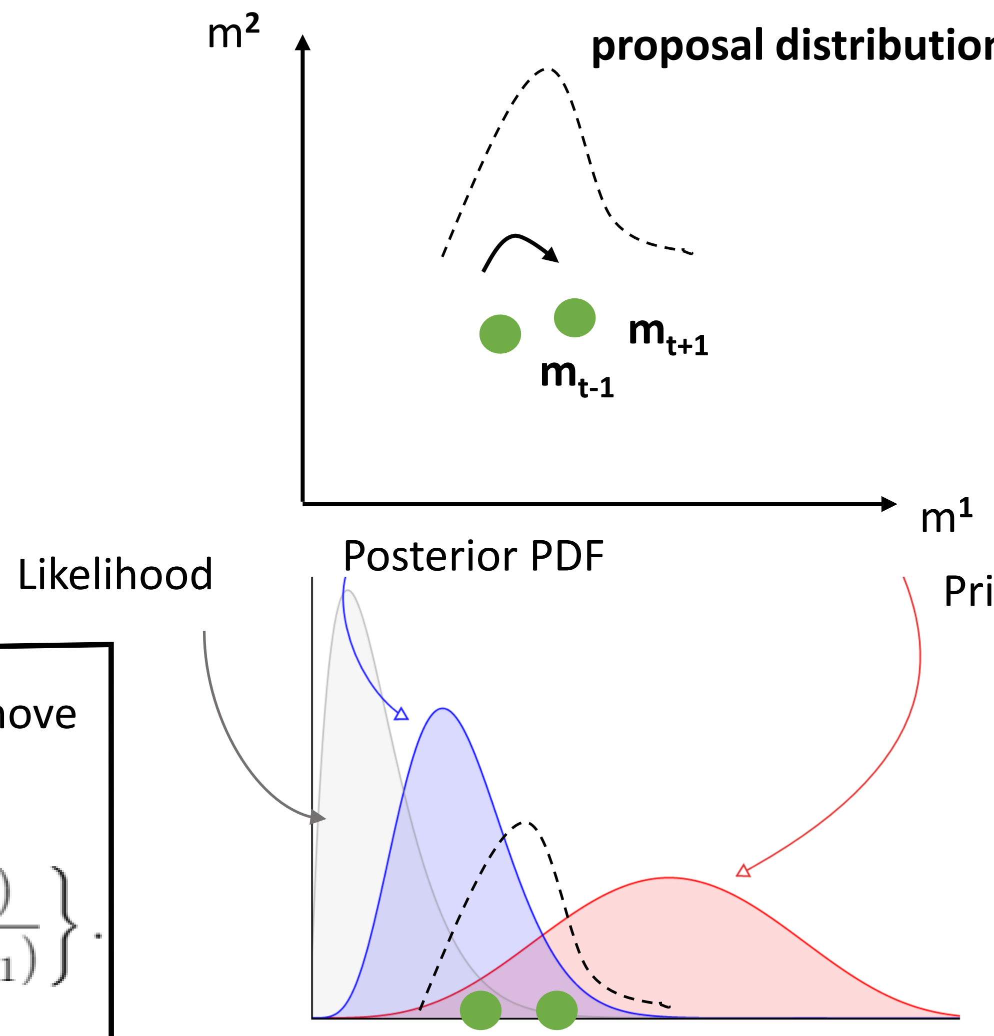
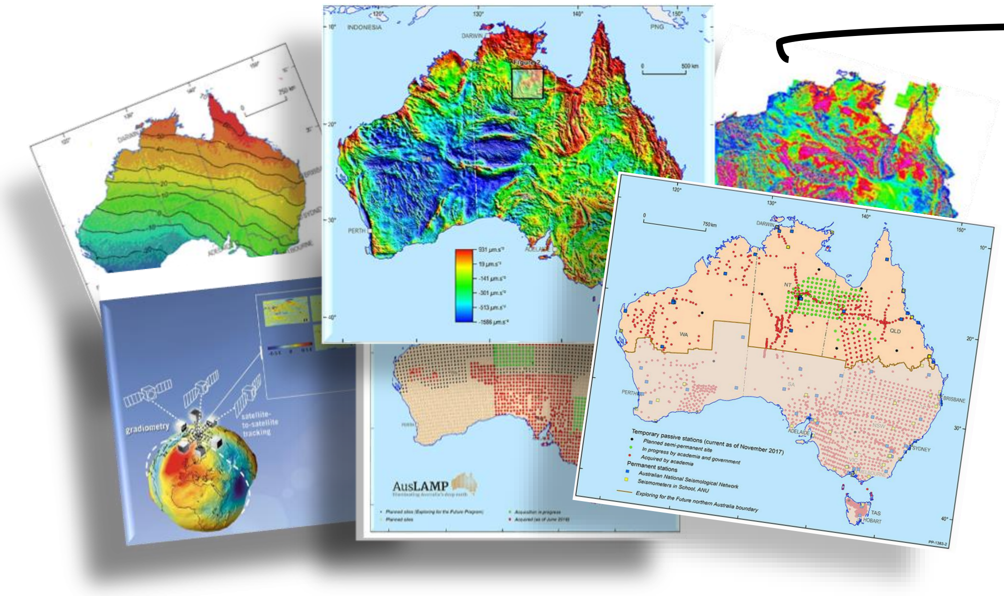


m=(m^1, m^2)

Compute forward solutions for every sample!



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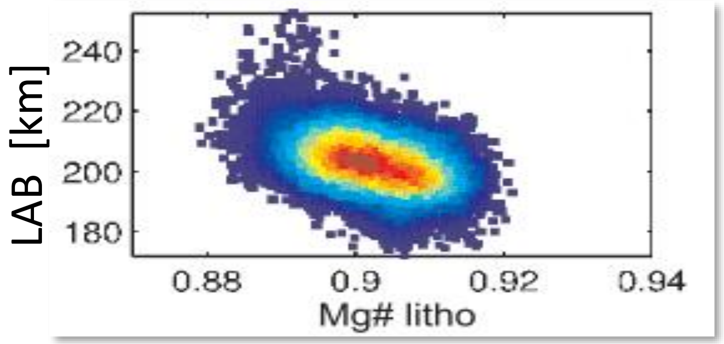
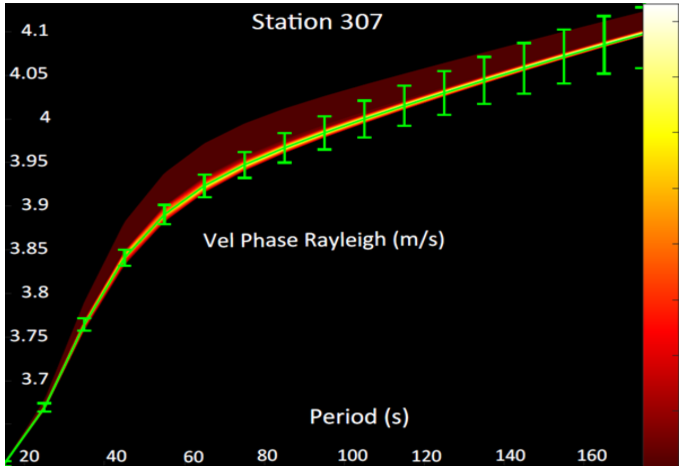




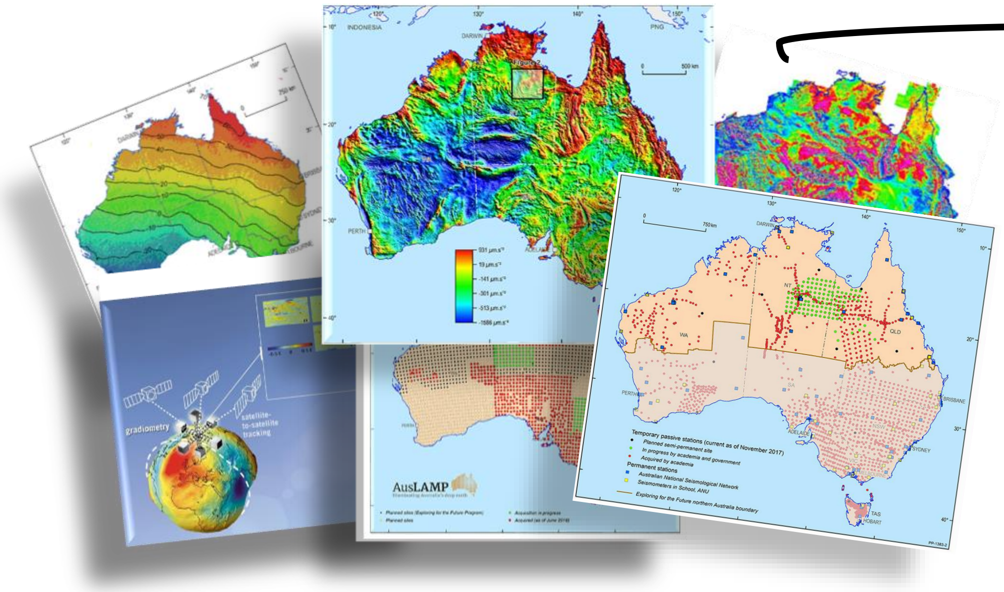
Advantages

- **Extensive information** about unknown parameters
- Inversion results are almost **independent of initial values**

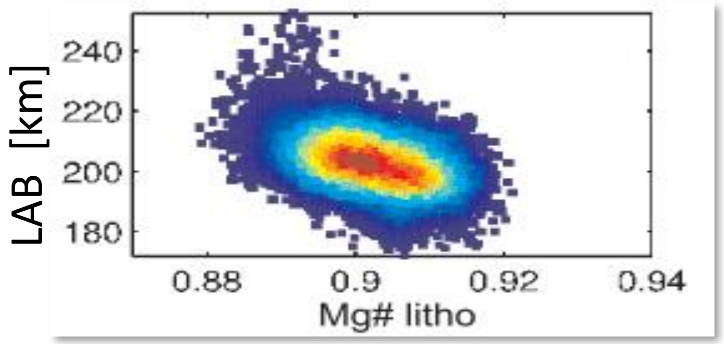
➘ Global and robust

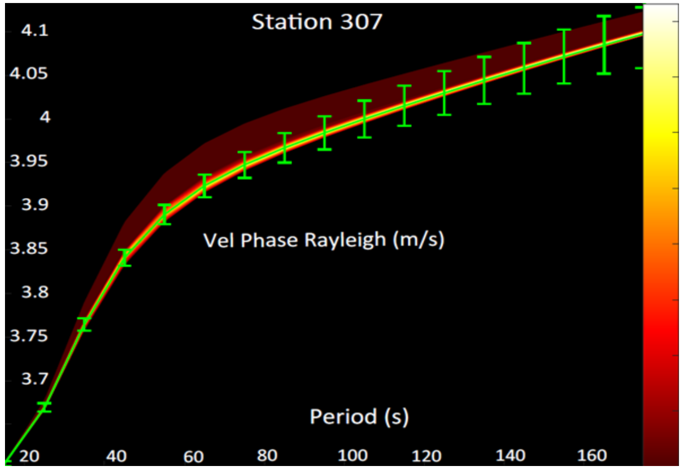
Result: a lot of models that are likely to fit the data!

Posterior probability distributions (PDFs) over data and parameters



Probabilistic Inversion





Advantages

- **Extensive information** about unknown parameters
- Inversion results are almost **independent of initial values**

➔ Global and robust

Result: a lot of models that are likely to fit the data!

Disadvantages

- Needs to run forward solutions **millions of times**

➔ Computationally **expensive**

➔ Application **limited** to problems where **fast forwards** are available

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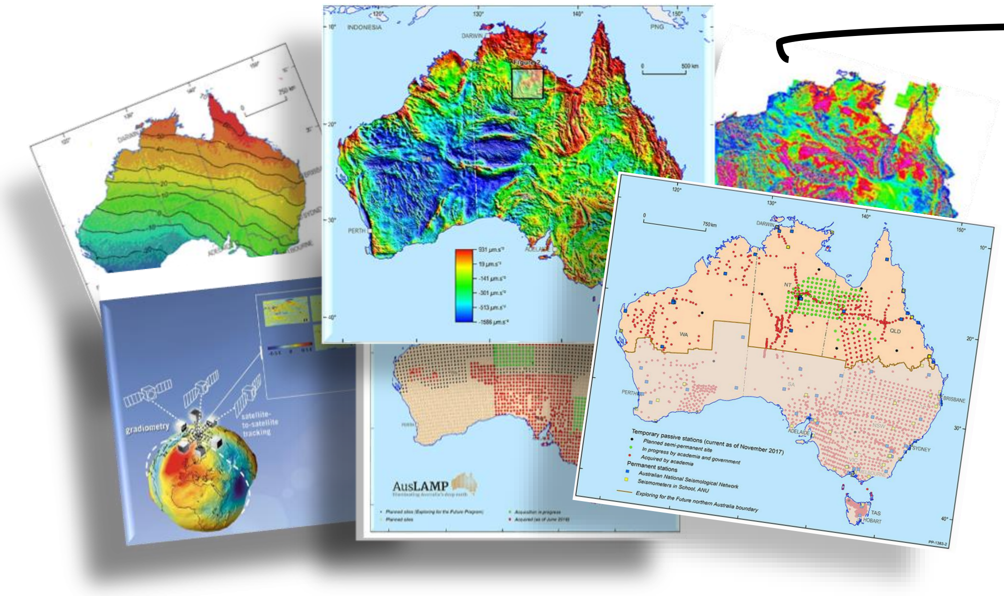
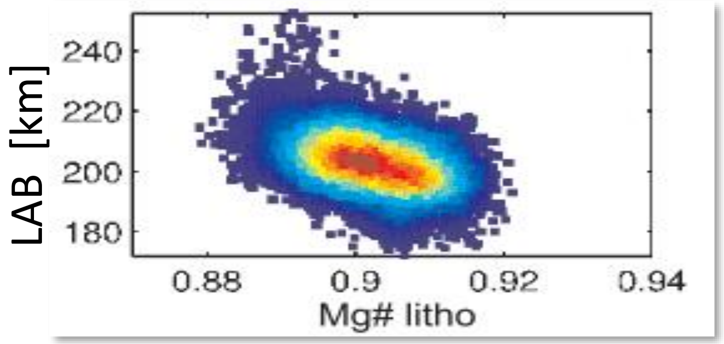
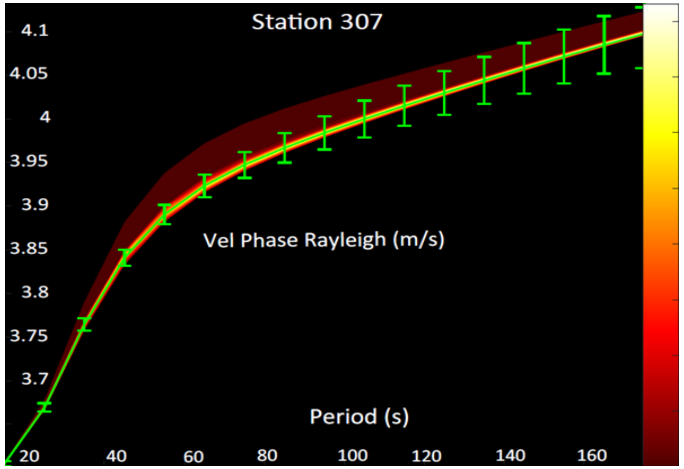


Probabilistic Inversion





Posterior probability distributions (PDFs) over data and parameters

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3D MT forwards really expensive!
Never used in a probabilistic inversion!

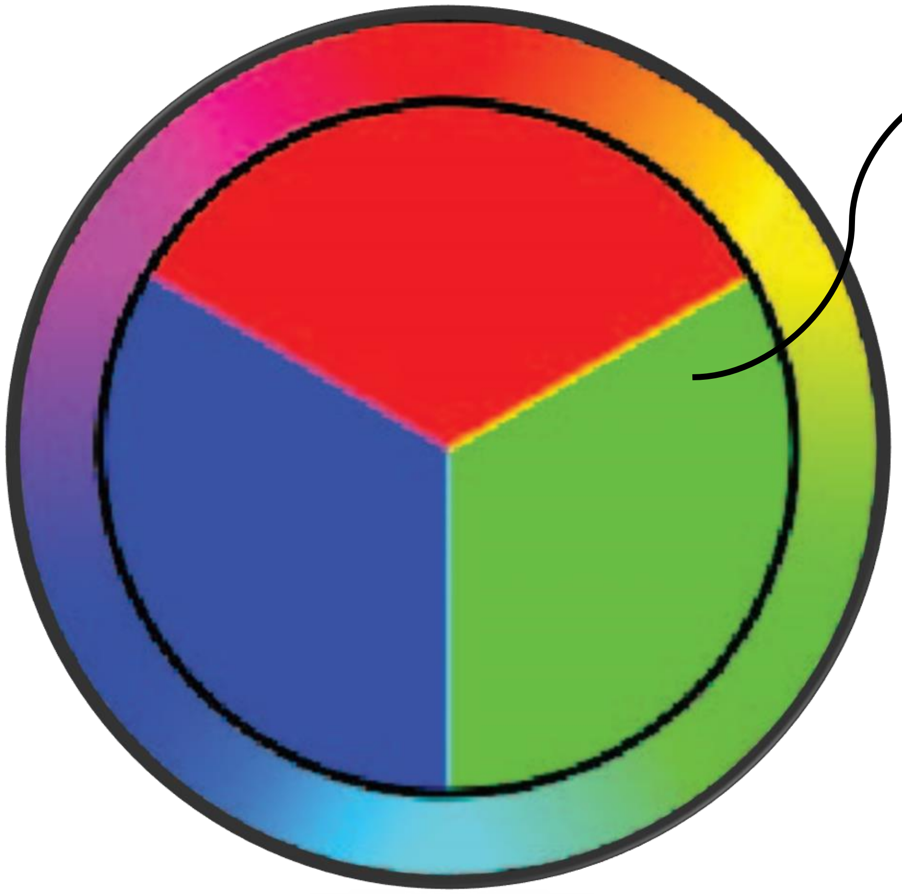
How do we compute fast forwards and include **3D MT into joint probabilistic inversions?**

How do we compute fast forwards and include **3D MT into joint probabilistic inversions?**

**Reduce Basis Methods!
How do they work?**

RB+MCMC approach for **3D MT**

Reduced basis, in a simple way



Primary or **basis** colours

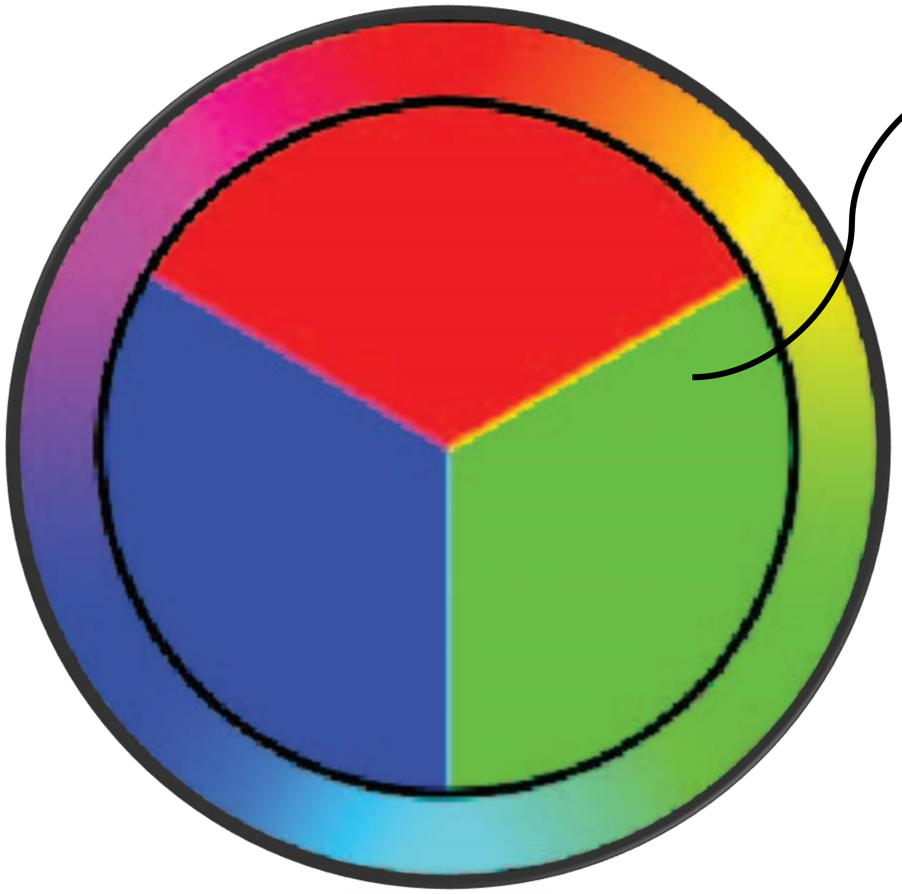
Red **Green** **Blue**

- If we want to obtain:
- Combine:

= 0.04 + 0.06 + 0.9

RB+MCMC approach for 3D MT

Reduced basis, in a simple way



Primary or **basis** colours

Red Green Blue → **Bases:** full forward solutions for certain conductivity models

- If we want to obtain: → Approximation of the full forward solution: **RB solution**…
- Combine:

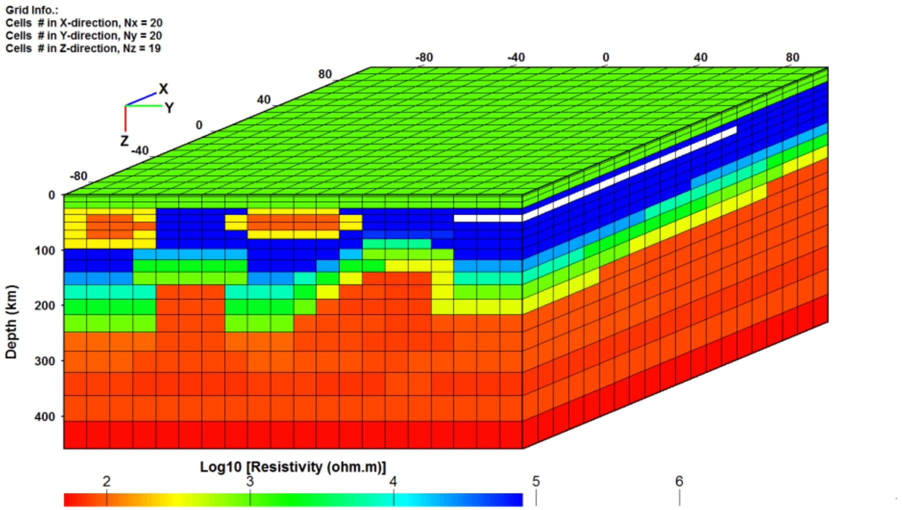
= 0.04 + 0.06 + 0.9

For fast **3D MT** forwards:

…as a combination of the **bases**

RB+MCMC approach for **3D MT**

Initial 3D conductivity model



Reduced basis scheme

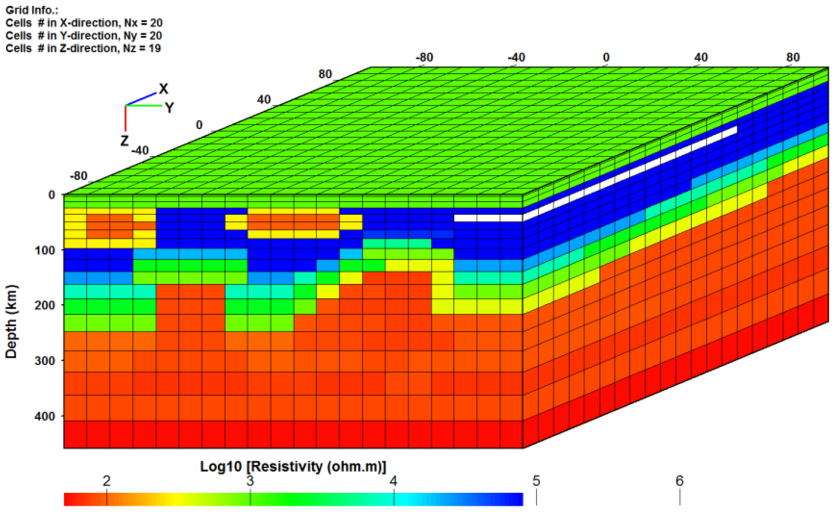
3D MT
Forward

Compute full forward

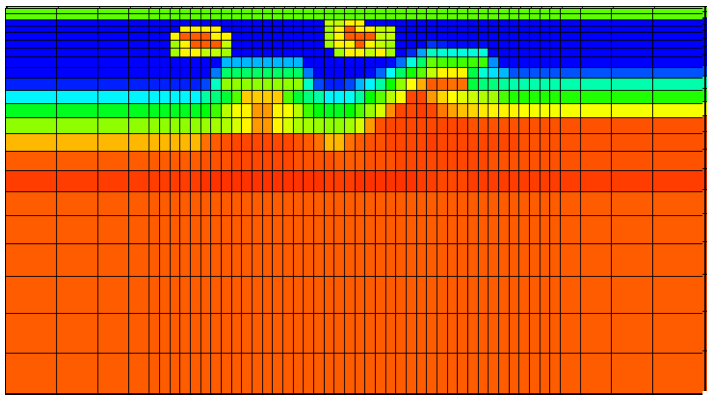
Base 1

RB+MCMC approach for **3D MT**

Initial 3D conductivity model



Random 3D conductivity fields in MT mesh



3D MT **Forward**

Reduced basis scheme

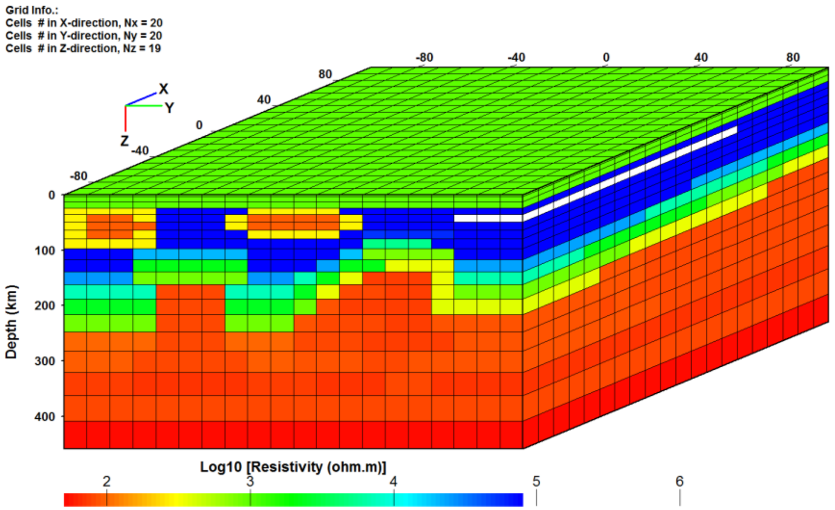
Different enough?

No

RB solution using

RB+MCMC approach for **3D MT**

Initial 3D conductivity model



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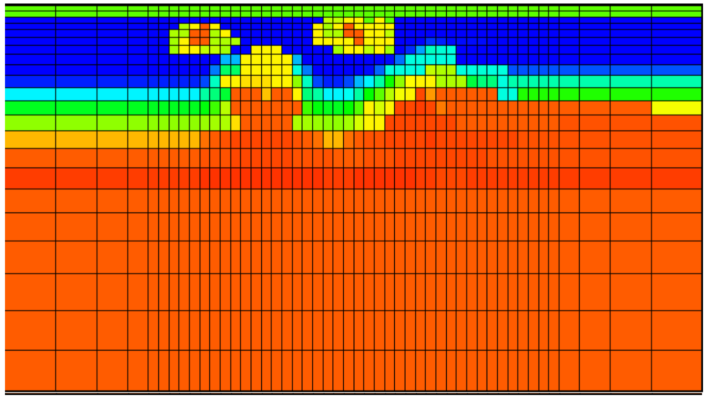

3D MT **Forward**

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Compute full forward

Base 1

Base 2

RB+MCMC approach for **3D MT**

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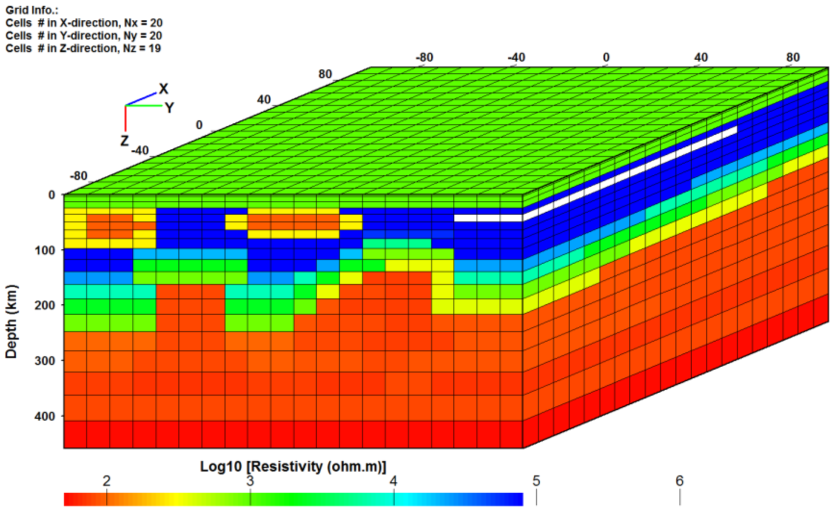

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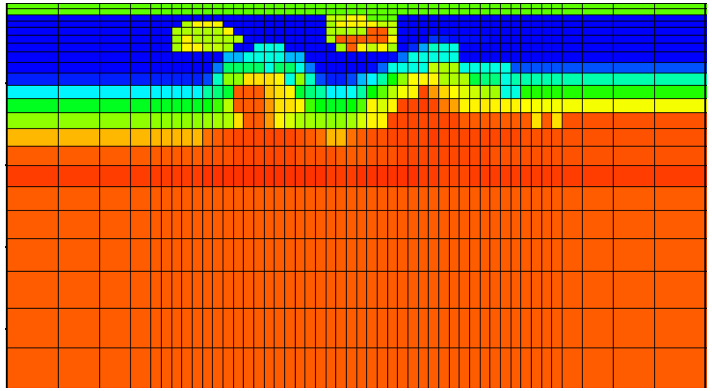

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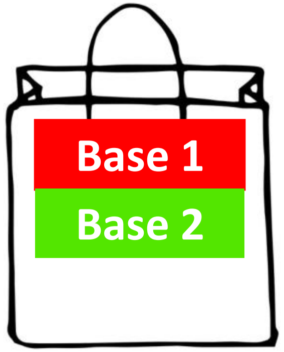
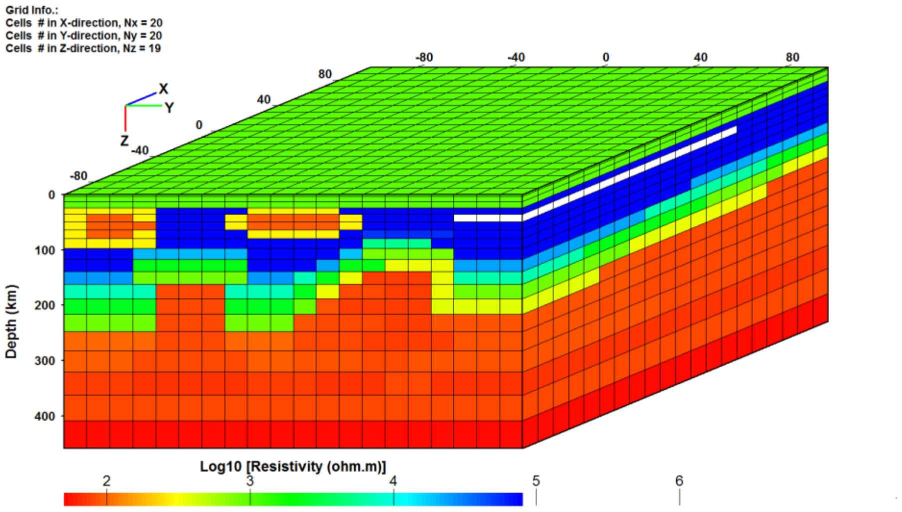


RB+MCMC approach for **3D MT**

Initial 3D conductivity model



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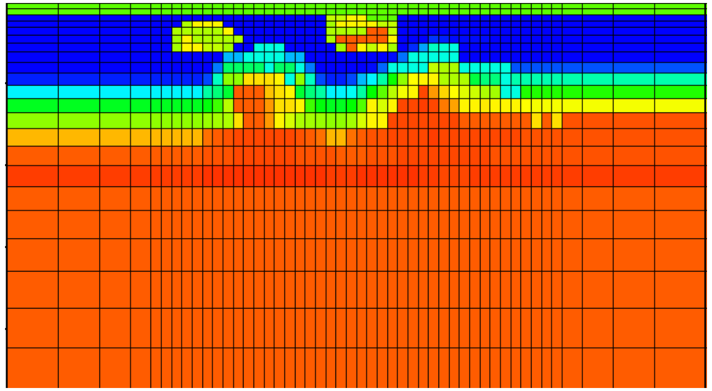

3D MT **Forward**

Reduced basis scheme

Different enough?

Yes → **Compute full forward**

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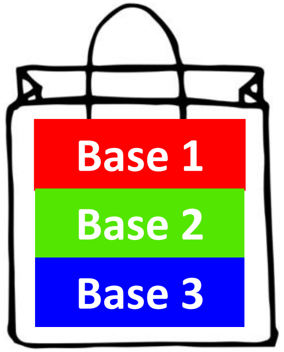


Elevation

Gravity anomalies

SHF

3D Multi-Observable **Probabilistic Inversion**

Geoid

R & L

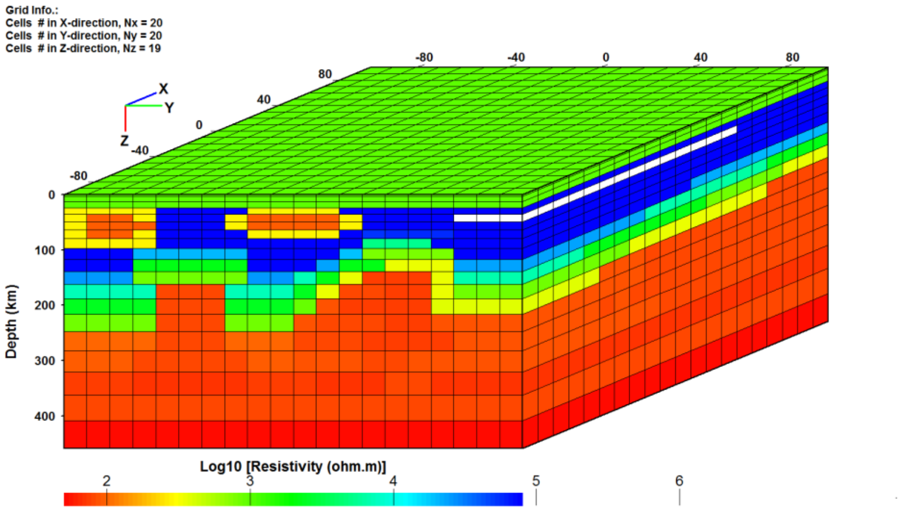
Receiver Functions

Seismic

Xenoliths

RB+MCMC approach for **3D MT**

Initial 3D conductivity model



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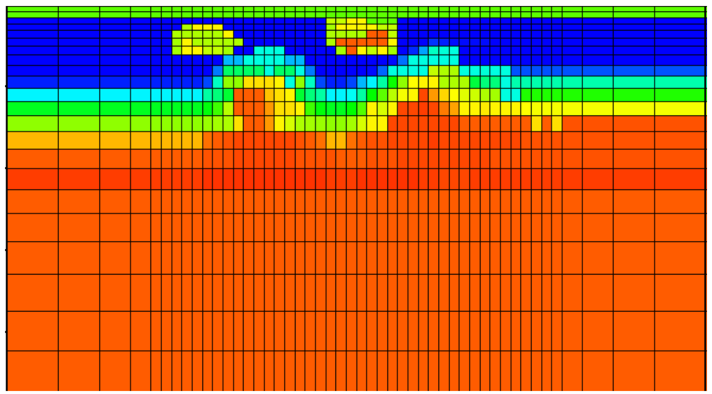

3D MT **Forward**

Reduced basis scheme

Different enough?

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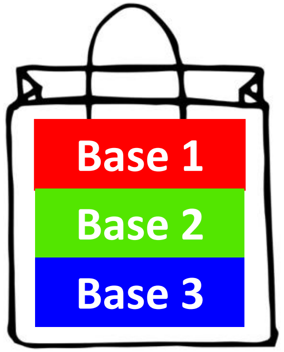
No → **RB solution using**



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3D Multi-Observable **Probabilistic Inversion**

3D MT

R & L

Receiver Functions

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NO need of expensive forward ✖

Reduced Basis Solution ✔

3D MT into Multi-Observable **Probabilistic Inversion**

3D MT forward problem

$$\sigma(\mathbf{x},\theta)\mathbf{E} - \nabla \times \mathbf{H} = -F \qquad (1)$$

$$i\omega\mu_0\mathbf{H} + \nabla \times \mathbf{E} = 0 \qquad (2)$$

$$(1-i)P_\tau a\mathbf{E} + \nu \times \mathbf{H} = 0 \quad \text{on } \partial\Omega \equiv \Gamma, \qquad (3)$$

Using the secondary field formulation of Douglas et al. (1999, 2000) and the absorbent boundary conditions defined by Sheen (1997), the MT forward problem in 3D is

3D MT into Multi-Observable **Probabilistic Inversion**

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- **Discretised** problem to solve:

$$\mathbb{K}(\theta)\mathbf{U}(\theta) = \mathbf{F}(\theta) \quad (4)$$

where $\mathbb{K}$ is the stiffness matrix and $\mathbf{F}$ is the nodal vector forces.

size of $\mathbf{K}$: $N_{FE} \times N_{FE}$

1000000 x 1000000

Full forward (high-fidelity) solutions are sought via an optimized version of the finite element (FE) code developed by Zyserman & Santos (2000). We use the parallel solver MUMPS

U is a vector containing the unknown coefficients for the electric field in the whole domain

3D MT into Multi-Observable **Probabilistic Inversion**

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Reduced Basis strategy

- Generate a space of N_{RB} linearly independent solutions or ***snapshots*** of (4)

$$\mathcal{U}_{\mathcal{RB}} = span\{\mathbf{u}_{h(1)}, \mathbf{u}_{h(2)}, ..., \mathbf{u}_{h(RB)}\} \subset \mathcal{U}_h$$

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$$\mathbf{U_{RB}} = \sum_{i=1}^{N_{RB}} a_i \mathbf{U}_i = \mathbb{U}_{\mathbb{RB}}\mathbf{a}$$

$$\mathbb{U}_{\mathbb{RB}} = [\mathbf{U}_1, \mathbf{U}_2, ..., \mathbf{U}_{N_{RB}}]^{N_{FE}\times N_{RB}}$$

$$\mathbf{a}^T = [a_1, a_2, ...a_{N_{RB}}]$$

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$N_{RB} \ll N_{FE}$

100 x 100

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$$(\mathbb{U}_{\mathbb{RB}}{}^T\mathbb{K}(\theta)\mathbb{U}_{\mathbb{RB}})\mathbf{a} = \mathbb{U}_{\mathbb{RB}}{}^T\mathbf{F}(\theta)$$

$$\boxed{\mathbb{K}_{\mathbb{RB}}(\theta)\mathbf{a} = \mathbf{F_{RB}}(\theta)}$$

3D MT into Multi-Observable Probabilistic Inversion

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$$\mathbb{K}_{\mathbb{RB}}(\theta)\mathbf{a} = \mathbf{F_{RB}}(\theta)$$

How good is the RB solution?

$$\mathbf{E_{RB}} := \mathbf{U_{RB}} - \mathbf{U}$$

$$\mathbb{K}\mathbf{E_{RB}} := \mathbb{K}\mathbf{U_{RB}} - \mathbb{K}\mathbf{U}$$

$$\mathbf{R_{RB}} := \frac{\mathbb{K}\mathbf{E_{RB}}}{\mathbf{F}} = \frac{|\mathbb{K}\mathbf{U_{RB}} - \mathbf{F}|}{|\mathbf{F}|}$$

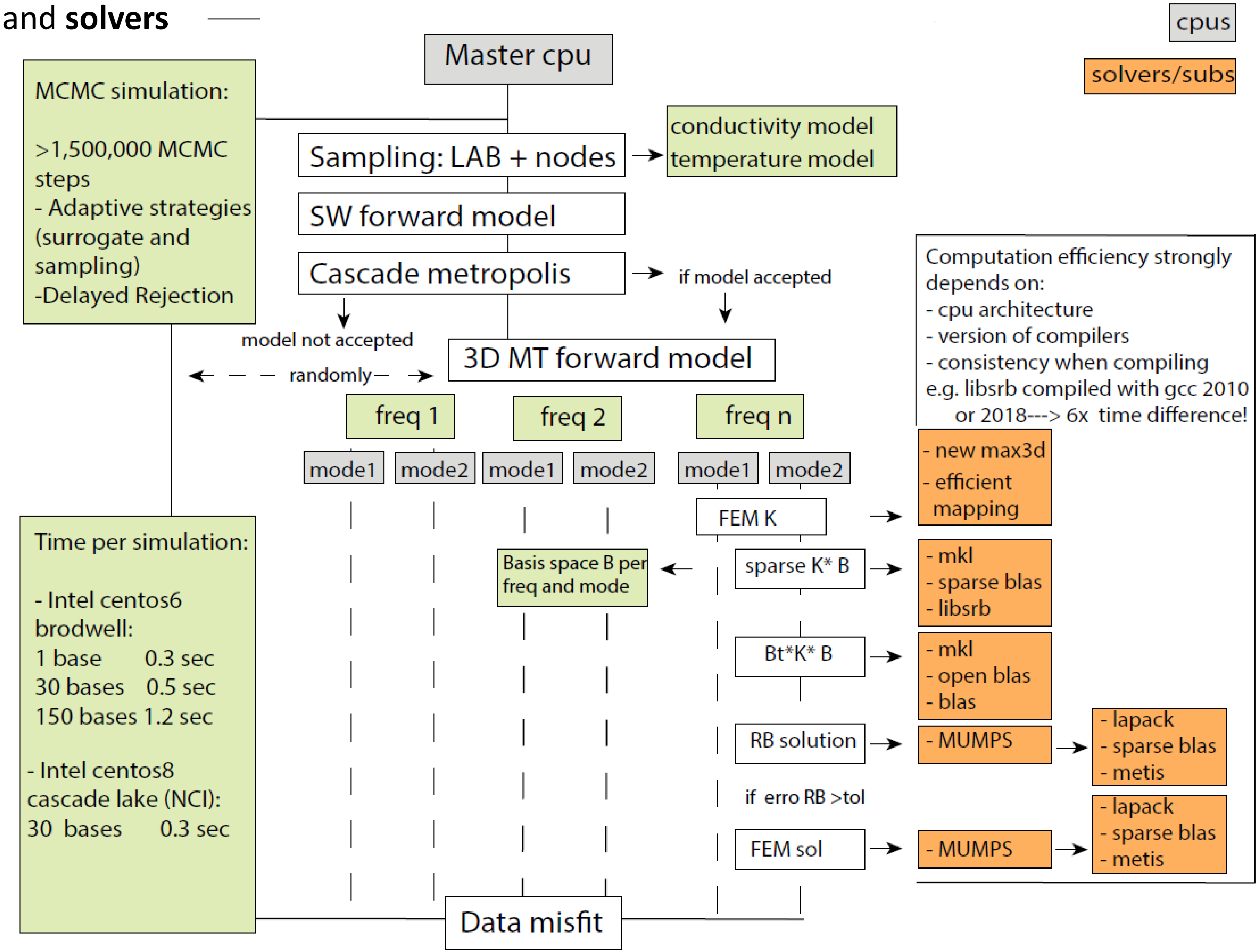
$$\mathbf{R_{RB}} \ll \text{tol}$$

We have included:

- Variable tolerance
- Orthonormalization

How do we implement the **RB approach** for the joint **MT+SW** inversion?

Parallel implementation and **solvers**



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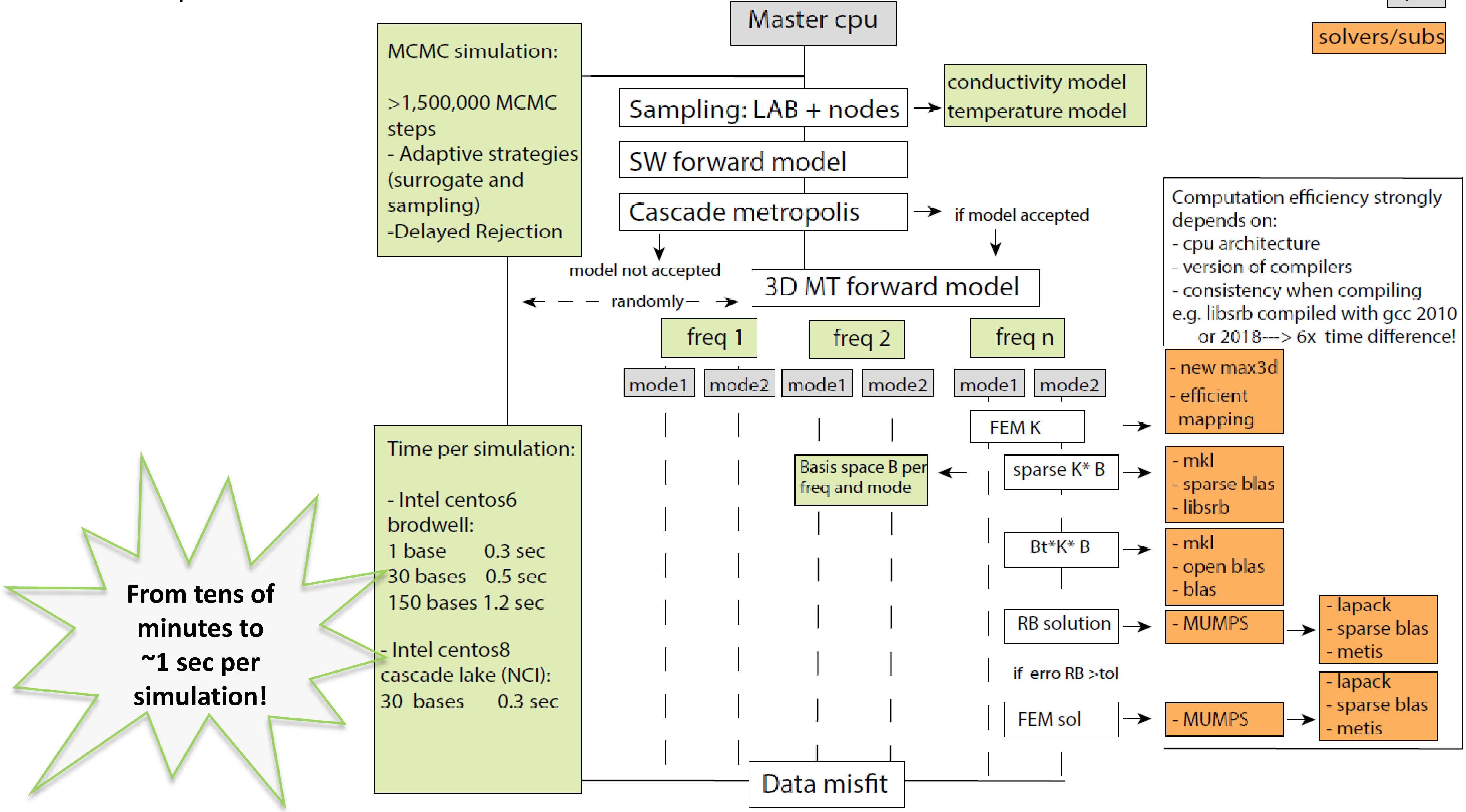
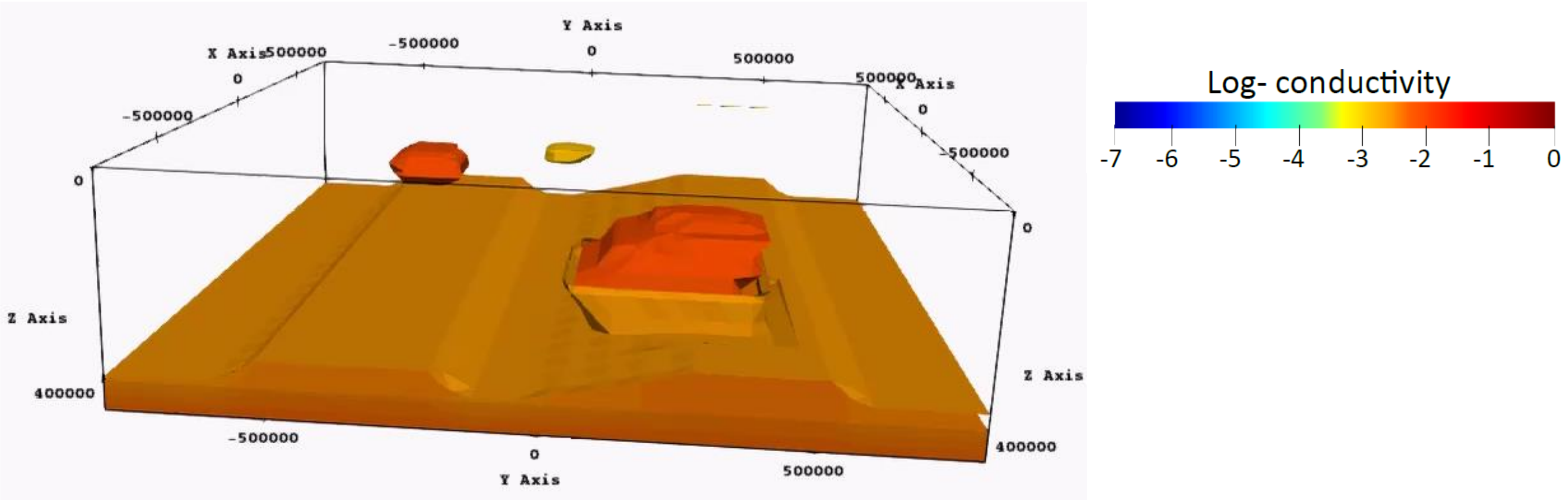


How do we **parameterise** our models for the joint **MT+SW** inversion?

Parameterisation : background + conductivity anomalies

Full conductivity model



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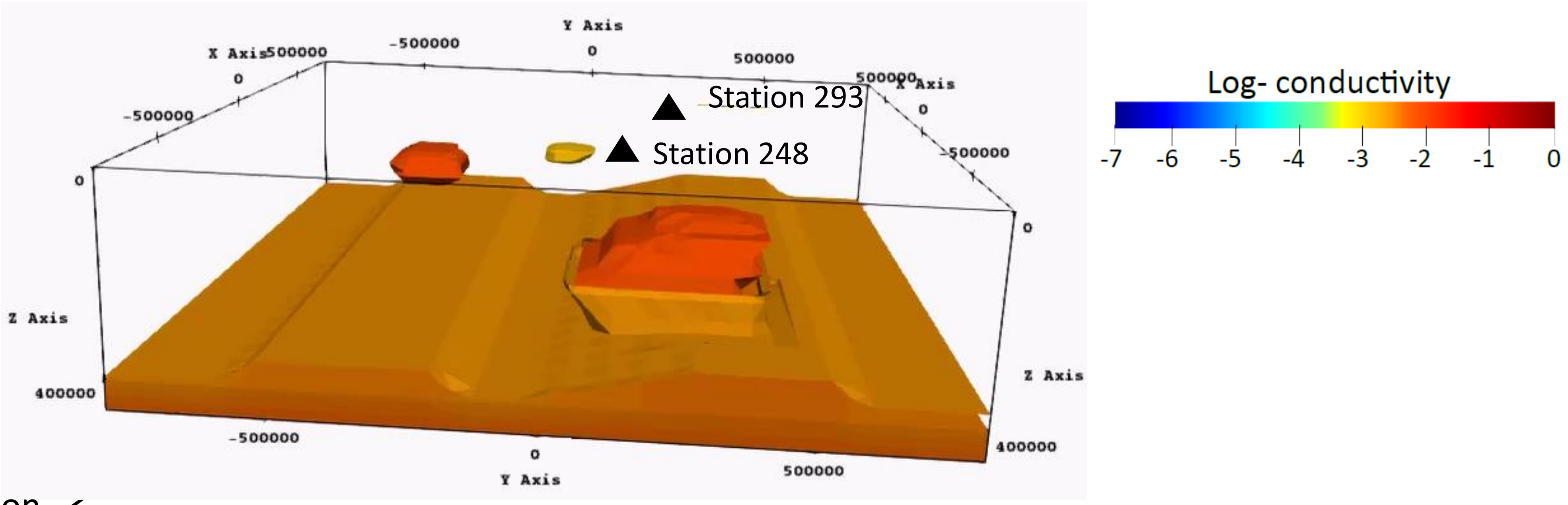
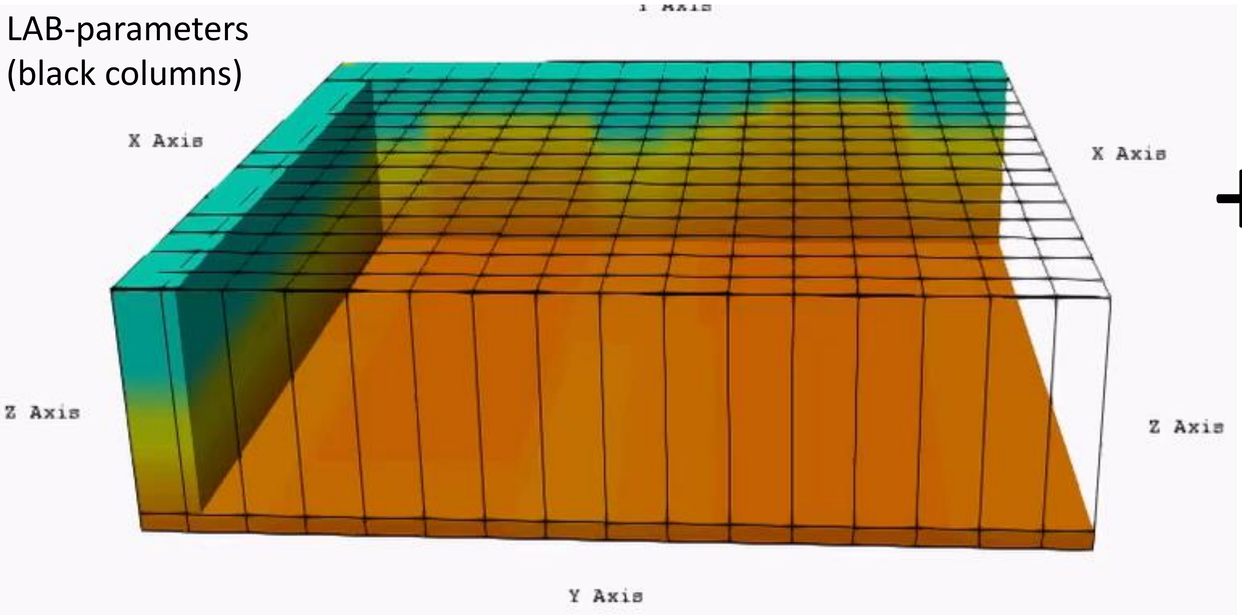


Large scale thermochemical background: temperature, pressure and bulk composition

LAB-parameters (black columns)



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Parameterisation : background + conductivity anomalies

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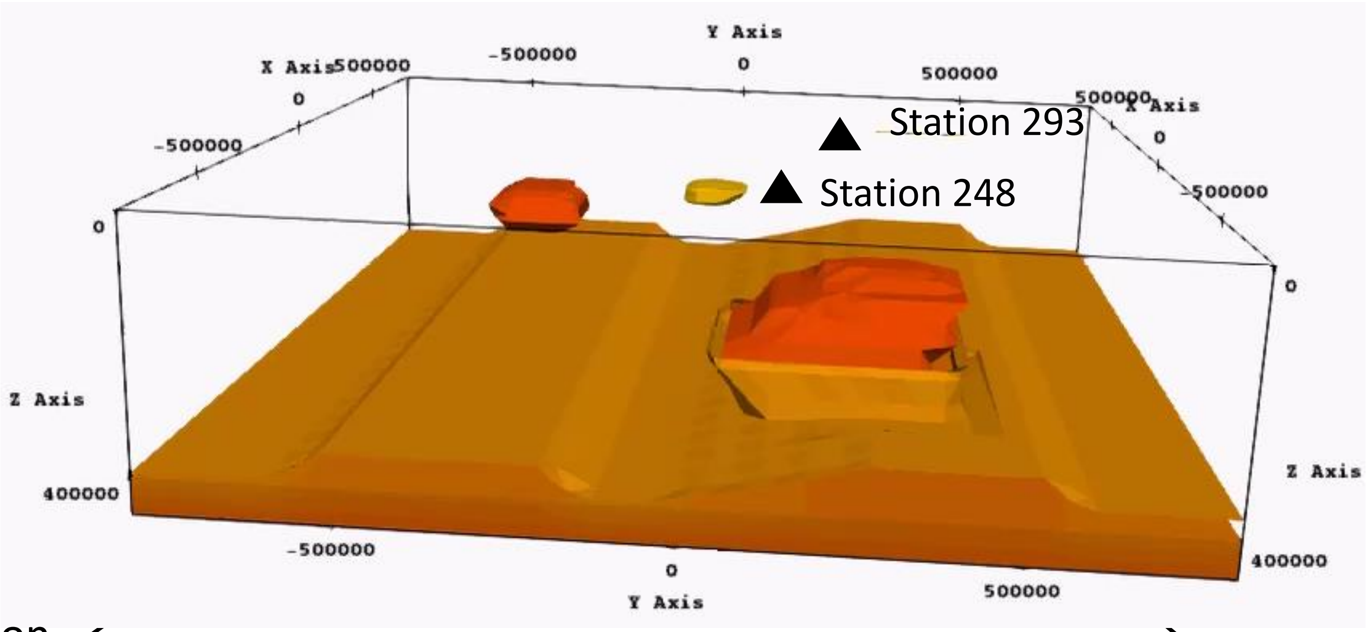


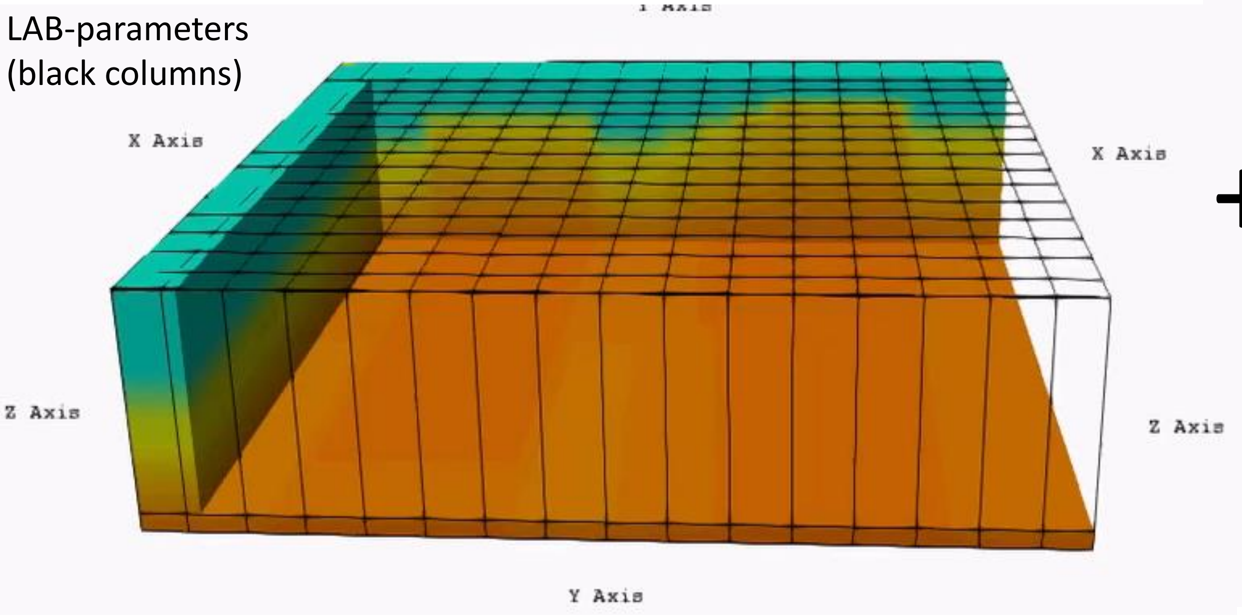
Log- conductivity

-7 -6 -5 -4 -3 -2 -1 0

Large scale thermochemical background: temperature, pressure and bulk composition

Small scale conductivity anomalies: as fluid pathways, melt-rich regions, hydrogen-rich domains, anomalous mineral assemblages

LAB-parameters (black columns)



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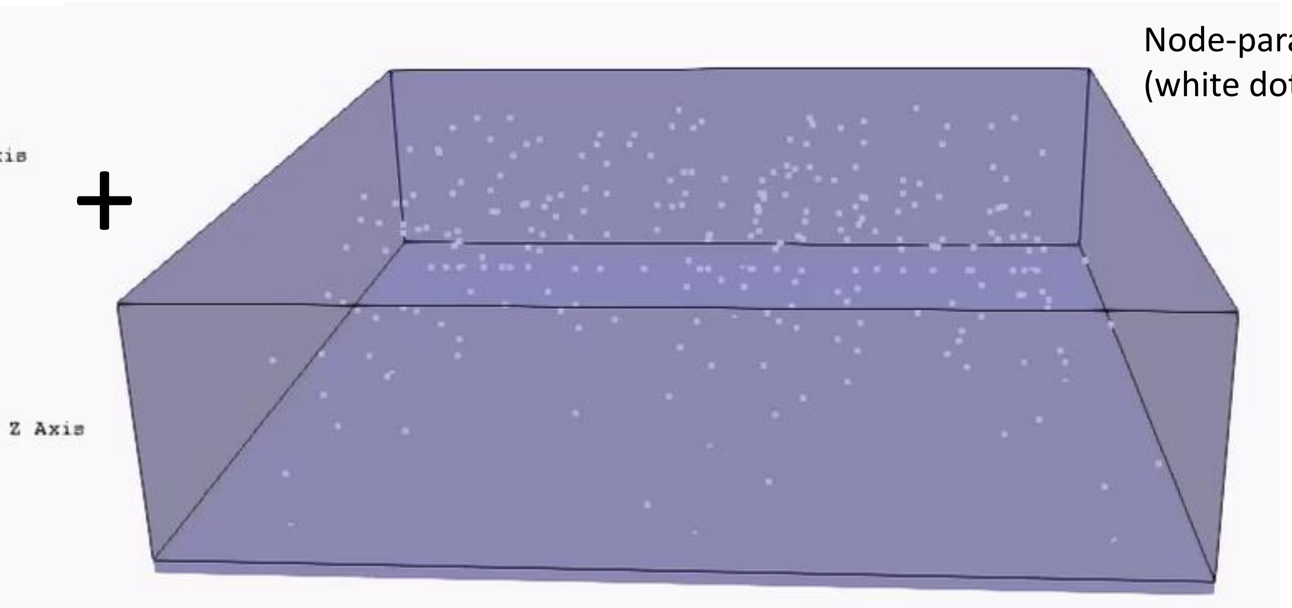
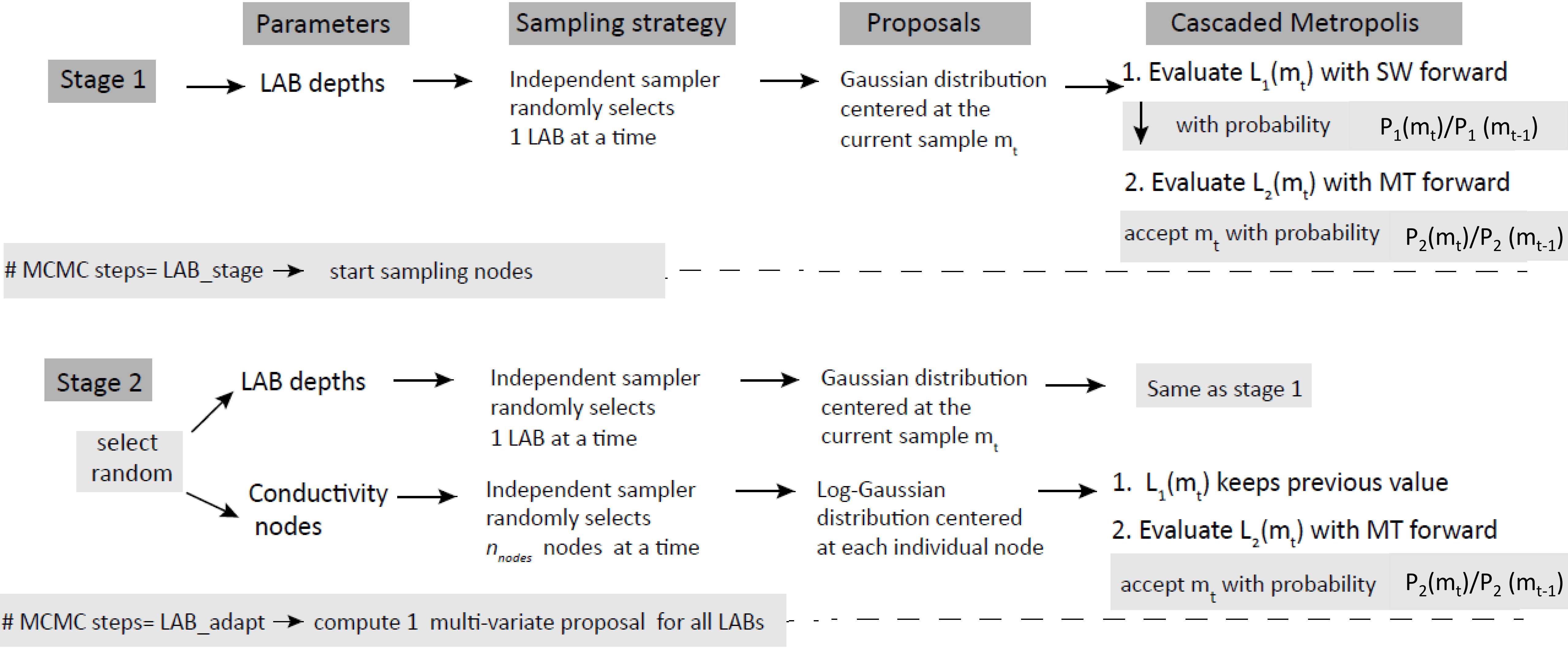
Node-parameters (white dots)



Sampling strategy

	Parameters	Sampling strategy	Proposals	Cascaded Metropolis
Stage 1 →	LAB depths →	Independent sampler randomly selects 1 LAB at a time →	Gaussian distribution centered at the current sample m_t →	1. Evaluate $L_1(m_t)$ with SW forward ↓ with probability $P_1(m_t)/P_1(m_{t-1})$ 2. Evaluate $L_2(m_t)$ with MT forward accept m_t with probability $P_2(m_t)/P_2(m_{t-1})$

Sampling strategy



Sampling strategy

Parameters	Sampling strategy	Proposals	Cascaded Metropolis

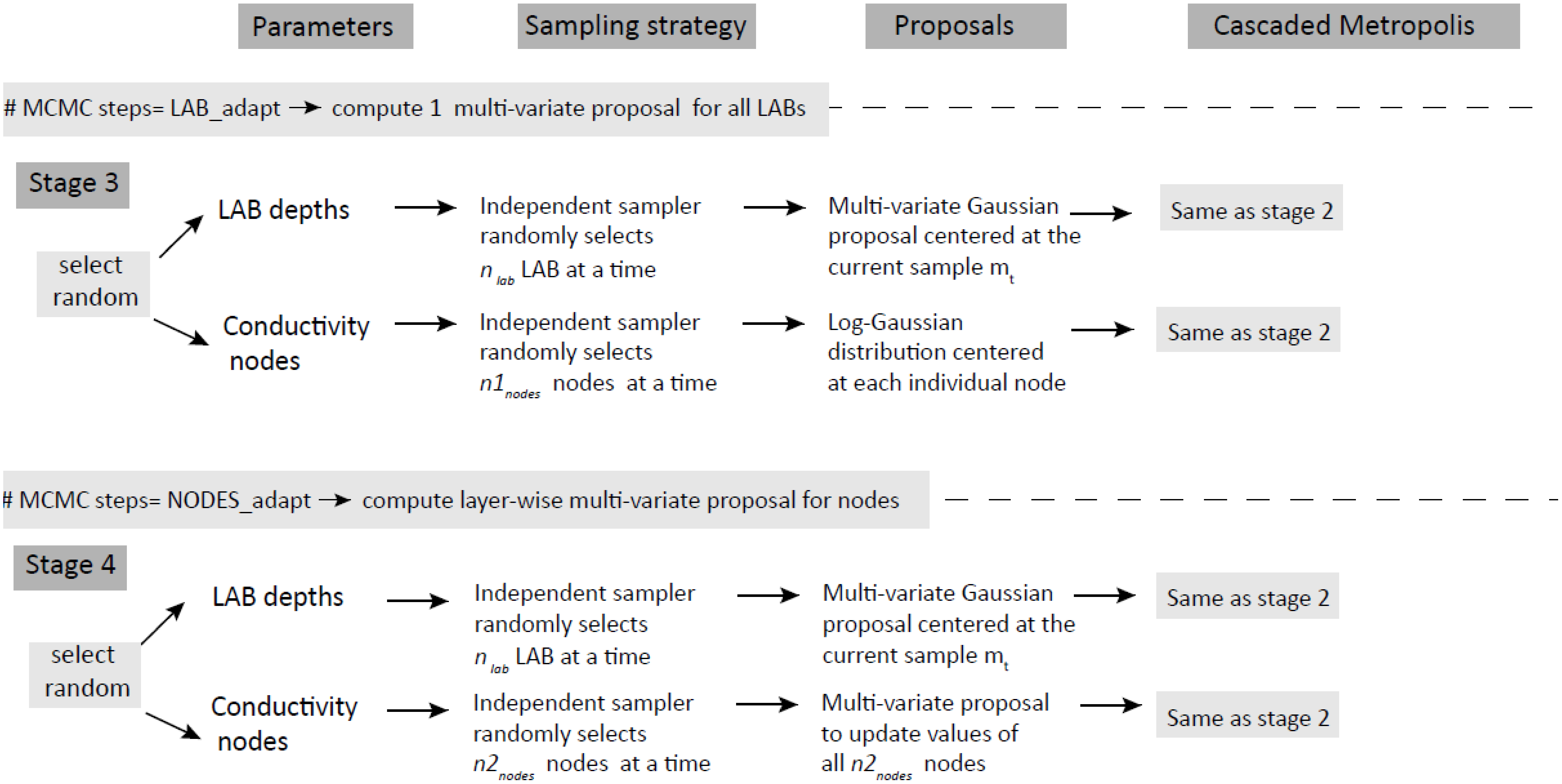
MCMC steps= LAB_adapt → compute 1 multi-variate proposal for all LABs

Stage 3

select random

Parameters	Sampling strategy	Proposals	Cascaded Metropolis
LAB depths	Independent sampler randomly selects n_{lab} LAB at a time	Multi-variate Gaussian proposal centered at the current sample m_t	Same as stage 2
Conductivity nodes	Independent sampler randomly selects $n1_{nodes}$ nodes at a time	Log-Gaussian distribution centered at each individual node	Same as stage 2

Sampling strategy



Synthetic Example

Synthetic **Data**

- The MT synthetic data are full impedance tensor computed for 12 periods between 3.2 and 10000 seconds at 400 stations.

- The data errors are assumed to be uncorrelated and normally distributed.

- The standard deviation is assumed as 5% of max(|Zxx|,|Zxy|) for the components Zxx and Zxy of the impedance tensor, and 5% of max(|Zyy|,|Zyx|) for the components Zyy and Zyx.

- The SW data are the Rayleigh wave phase velocities for periods between 15 and 175 seconds, computed at the locations of the MT stations.

- We assume normally distributed data errors with a standard deviation of 1% of the velocity in meters.

Data Misfits

$$\phi_{SW} = -\frac{1}{2}\sum_{i=1}^{N_{sta}}\sum_{j=1}^{N_{per}}\left(\frac{g_{ij}-d_{ij}}{std_{ij}}\right)^2$$

$$\phi_{MT} = -\frac{1}{2\cdot N_{dat}}\sum_{i=1}^{N_{sta}}\sum_{j=1}^{N_{per}}\left(\frac{g_{ij}-d_{ij}}{std_{ij}}\right)^2$$

N_{sta} and N_{per} are the number of stations and periods for each dataset;

d_{ij} and g_{ij} correspond to the observed and computed data (with the MT or the SW forward) for station i and period j,

std_{ij} is the standard deviation for data d_{ij}.

N_{dat} is the total number of MT data used for each station and frequency

Model setup

- The inversion area is sub-divided into 324 columns of size 80×80×460 km
- 1155 conductivity nodes sparsely located within the inversion volume (1440×1440×410 km)
- The vector of model parameters contains **324 LAB values and 1155 nodal conductivity values (1479 parameters)**

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- The vector of model parameters contains **324 LAB values and 1155 nodal conductivity values**

Prior and proposal distributions

- The priors for the LAB depths are uniform distributions defined in a range of ±70km, centered on the true value of each column.
- The proposals used in the first stage are Gaussian distributions centered on the current sample with a standard deviation of 20 km.
- For the conductivity nodes, we use Gaussian prior distributions centered on the background conductivity value (in log-scale) with a standard deviation of 1.5 $\log_{10}$ (S/m).
- The initial proposal distributions are log-normal centered on the current node value and standard deviation of 0.9 $\log_{10}$ (S/m).

Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

Large scale example

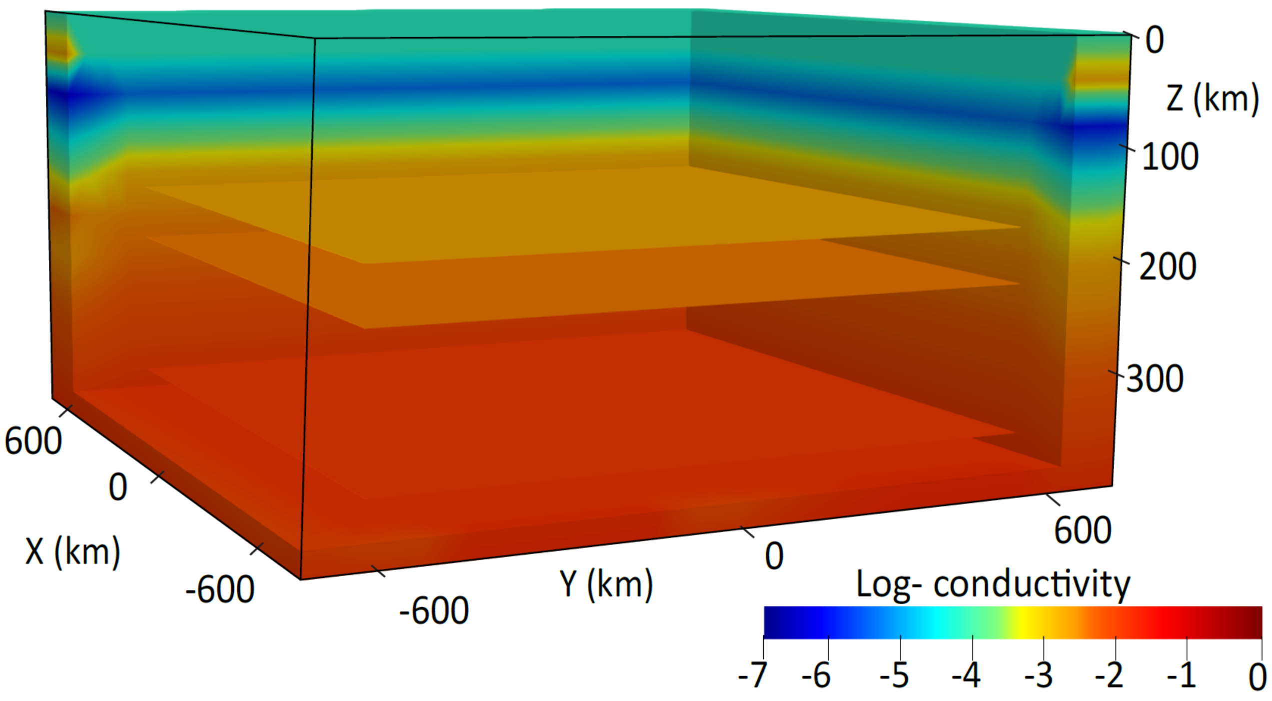
- Model size=

  1200x1200x460 km
- 12 frequencies
- 400 stations

Parameters:

- 324 LAB + 1155 conductivity nodes

Initial model. All LAB at 180km depth



RUN **1,000,000 MCMC** steps

- tol1= 0.068
- tol2 =0.058
- 2 processors (Intel(R) Xeon(R) CPU E5-2680 v3 @ 2.50GHz) per frequency
- Inversion took 14 days, with an average of 1.2 sec per simulation

Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

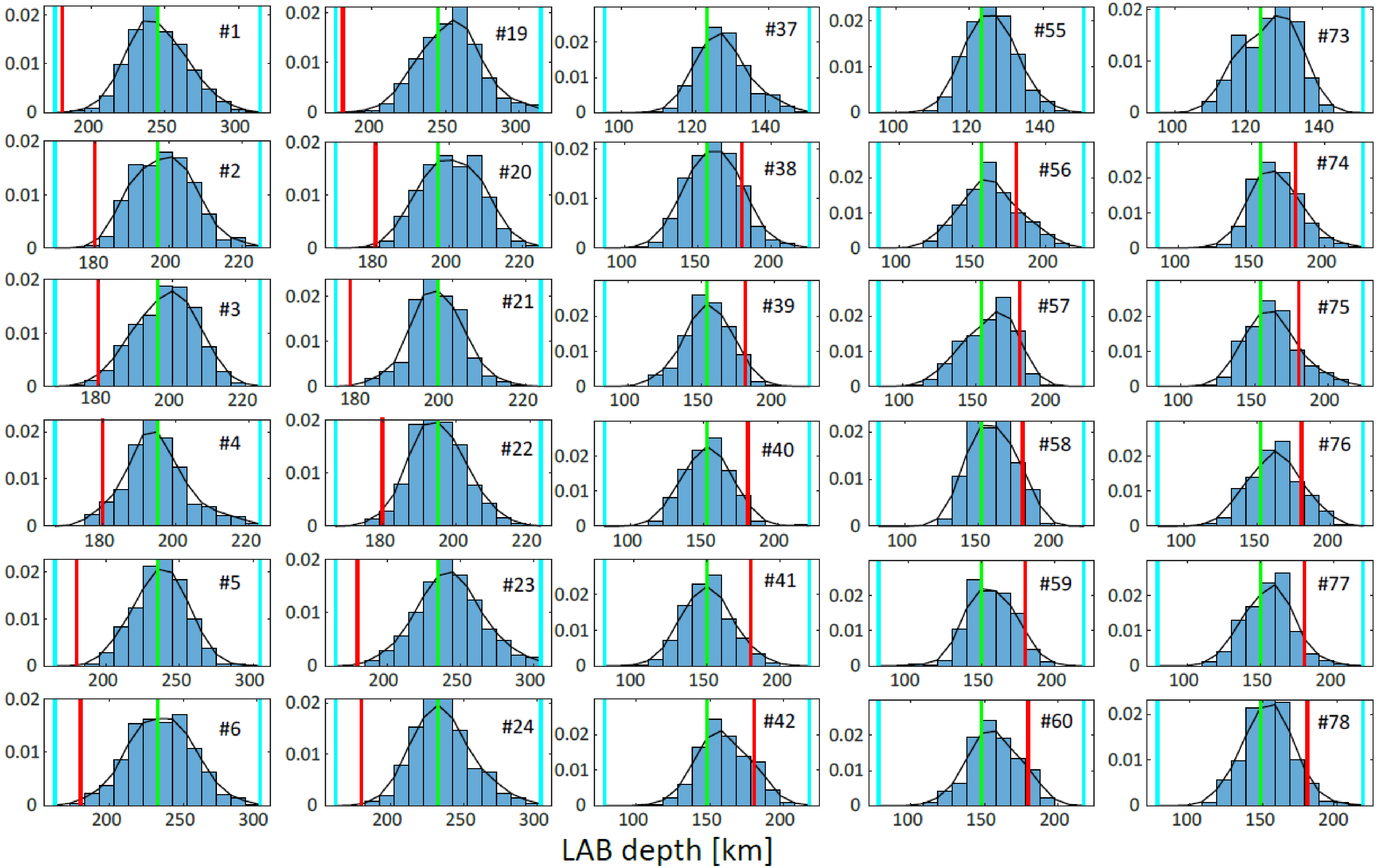
Large scale example

- Model size= 1200x1200x460 km
- 12 frequencies
- 400 stations

Parameters:

- 324 LAB
- 1155 nodes

LAB depth PDFs
(after 1,000,000 simulations)



Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

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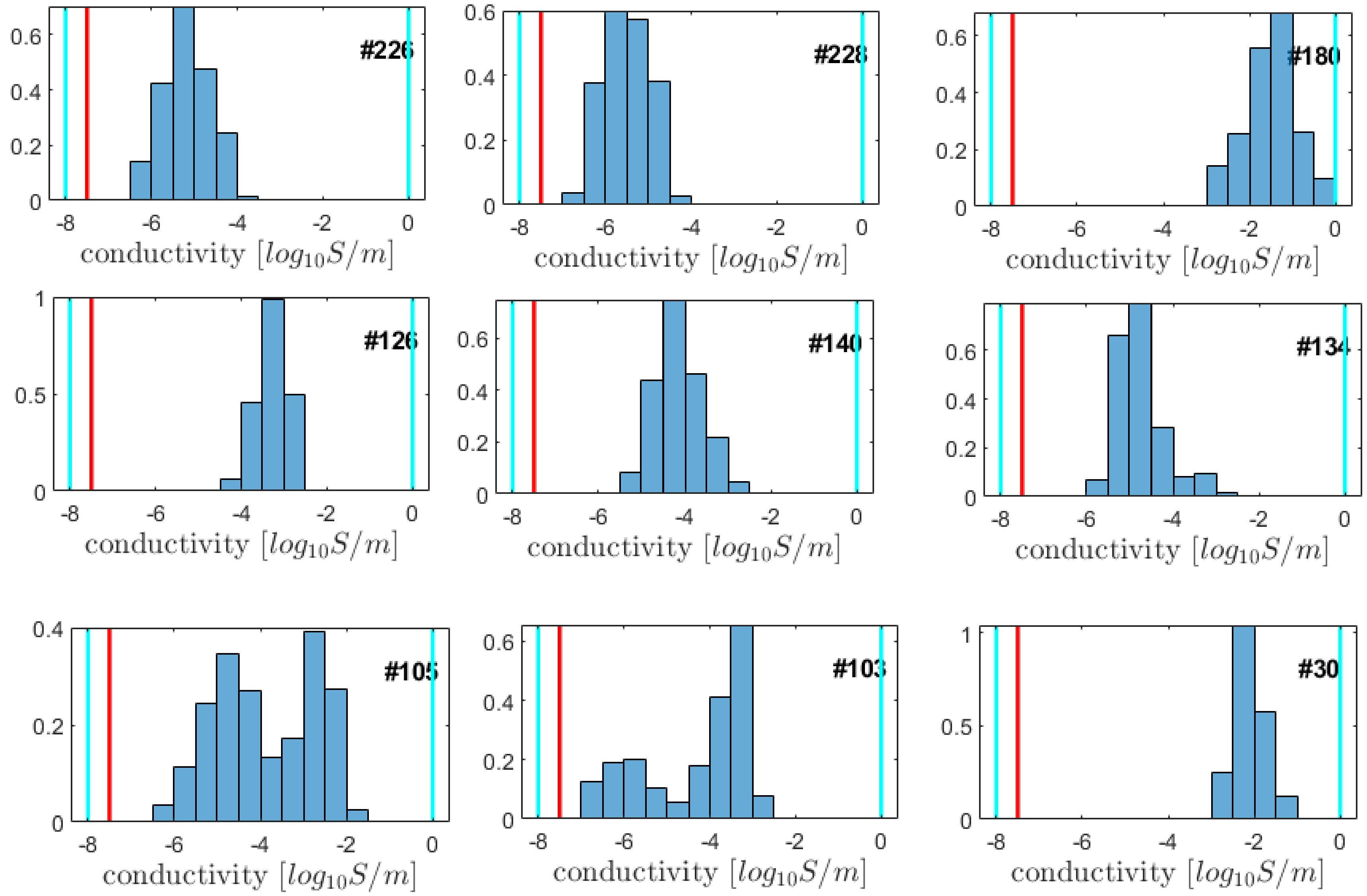
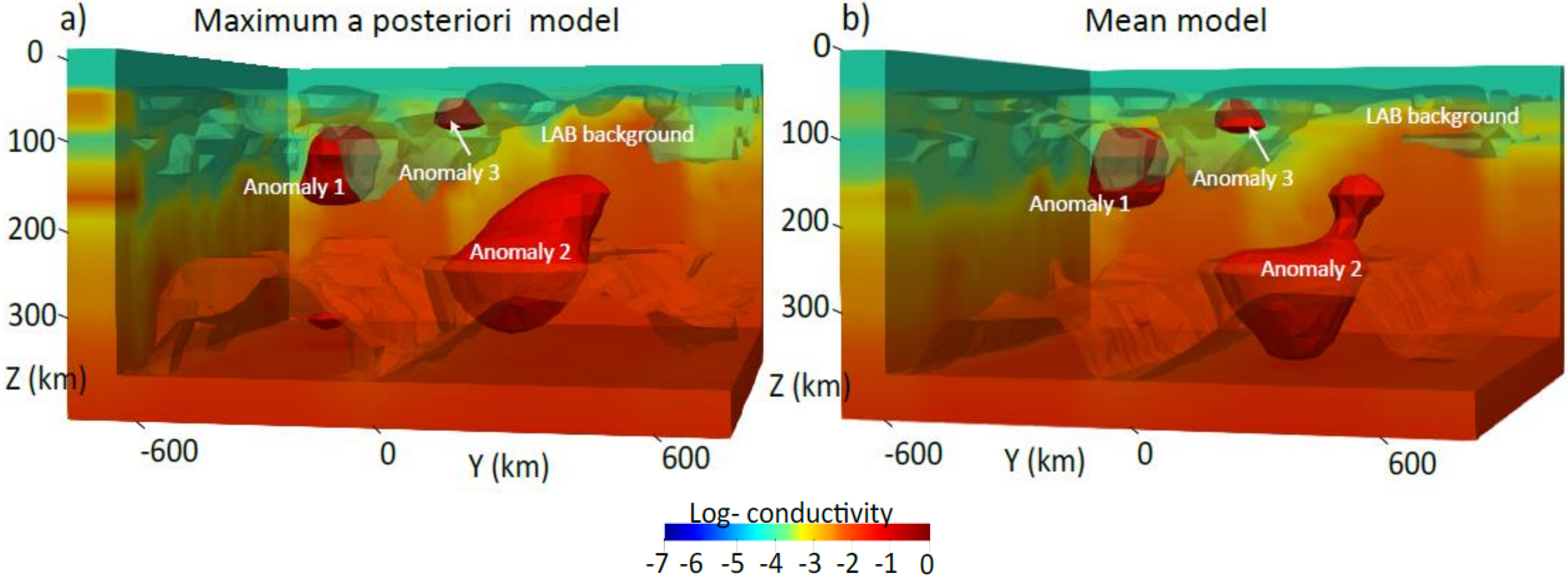


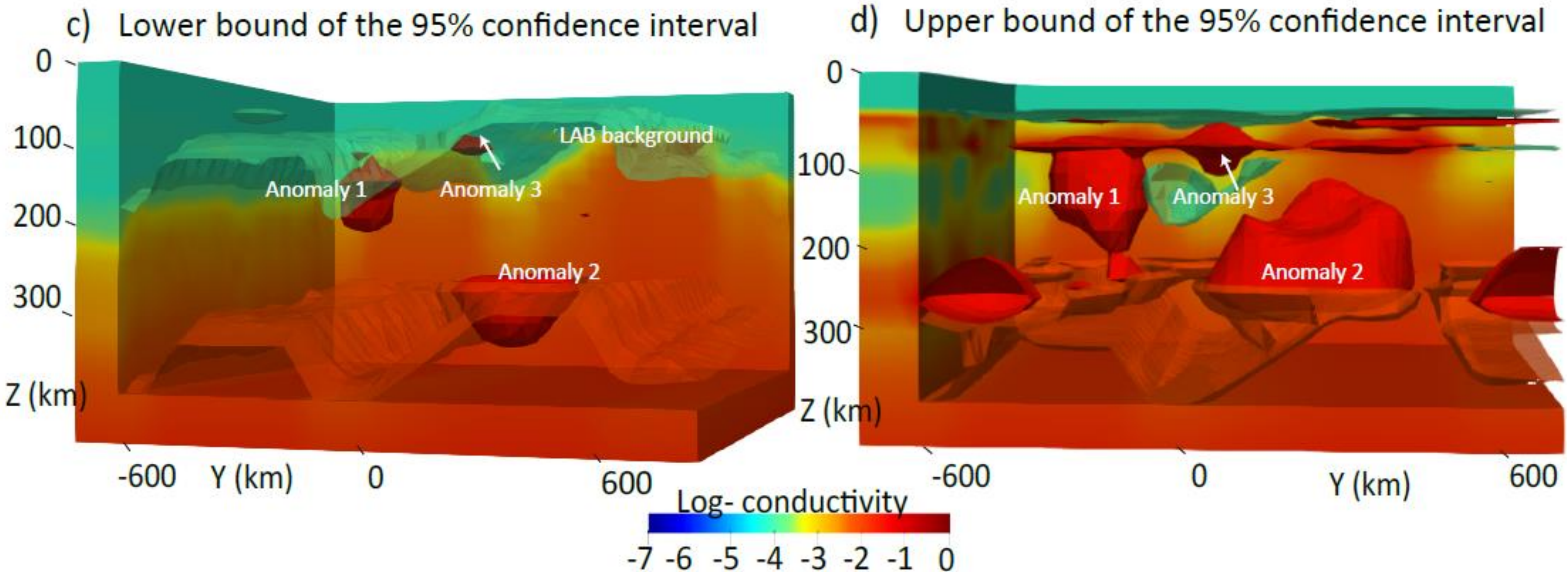
Conductivity nodes PDFs

(after 1,000,000 simulations)

Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

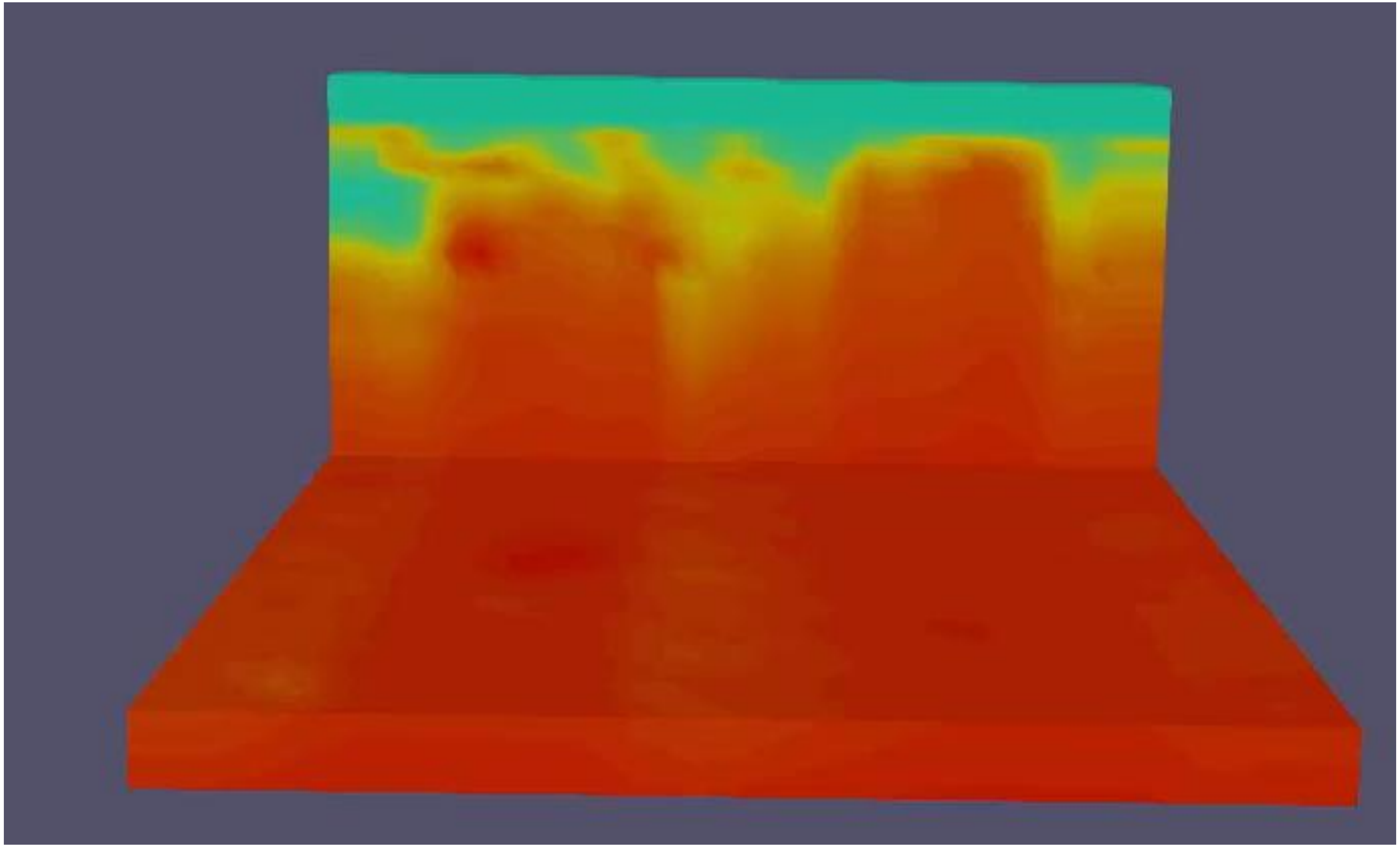


Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**



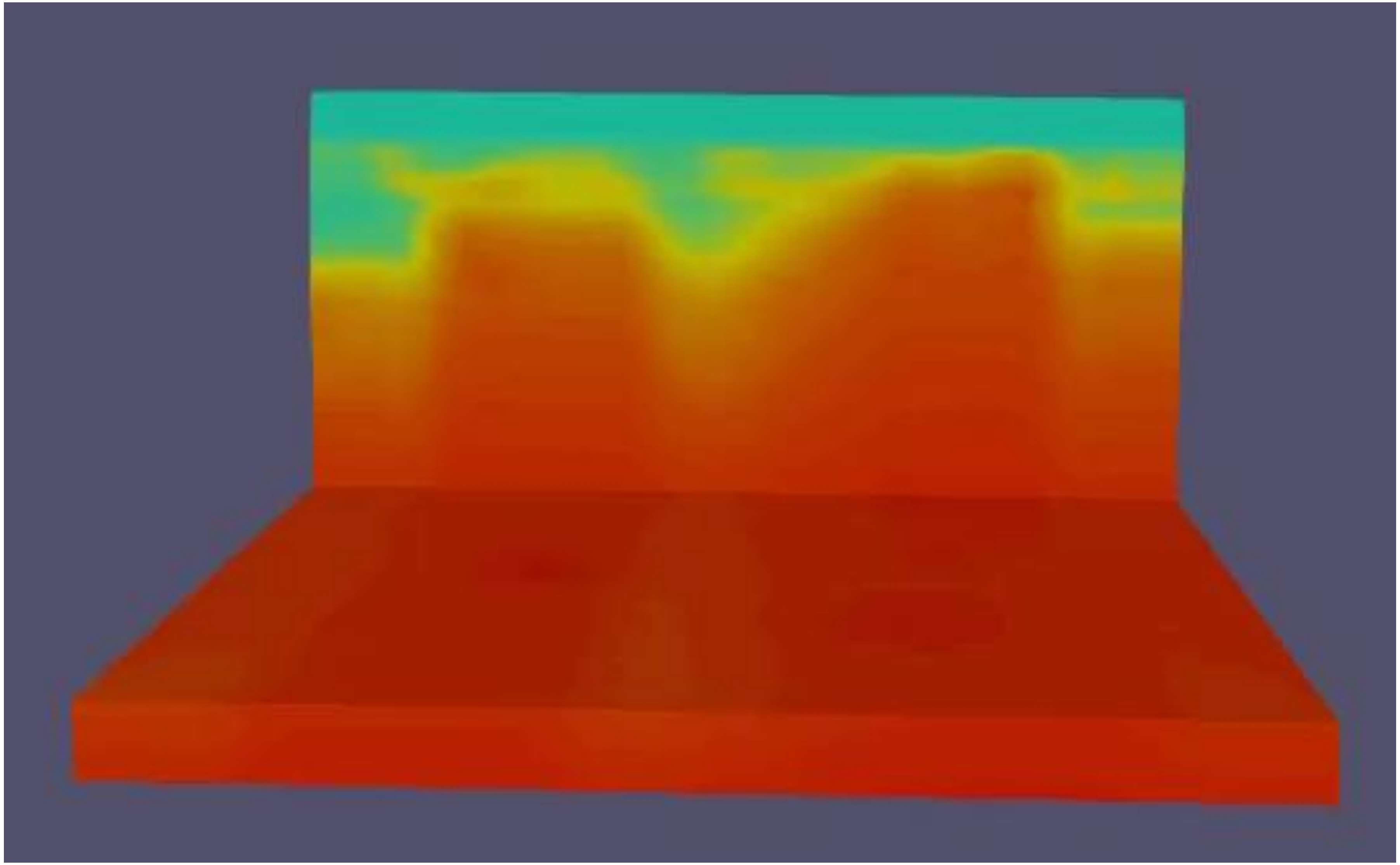
Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

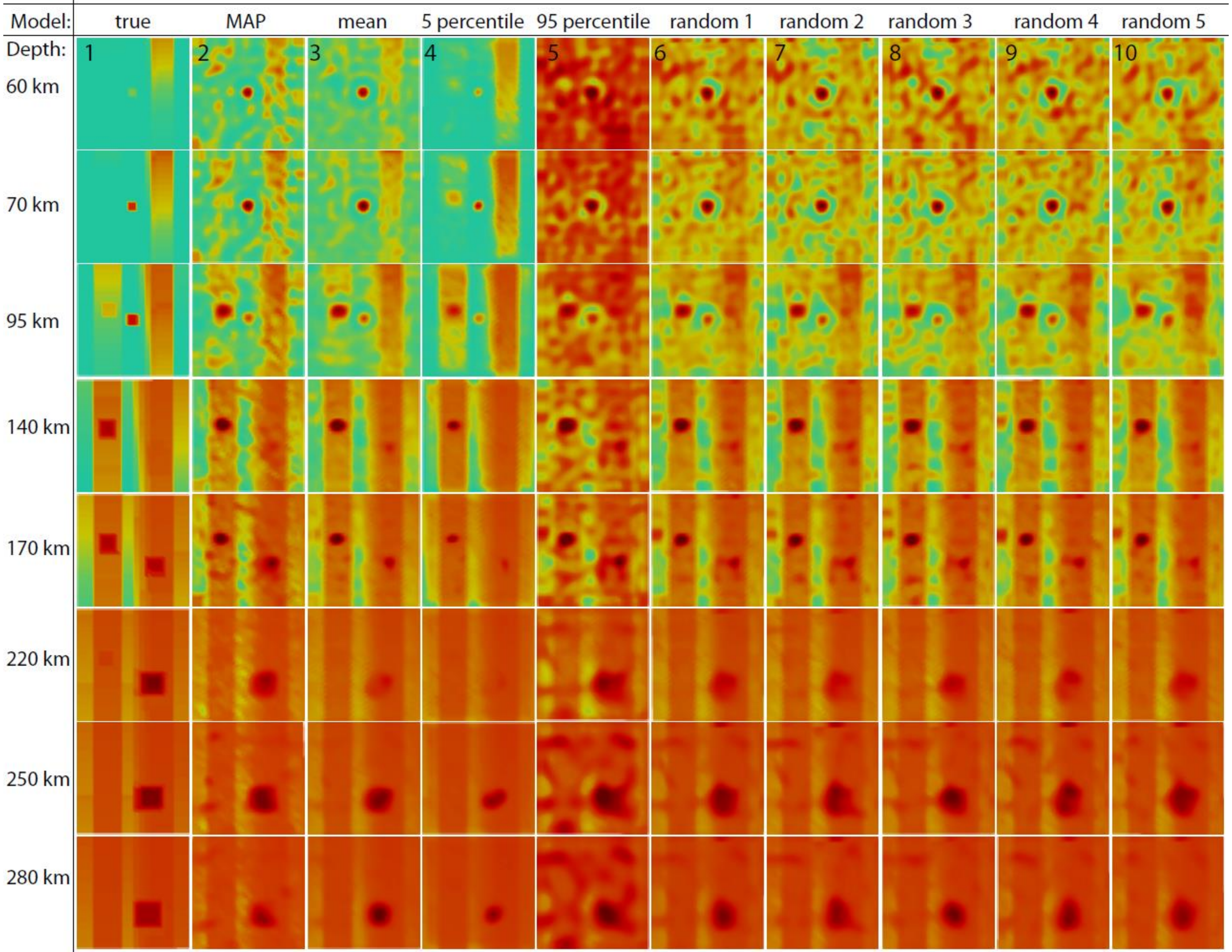
Best model after 1,000,000 simulations



Joint **probabilistic** inversion of **3D MT** and **SW synthetic data**

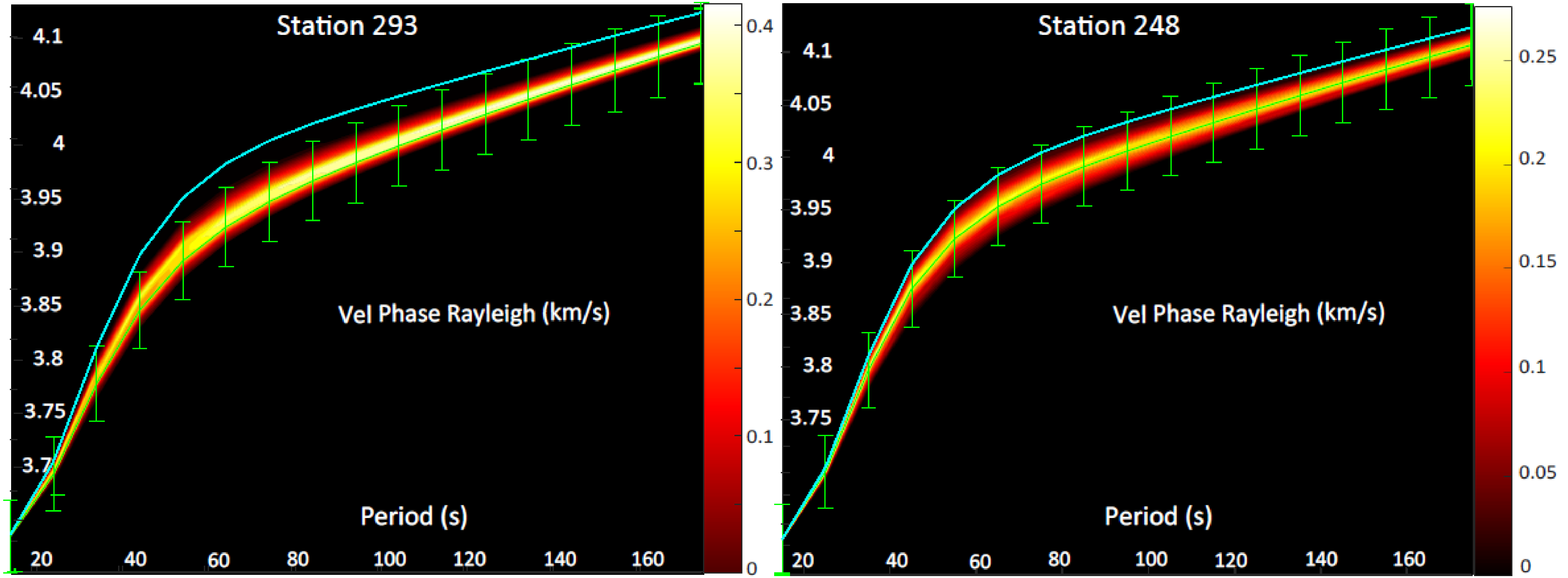
Mean model after 1,000,000 simulations



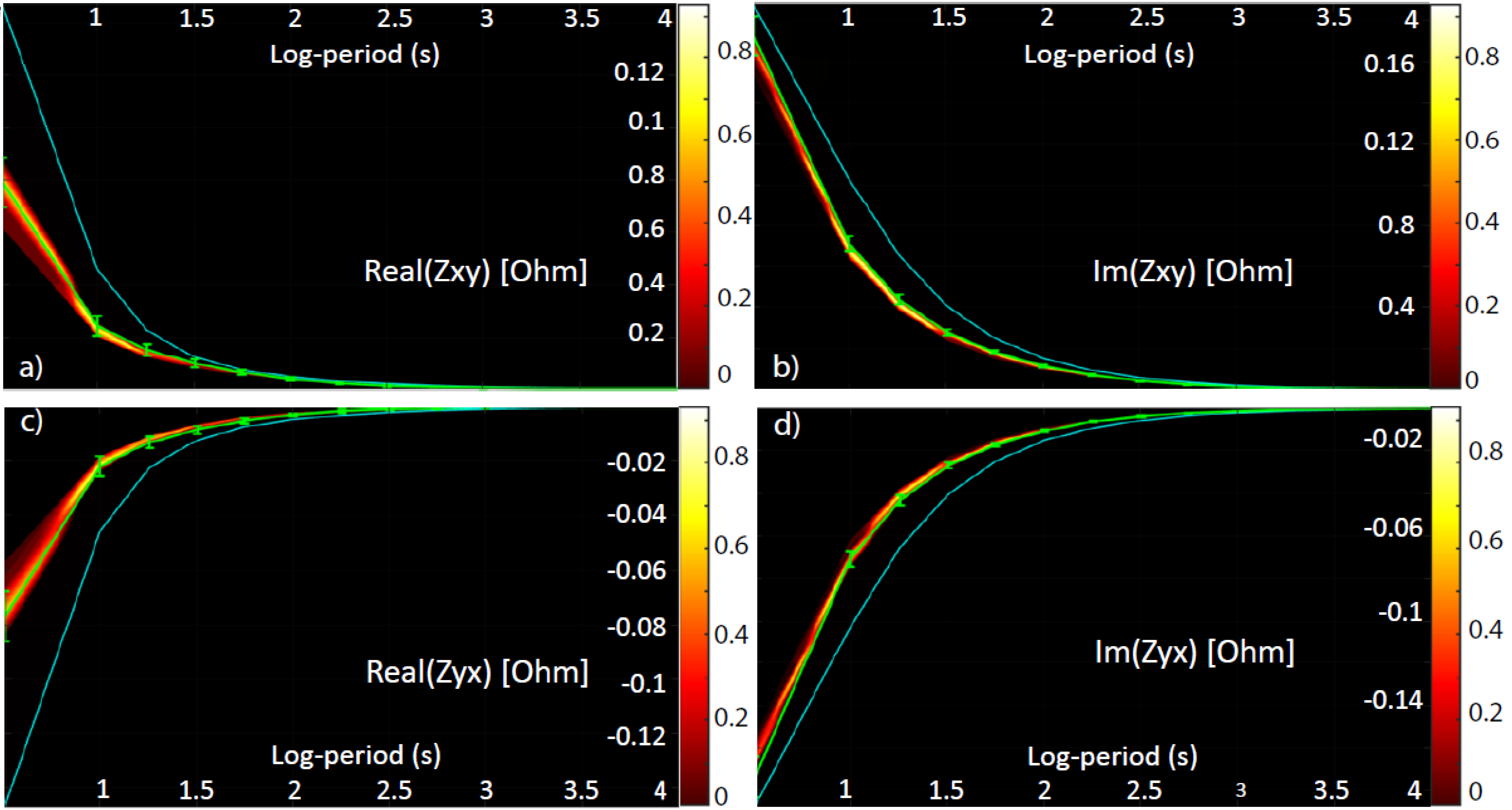


Model:
true
MAP
mean
5 percentile
95 percentile
random 1
random 2
random 3
random 4
random 5
Depth:
60 km
70 km
95 km
140 km
170 km
220 km
250 km
280 km
1
2
3
4
5
6
7
8
9
10

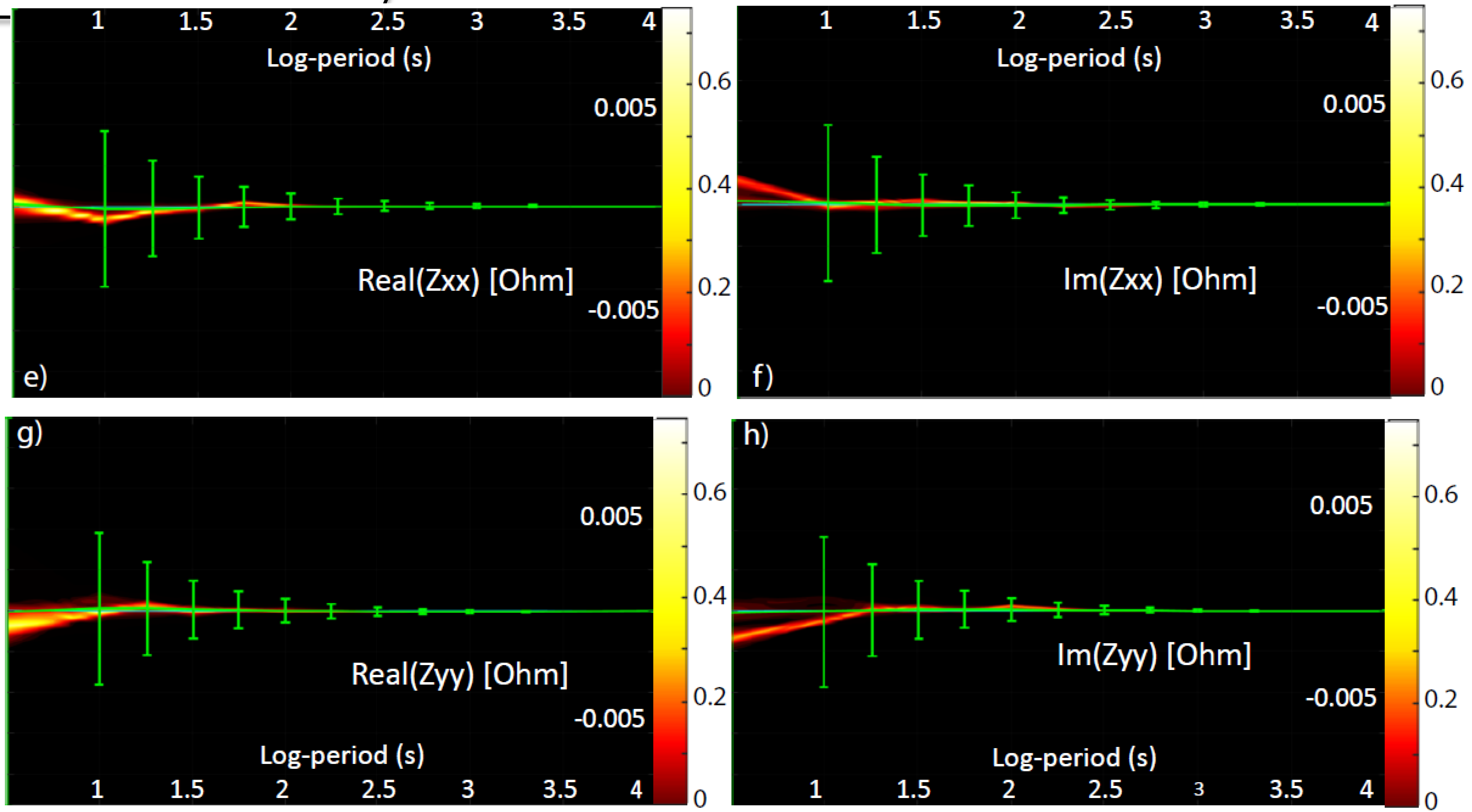
Surface-waves data pdfs



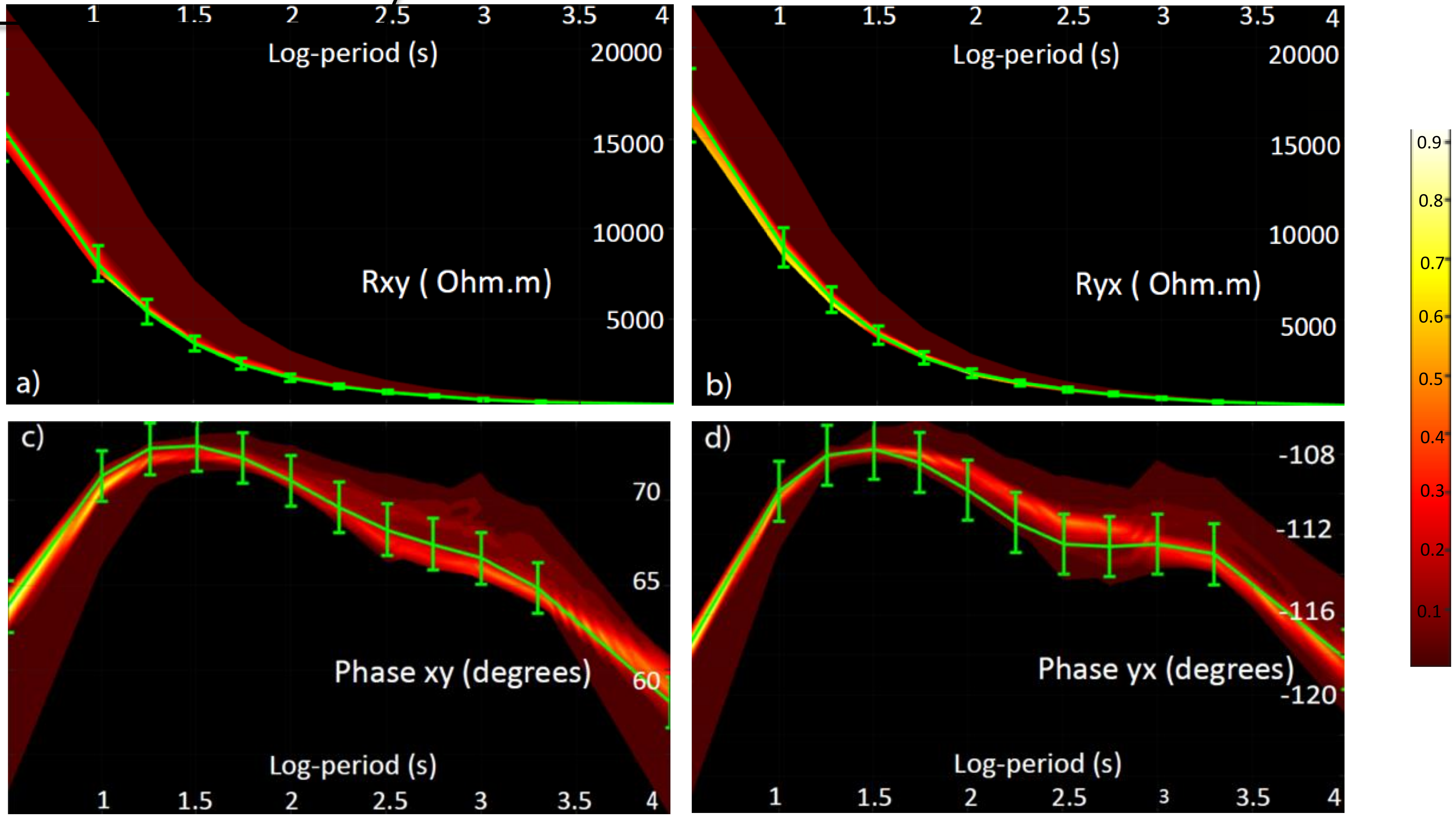
MT data pdfs: station 293

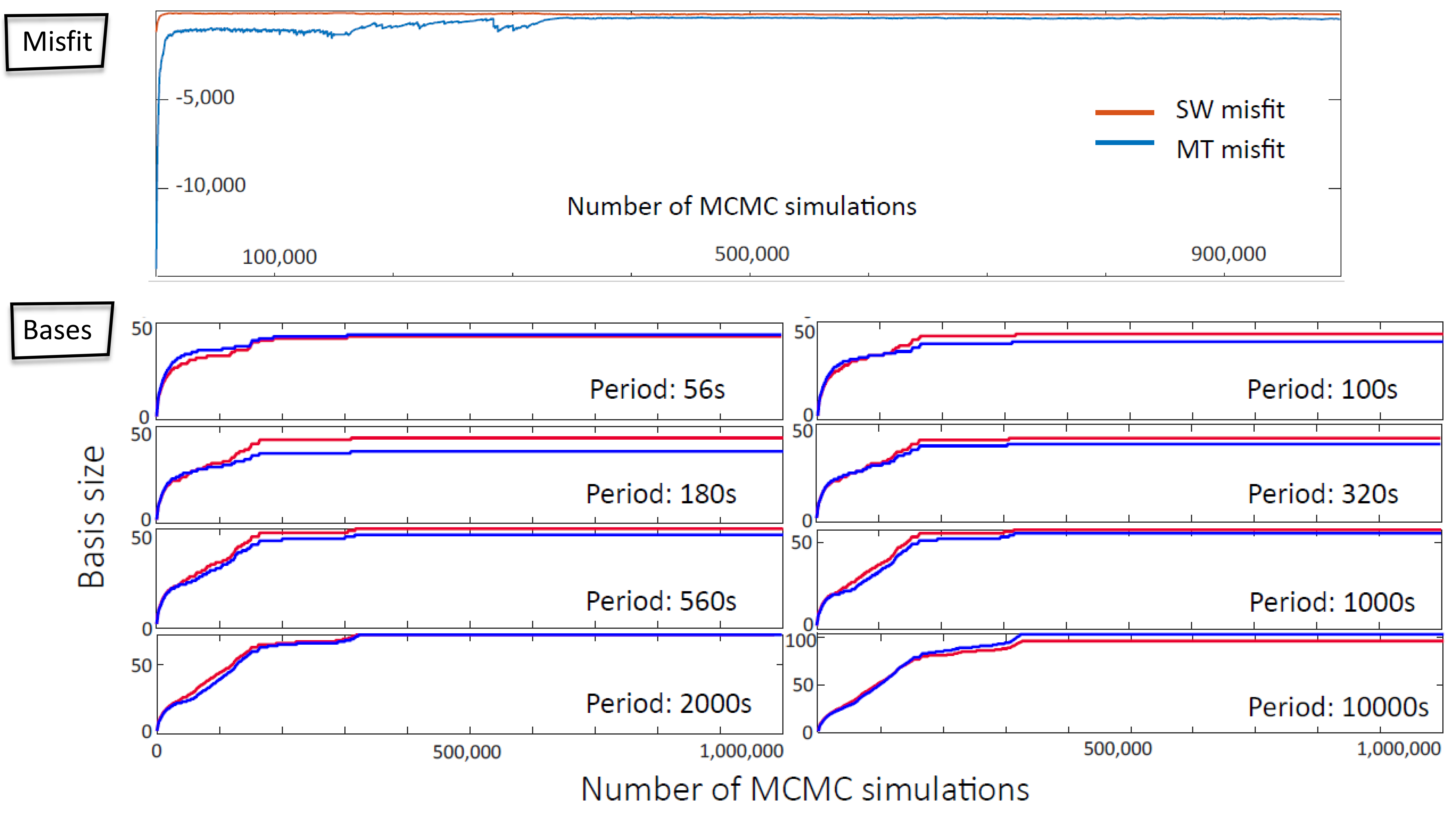


MT data pdfs: station 293



MT data pdfs: station 244





Misfit
-5,000
-10,000
Number of MCMC simulations
100,000
500,000
900,000
SW misfit
MT misfit
Bases
Basis size
Period: 56s
Period: 100s
Period: 180s
Period: 320s
Period: 560s
Period: 1000s
Period: 2000s
Period: 10000s
0
50
100
500,000
1,000,000
Number of MCMC simulations

Recent publications

1. Manassero, M.C., Afonso, J.C., Zyserman, F., Zlotnik, S. and Fomin, I., 2020.
A reduced order approach for probabilistic inversions of 3-D magnetotelluric data I: general formulation. *Geophysical Journal International*, *223*(3), pp.1837-1863

2. Manassero, M.C., Afonso, J.C., Zyserman, F.I., Zlotnik, S. and Fomin, I., 2021.
A Reduced Order Approach for Probabilistic Inversions of 3D Magnetotelluric Data II: Joint inversion of MT and Surface-Wave Data. *Journal of Geophysical Research: Solid Earth* (**in review)**

3. Manassero, M.C., Kirkby, A., Afonso, J.C, Czarnota, K., 2021. A Reduced Order Approach for Probabilistic Inversions of 3D Magnetotelluric Data III: Joint inversion of MT and Surface-Wave Data in the Tasmanides, southeast Australia (**in preparation)**

Conclusions

Parallel-in-parallel structure

+

Reduced Basis methods

+

Adaptive strategies (MCMC and the surrogate model

→

Fast 3D MT forwards: less than **1 sec**!

Conclusions

Parallel-in-parallel structure

\+ **Reduced Basis** methods

\+ **Adaptive strategies** (MCMC and the surrogate model

→ **Fast 3D MT forwards**: less than **1 sec**!

\+ Efficient parameterisation of **background + conductivity anomalies**

↓

Develop the **1st** numerical platform **(RB+MCMC)** to **jointly** invert **3D MT** data and **seismic data** in a **probabilistic** way

Conclusions

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→

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+

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Develop the **1st** numerical platform **(RB+MCMC)** to **jointly** invert **3D MT** data and **seismic data** in a **probabilistic** way

Future work

- Inversion using **field MT and SW data**
- Efficient sampling MCMC strategies (**trans-dimensional** scheme, parallel tempering, etc..)

Conclusions

Parallel-in-parallel structure + **Reduced Basis** methods + **Adaptive strategies** (MCMC and the surrogate model → **Fast 3D MT forwards**: less than **1 sec**!

+ Efficient parameterisation of **background + conductivity anomalies**

↓

Develop the **1st** numerical platform **(RB+MCMC)** to **jointly** invert **3D MT** data and **seismic data** in a **probabilistic** way

Thank you!

Future work

- Inversion using **field MT and SW data**
- Efficient sampling MCMC strategies (**trans-dimensional** scheme, parallel tempering, etc..)

Any questions??

e-mail:
maria-constanza.manassero@mq.edu.au